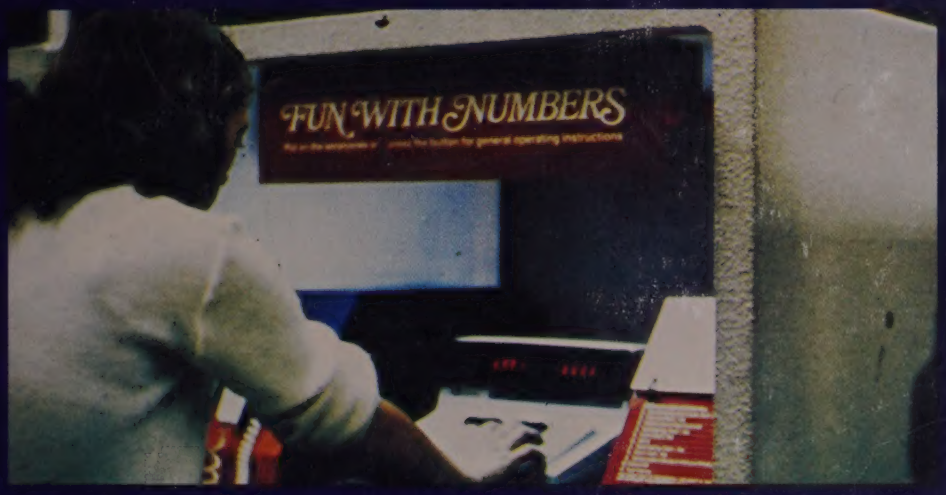
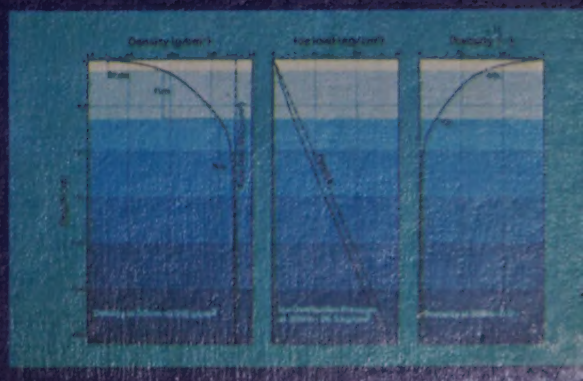
MATH PROBE

I

KRYSAK BYE ELLIOTT



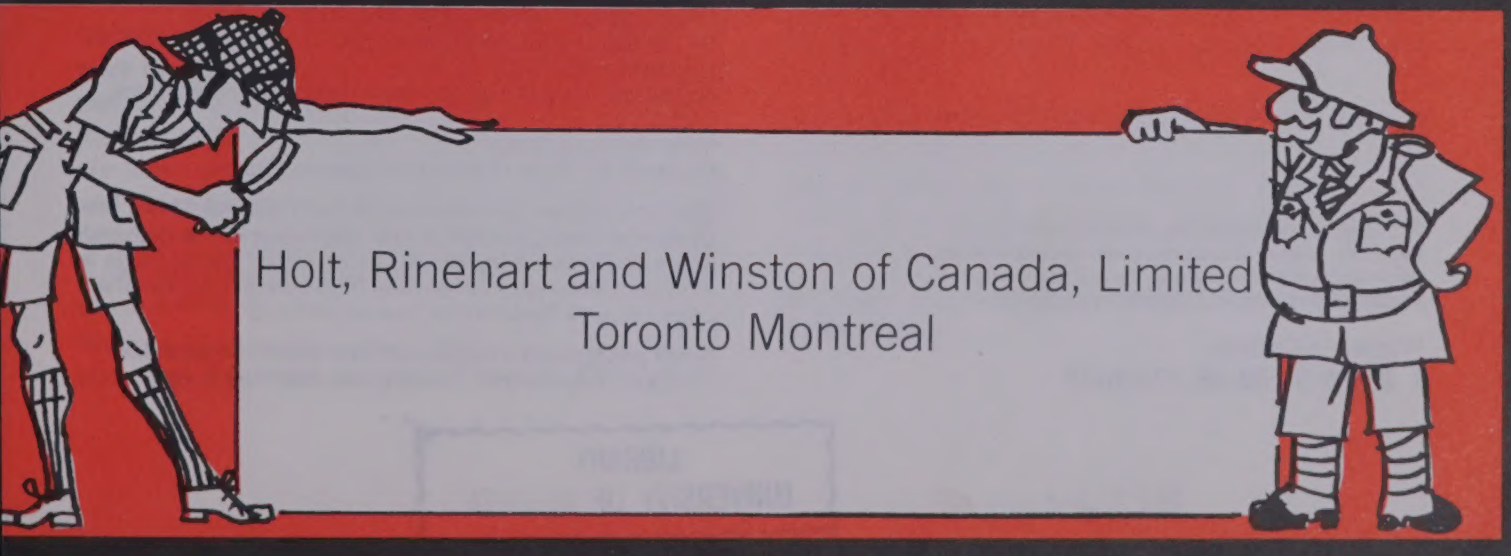


Math Probe

1

KRYSAK BYE ELLIOTT

HOLMES, WHAT IS THIS NEW CASE YOU'VE TAKEN ON? IT SEEMS TO HAVE YOU PUZZLED!
A VERY COMPLICATED ONE, WATSON. SOME NUMBERS JUST AREN'T HERE. WE MUST FIND THEM AND IDENTIFY THEM!
1 2 3 4 5 ...



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Math Probe 1

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ISBN 0-03-921234-3

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Printed in Canada
1 2 3 4 5 79 78 77 76 75

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A Note From the Authors

To the Students:

This book is written to you, the student. We have tried to put mathematics in situations which you will find interesting. A number of questions are asked just as if we were talking to you—and often no answers are given immediately. You should think about these questions, try to answer them, discuss them, and then read on.

The Projects are designed to be done by groups of two or more students working co-operatively, each doing his part and sharing the results. The Projects are to relate mathematics to relevant situations and to put mathematics to work. It is important that you discuss mathematics with each other and with your teacher. You will find you learn a great deal during these discussions.

Certain exercises are designed primarily for you to work by yourself, though you may wish to check your conclusions with your classmates. While at times mathematics is something you must do alone, there are times when it is more efficient, more interesting, and more fun to work with others. But there are occasions when you must practise alone. Mathematics is like most sports—you only become good at it when you practise.

No book is complete within itself. This is true of this book. You may want to research further some of the topics introduced in the special sections. These sections are a vital part of this program and should not be omitted. Each question relates to what you have studied or will study.

While specific books are mentioned in "Using the Library" sections, many of the answers and reports can be compiled from other books. *Suggestion:* Look ahead—start these assignments early. Do not wait until the end of the chapter to start using the library.

We have used the Sherlock Holmes theme to emphasize the point that mathematics is similar to a detective or mystery story. Sherlock Holmes was a fictitious detective, created by Sir Arthur Conan Doyle. Many of the quotations used in this book are taken from *The Hound of the Baskervilles*, which is available in the Looking Glass Library, published by Random House. In a detective or mystery story, clues are provided in many disguises. You must probe the clues searching for patterns and facts, then you must put these facts and patterns together with what you already know and reach a logical conclusion or solution. If you cannot find a solution, then either review or more facts are necessary.

An optional appendix on measurement (Chapter 10) has been included for those students who may need experience in metric measurement. There is also some more advanced, optional material on the force of gravity and pressure.

We hope you will enjoy probing the mysteries of mathematics.

M.P.B. W.P.K. H.A.E.

2515238

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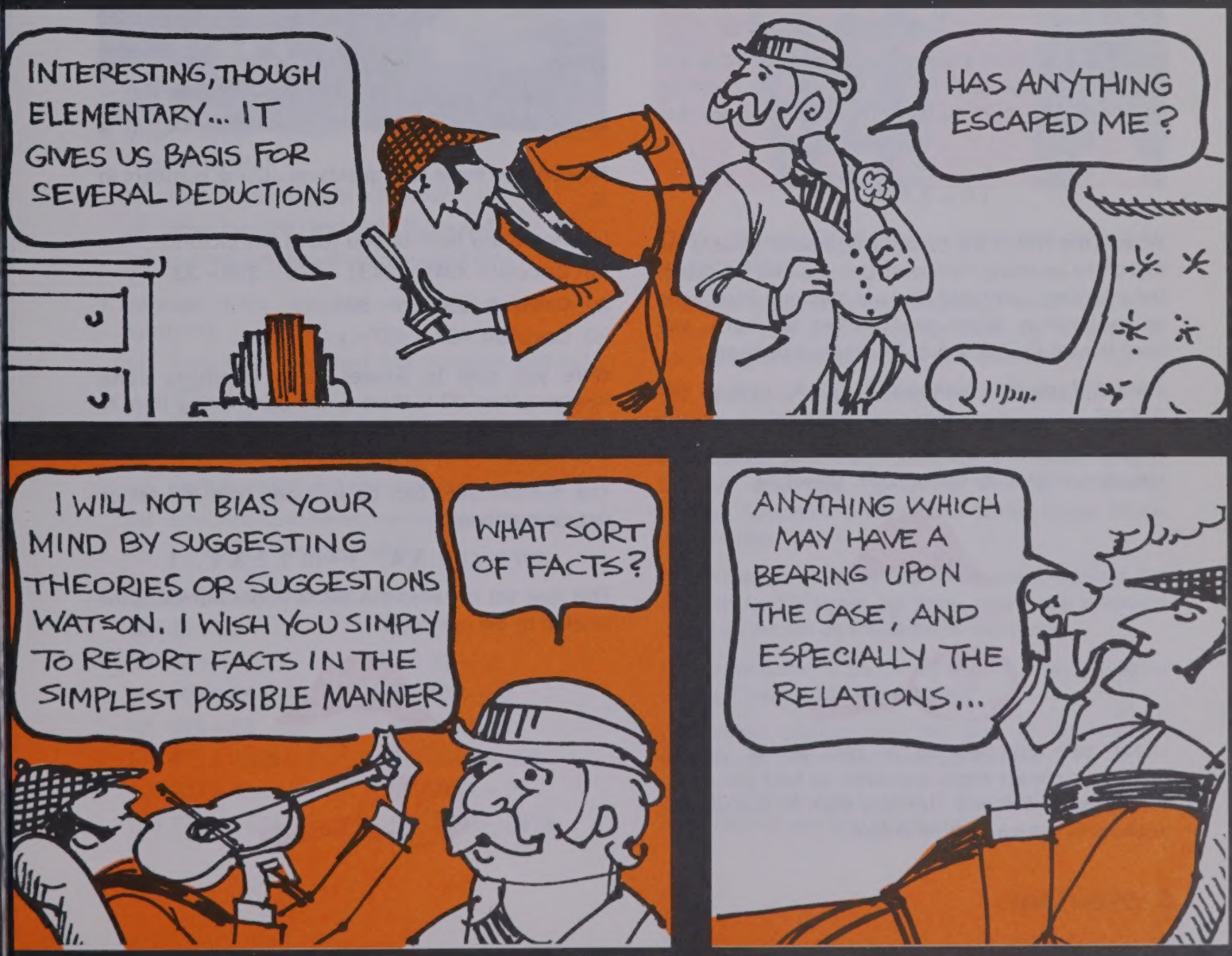

INTERESTING, THOUGH ELEMENTARY... IT GIVES US BASIS FOR SEVERAL DEDUCTIONS
HAS ANYTHING ESCAPED ME?
I WILL NOT BIAS YOUR MIND BY SUGGESTING THEORIES OR SUGGESTIONS WATSON. I WISH YOU SIMPLY TO REPORT FACTS IN THE SIMPLEST POSSIBLE MANNER
WHAT SORT OF FACTS?
ANYTHING WHICH MAY HAVE A BEARING UPON THE CASE, AND ESPECIALLY THE RELATIONS...

1·1 The Story Begins

How many birds do you see in the picture? What did you do to get the answer? Did you use the counting numbers? The set of counting numbers is shown this way:

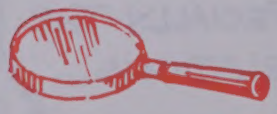
$$\{1, 2, 3, 4, 5, \ldots\}.$$

What is the first of the counting numbers? What is the last of the counting numbers? Can you find the last of the counting numbers? Discuss this and then write an explanation. When you counted the birds, you used the set $\{1, 2, 3, 4, 5, 6, 7\}$. This is a *finite set*.

The set of counting numbers $\{1, 2, 3, 4, \ldots\}$ is an *infinite set*.

Another name for the set of counting numbers is *natural numbers*. Its symbol is N. Therefore,

$N = \{1, 2, 3, 4, \ldots\}$.



Using *only* numbers in N (the set of natural numbers), answer these questions as best you can—remember to use only numbers from N. Discuss the answers in groups of three or four.

Find the number of:

(a) blonde girls (boys) in your class,
(b) students in your class,
(c) hours before home time,
(d) days before the end of the school term,
(e) classes you have skipped this week,
(f) times you went ice skating last July.

No doubt you had difficulty answering all the questions. Why?

Now answer these four questions—using numbers in N.

(a) How many birds do you see in this picture?
(b) Calculate: $3945 - 2431 - 879 - 398 - 237 = y$.
(c) Calculate: $394 + n = 394$.
(d) Calculate: $437 - 437 = x$.

Were you able to answer these questions using numbers from N? Is there a number missing from N that prevents you from answering these questions? What number is missing?

The number zero can be included with the set of natural numbers.

$$\{0\} \cup \{1, 2, 3, 4, \ldots\} = \{0, 1, 2, 3, 4, \ldots\}$$

This new set is called the set of whole numbers, indicated by the symbol W.

$W = \{0, 1, 2, 3, \ldots\}$

EXERCISE 1·1

1. Explain what you think is meant by:
   (a) infinite; (b) finite.
2. Use the dictionary to find the meaning of (a) finite; (b) infinite. Compare these definitions with your answers in Question 1.
3. Using n for the variable, write three number sentences which require the missing number zero in the answer. Discuss these with your classmates.
4. Give three reasons why you think zero is an important number. Discuss in class.
5. What whole number immediately follows:
   (a) two hundred and ninety-nine?
   (b) nine thousand and nine?
   (c) fifteen million four hundred and nineteen?
6. What whole number immediately precedes (comes before):
   (a) four thousand eight hundred?
   (b) three hundred and eight thousand?
   (c) one million sixty-three thousand nine hundred?
7. Arrange from smallest to largest (write in symbols):
   (a) thirty-eight thousand four hundred and sixty-two;
   (b) fifteen thousand and seven;
   (c) two million seven hundred;
   (d) eight hundred and one thousand;
   (e) nine thousand seven hundred and twelve;
   (f) seven thousand and ninety.
8. Do the following for practice.
   (a) $38 + 83$ (b) $65 - 27$
   (c) 42×16 (d) $72 \div 12$
   (e) $409 + 344$ (f) $513 - 85$
   (g) 618×53 (h) $225 \div 15$
   (i) $471 + 3639$ (j) $2105 - 369$
   (k) 207×428 (l) $1431 \div 27$
   (m) $7354 + 6297$ (n) $5248 - 2673$

This is the international logo used to promote anti-litter campaigns.

The new logo has a clear message which demands action. It can also easily be used by industry on packaging and by government on litterbarrels.

The symbol is being introduced throughout Western Canada during 1975 Anti-Litter Week.

The symbol is already being used in Great Britain, Ireland, Bermuda, Belgium, France, Germany, Norway, Switzerland, the United States, South Africa, and Finland.

If each person in your city threw one chocolate bar wrapper on the street per week, how many wrappers would be thrown on the street in one year?

If each wrapper is 15 cm by 20 cm, what would be the total area of all the wrappers?

How many playgrounds, the size of your school playground, could these wrappers cover?

If each wrapper has a mass of 5 g, what would be the total mass of one year's pollution?

Project

Do you have difficulties with multiplication and division? If your answer is "Yes", then this simple instrument will help you. It is called a *circular slide rule* or just a *slide rule*.

START

TRACE THE TWO CIRCULAR SCALES.

CUT OUT AND GLUE ON BRISTOL BOARD.

CUT BRISTOL BOARD TO SHAPE.

OBTAIN THIN PLASTIC AND CUT FOR CURSOR.

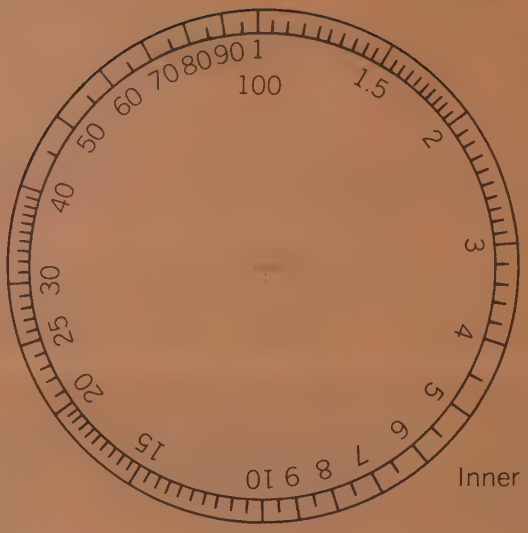
PLACE DOTTED LINE ALONG CURSOR.

MAKE SMALL HOLES AT EACH +.

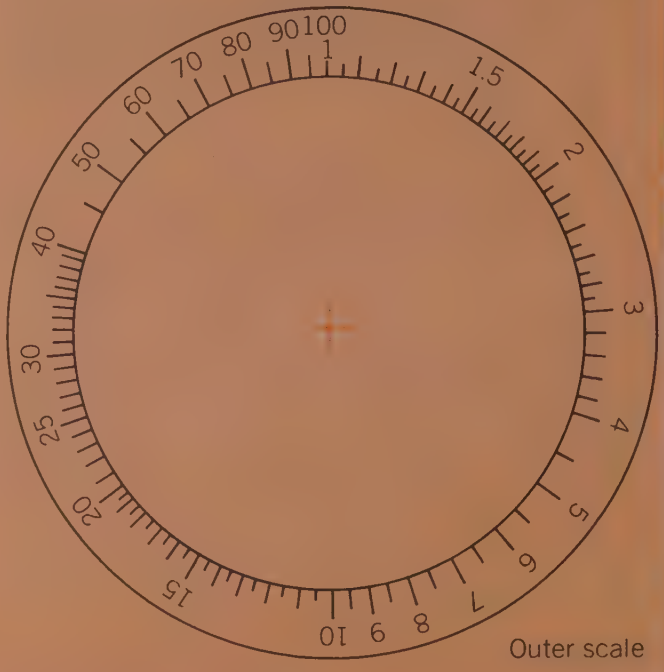
PLACE PAPER FASTENER THROUGH HOLE AND ASSEMBLE.

SLIDE RULE IS READY FOR USE.

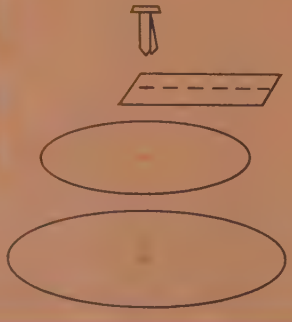
STOP

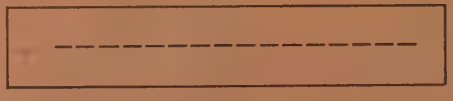


Inner scale



Outer scale



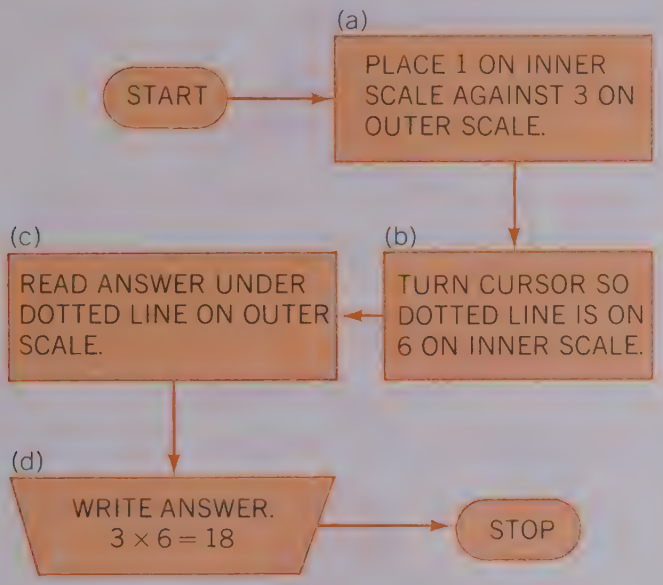


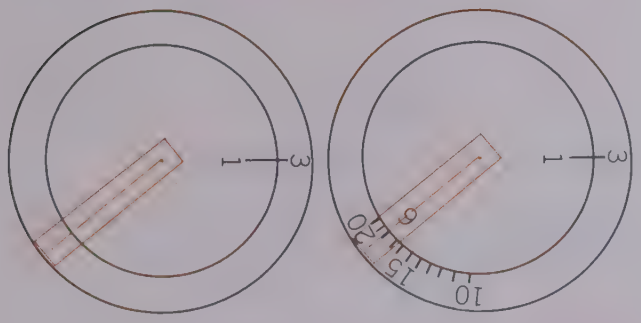
Cursor (thin plastic)

Paper fastener

1·2 Using Your Circular Slide Rule for Multiplication

Problem: Multiply 3×6.





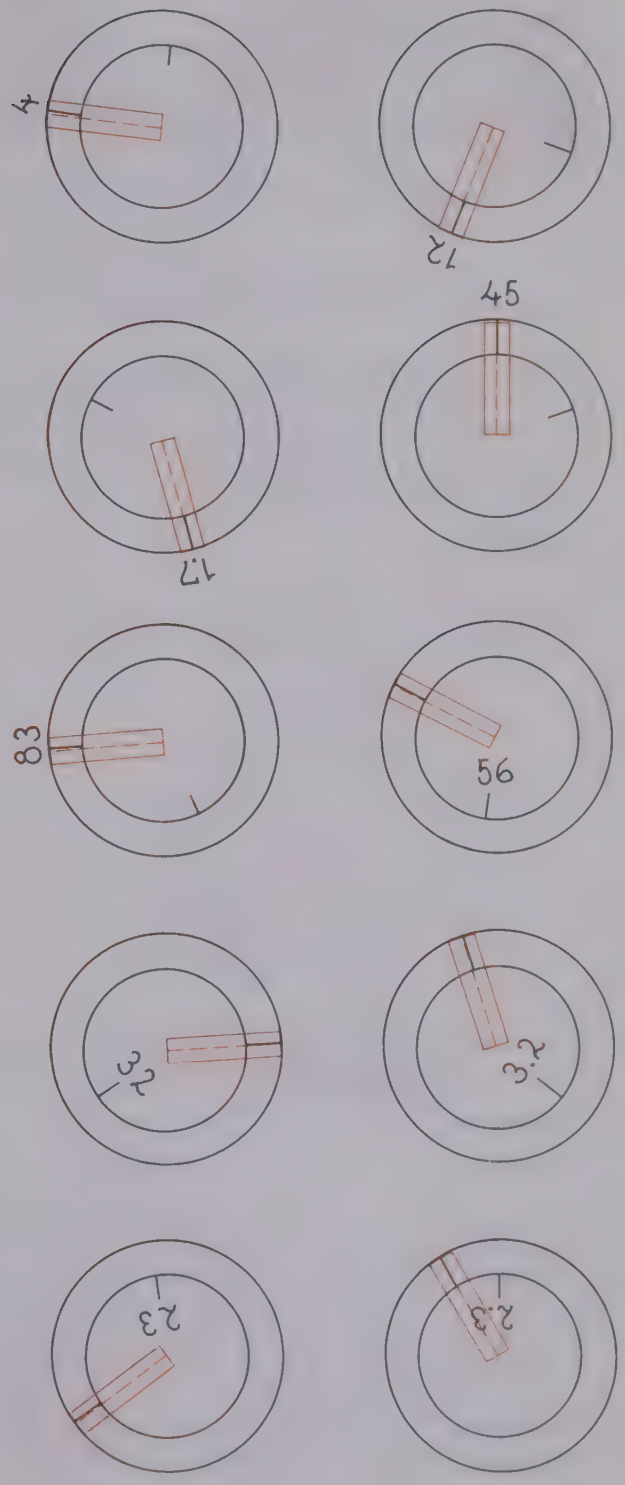
EXERCISE 1·2

1. Multiply using your circular slide rule:

   12×6 $\quad 9 \times 7$ $\quad 9 \times 9$ $\quad 3 \times 9$

2. On your slide rule, 3 may mean 3, 30, 300, 3000, etc. The number 25 may mean 25, 250, 2500, etc.

Give three values for each of the following settings:



3. Work in pairs. Each person places the cursor of his slide rule on a number. Exchange slide rules and read the number. Check with each other. Repeat several times.

4. Work in pairs. Use the cursor to locate these numbers on your slide rule. Then compare your answers. Discuss any answers that are different.

   (a) 18 (b) 28 (c) 36 (d) 140

   (e) 81 (f) 360 (g) 4500 (h) 2000

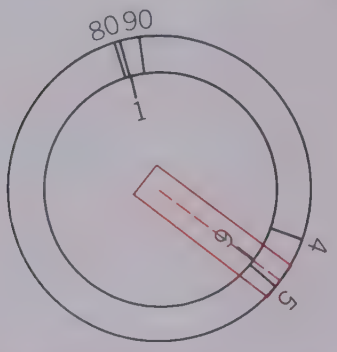
5. On how many places on your slide rule can you locate 350? Discuss with your partner.

6. When you multiply using your slide rule, you must always estimate your answer so that you know how to interpret the reading on your slide rule. Use your slide rule to calculate each of the following. The first one is done for you.

   (a) 82×6

   Solution

   82×6 is approximately $80 \times 6 = 480$.



   The answer is between 400 and 500. Hence, it is about 490 according to the slide rule.

   (b) 70×5 (c) 70×50

   (d) 35×8 (e) 35×80

   (f) 15×15 (g) 25×40

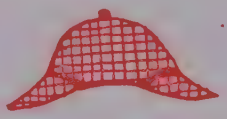
   (h) 75×8 (i) 75×80

7. Work in pairs. Make up ten problems (that can be done on your slide rule) and give them to your partner to do. Then check his answers.

8. Make up a multiplication problem that you think will stump your partner. Does it?

1·3 Properties of the System of Whole Numbers

Closure Property (under addition)



When any two whole numbers are added, the sum is always a whole number.

$5 + 8 = 13$

Closure Property (under multiplication)

When any two whole numbers are multiplied, the product is always a whole number.

$4 \times 9 = 36$

We say that the set of whole numbers is closed under the operations of addition and multiplication.

Do Questions 1 and 2 in Exercise 1·3.

Commutative Property (under addition)

23	Add	23	Add	What do
18 ↓	down.	18 ↑	up.	you notice?

This is one way of checking addition.

When a whole number m is added to another whole number x, the sum is the same as when x is added to m.

$$m + x = x + m$$
$$5 + 8 = 8 + 5$$

Can you find an example for which this is not true?

Commutative Property (under multiplication)

Multiply:

	37	26	What do
	26	37	you notice?

This is one way of checking multiplication.

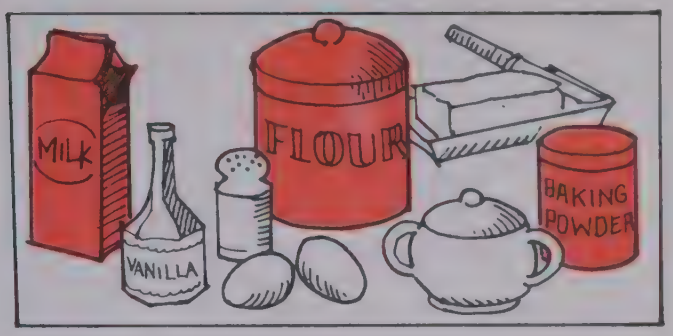
When a whole number h is multiplied by another whole number y, the product is the same as when y is multiplied by h.

$$h \times y = y \times h$$
$$6 \times 3 = 3 \times 6$$

Can you find an example for which this is not true? We say the commutative property for whole numbers holds true under addition or under multiplication.

STOP

Do Questions 3 and 4 in Exercise 1·3.



Does the commutative property hold for the mixing of the ingredients to bake a cake?

Associative Property of Addition and Multiplication

In finding the sum or the product of any three whole numbers a, b, and c, the result is the same regardless of the order in which they are grouped.

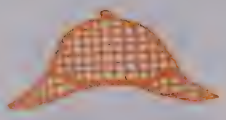
$$(a + b) + c = a + (b + c)$$
$$(3 + 8) + 5 = 3 + (8 + 5)$$
$$11 + 5 = 3 + 13$$
$$16 = 16$$
$$(a \times b) \times c = a \times (b \times c)$$
$$(4 \times 7) \times 3 = 4 \times (7 \times 3)$$
$$28 \times 3 = 4 \times 21$$
$$84 = 84$$

This illustrates the associative property for whole numbers. It holds true under the operations of addition or multiplication.

STOP

Do Question 5 in Exercise 1·3.

Identity Element



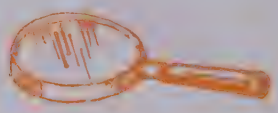
If, in a set of numbers, there is one special number which does not change the value of any other number of the set under one of the operations, then the special number is called an *identity element* for that operation.

Zero is the additive identity element.

$3 + 0 = 3$ $\qquad$ $17 + 0 = 17$

One is the multiplicative identity element.

$5 \times 1 = 5$ $\qquad$ $14 \times 1 = 14$



Do Question 6 in Exercise 1·3.

Distributive Property

The distributive property is expressed as multiplication over addition or multiplication over subtraction.

If a, b, and c are three numbers of a set, then the distributive property can be stated as follows:

(i) $a(b + c) = ab + ac$

or

(ii) $a(b - c) = ab - ac$

Substituting whole numbers for a, b, and c in case (i), we show that for $a = 3$, $b = 5$, $c = 7$:

$$3(5 + 7) = 3(5) + 3(7).$$

Since $\qquad 3(5 + 7) = 3(12) = 36$

and $\qquad 3(5) + 3(7) = 15 + 21 = 36$

and because any other substitutes for a, b, and c from the whole numbers will work, we say that the distributive property of multiplication over addition holds true for whole numbers.

EXERCISE 1·3

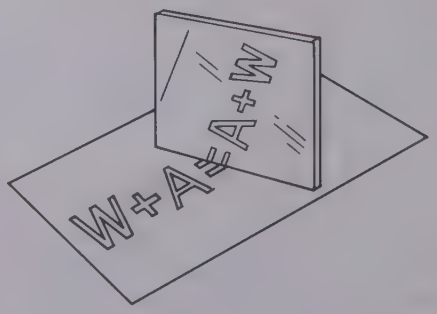
1. Consider the operation of subtraction on the set of whole numbers; e.g., $18 - 9 = \square$. Can you find an example in which the answer is not a whole number? Is subtraction on the set of whole numbers closed?

2. Consider the operation of division on the set of whole numbers; e.g., $24 \div 3 = \square$. Can you find an example in which the answer is not a whole number? Is division on the set of whole numbers closed?

3. Consider the operations (a) subtraction, (b) division, on the set of whole numbers. Does the commutative property hold for each of the operations? *Hint!* Make replacements for m and n in each case.

$$n - m \stackrel{?}{=} m - n \qquad\qquad m \div n \stackrel{?}{=} n \div m$$

4. Here we see a reflection:



This picture illustrates the commutative property. What other letters of the alphabet will work in this situation? Discuss the ways in which this experiment (a) illustrates, (b) does not illustrate, the commutative property.

5. Does the associative property under subtraction and division hold? *Hint!* Make replacements for *a*, *b*, and *c* in each case:

$(a-b)-c \stackrel{?}{=} a-(b-c)$ $\quad$ $(a \div b) \div c \stackrel{?}{=} a \div (b \div c)$

Can you find any examples in which they do hold? Discuss.

6. Is $a+b=b+a$ if $a=43$, $b=74$?

Show $(a+b)$ is a natural number if $a=16$, $b=19$.

Is $(a-b)$ a whole number if $a=8$, $b=13$?

Is $2 \times a = a \times 2$ when a is a natural number? when a is a whole number?

Is $a(b-c)=ab-ac$ for $a=8$, $b=9$, $c=7$? for $a=7$, $b=9$, $c=8$?

Is $(4a+6b)+9c=4a+(6b+9c)$ for $a=12$, $b=13$, $c=14$?

Is $39+16+47=39+47+16$?

7. Match each property with two examples. (You should end up matching two examples with each property.)

(i) Closure (under addition).
(ii) Commutative (under multiplication).
(iii) Distributive (multiplication over subtraction).
(iv) Identity element (additive).
(v) Associative (under multiplication).
(vi) Distributive (multiplication over addition).
(vii) Identity element (multiplicative).
(viii) Commutative (under addition).
(ix) Closure (under multiplication).
(x) Associative (under addition).

(a) $6 \times 1 = 6$
(b) $a(bc)=(ab)c$
(c) $2(7-4)=14-8$
(d) $x(a+b)=xa+xb$
(e) $(11+3)+8=11+(3+8)$
(f) $z+0=z$
(g) $9+5=14$
(h) $3(b-c)=3b-3c$
(i) $4 \times (3 \times 5)=(4 \times 3) \times 5$
(j) $5 \times 3 = 15$
(k) $5 \times 7 = 7 \times 5$
(l) $1(x)=x$
(m) $3 \times 4 = 12$
(n) $m+n=n+m$
(o) $0+7=7$
(p) $6+2=8$
(q) $9+8=8+9$
(r) $hk=kh$
(s) $5(2+7)=10+35$
(t) $(p+q)+r=p+(q+r)$

8. For practice (without slide rule):

341	214	527	371	564
526	398	196	609	321
719	796	401	745	797
7632	3926	4003	6741	1643
−2597	−1997	−2776	−2003	−1591
345	763	420	702	411
×28	×51	×60	×101	×99
$35\overline{)714}$	$46\overline{)1234}$	$25\overline{)4321}$	$89\overline{)4567}$	

Estimate, in metric units, the capacity of the containers and dishes shown.

1·4 Review of Operations on the Whole Numbers

The four operations basic to our study are addition, subtraction, multiplication, and division.

Name of Operation		Name of Result
Multiplication $a \times b$	⟶	Product ab
Addition $a + b$	⟶	Sum $a + b$
Subtraction $a - b$	⟶	Difference $a - b$
Division $a \div b$	⟶	Quotient $\frac{a}{b}$

Example 1

(a) What is the sum of 27 and 34?
(b) Find the difference between 65 and 18.
(c) Give the product for 36 and 25.
(d) Determine the quotient when 660 is divided by 15.

Solution

(a) $27 + 34 = 61$
(b) $65 - 18 = 47$
(c) $36 \times 25 = 900$
(d) $660 \div 15 = 44$

Example 2

Find the values of the algebraic expressions when $x = 7$ and $y = 3$.

(a) $3x + 8y$
(b) $6x - 5y$
(c) $2x(7y)$
(d) $12x \div 4y$

Solution

(a) $3x + 8y = 3(7) + 8(3)$
$= 21 + 24$
$= 45$

(b) $6x - 5y = 6(7) - 5(3)$
$= 42 - 15$
$= 27$

(c) $2x(7y) = 2(7)(7)(3)$
$= 14 \times 21$
$= 294$

(d) $12x \div 4y = 12(7) \div 4(3)$
$= 84 \div 12$
$= 7$

Example 3

Evaluate these expressions when $x = 6$ and $y = 2$.

(a) $4xy$ (b) $3[2x - 3y]$ (c) $\frac{5x + 3y}{3xy}$

Solution

(a) $4xy = 4(6)(2)$
$= 48$

(b) $3[2x - 3y] = 3[2(6) - 3(2)]$
$= 3(12 - 6)$
$= 3(6)$
$= 18$

Always work that in the inner brackets first.

(c) $\frac{5x + 3y}{3xy} = \frac{5(6) + 3(2)}{3(6)(2)}$
$= \frac{30 + 6}{36}$
$= \frac{36}{36}$
$= 1$

EXERCISE 1·4

1. (a) Perform the four operations on 18 and 6. Arrange the sum, difference, product, and quotient in the order from smallest to largest.

(b) Which result is the greatest? Is it always the largest of the four results when the operations on two numbers are compared? Support your answer with three examples.

(c) Which result is second largest? Is it always the second largest of the four? Support your answer with three examples.

(d) Is the quotient always the least? Find two pairs of numbers for which, in each pair, the quotient is less than or equal to the difference.

2. (a) Find the difference between the product and the sum of 8 and 4.

   (b) What is the sum of the difference and the quotient of 36 and 18?

   (c) Give the product of the sum and the quotient of 12 and 3.

3. Evaluate the following when $m = 3$.

Always perform multiplication before addition or subtraction.

   (a) $m + 4$ (b) $13m$ (c) $5m + 6$

   (d) $5 \times 3 - 2 \times m + 4 + 3 \times m$ (e) $4 \times 3 - 2 \times m$

   (f) $28m$ (g) $94 - 22m$ (h) $87 - 8m$

4. Evaluate the following when $n = 7$ and $k = 8$. (Divide last in each case.)

   (a) $\frac{6n}{14}$ (b) $\frac{3n-k}{13}$ (c) $\frac{3n-2k}{5}$ (d) $\frac{9n-3k}{13}$

   (e) $\frac{7k-5n}{3n}$ (f) $\frac{14n-7k}{2n}$ (g) $\frac{22k-16n}{2k}$

   (h) $n \cdot n + k \cdot k - n \cdot k$

5. Evaluate the following expressions for $a = 7$ and $b = 4$.

   (a) $3a - 2b$ (b) $ab - 2(a + b)$

   (c) $2ab + 4(2b - a)$ (d) $\frac{5a-8b}{3}$

   (e) $12(6a - 10b)$ (f) $\frac{a(9a+17b)}{3b}$

6. Find the value of each expression when $x = 16$ and $y = 15$.

   (a) $2(x + y)$ (b) $\frac{8xy}{120}$ (c) $\frac{6x-2y}{25}$

   (d) $\frac{2(3x+4y+24)}{xy}$ (e) $\frac{5xy}{32y}$

   (f) $\frac{15x}{8y} + 7(x - y) + \frac{3x-2y}{9}$

1·5 Using Your Circular Slide Rule to Divide

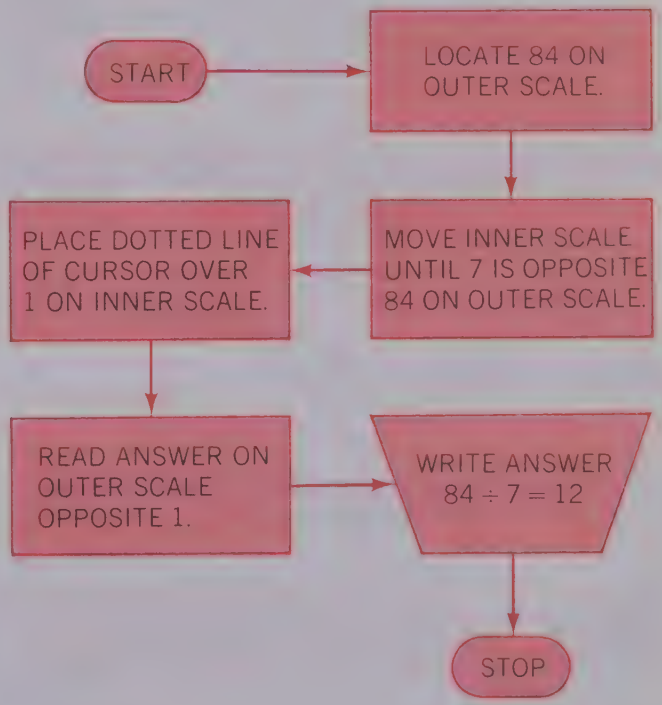
Example

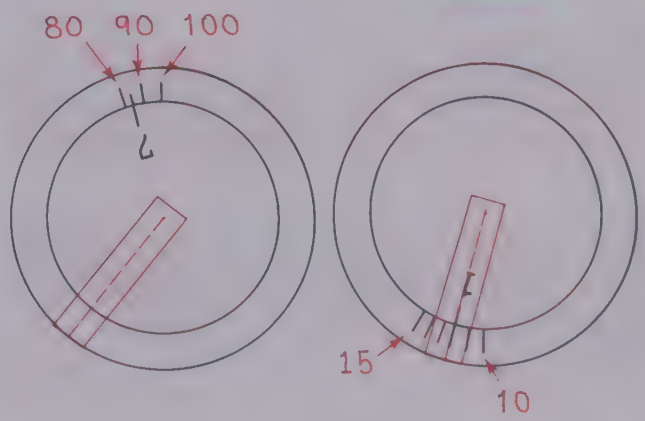
Divide $84 \div 7$.

Solution

Try this on your own. Remember, if $a \times 7 = 84$, then $84 \div 7 = a$.

If you need help, use this flowchart.


START
LOCATE 84 ON OUTER SCALE.
MOVE INNER SCALE UNTIL 7 IS OPPOSITE 84 ON OUTER SCALE.
PLACE DOTTED LINE OF CURSOR OVER 1 ON INNER SCALE.
READ ANSWER ON OUTER SCALE OPPOSITE 1.
WRITE ANSWER 84 ÷ 7 = 12
STOP



EXERCISE 1·5

1. Use your slide rule to divide:

(a) $56 \div 8$	(b) $63 \div 9$	(c) $96 \div 12$
(d) $165 \div 55$	(e) $216 \div 3$	(f) $480 \div 80$
(g) $189 \div 9$	(h) $357 \div 7$	(i) $462 \div 21$
(j) $735 \div 35$	(k) $943 \div 23$	(l) $1008 \div 18$
(m) $768 \div 64$	(n) $680 \div 85$	(o) $1541 \div 23$
(p) $2730 \div 65$	(q) $1870 \div 34$	(r) $1863 \div 81$
(s) $2280 \div 57$	(t) $2775 \div 37$	(u) $3120 \div 48$

2. Make up ten division problems (that can be solved on your slide rule) and give them to your partner to solve. Then check his answers.

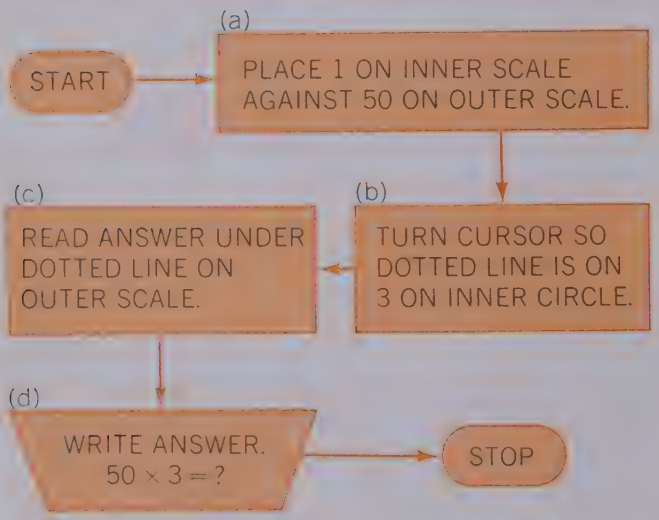
1·6 Using Your Slide Rule for Larger Numbers

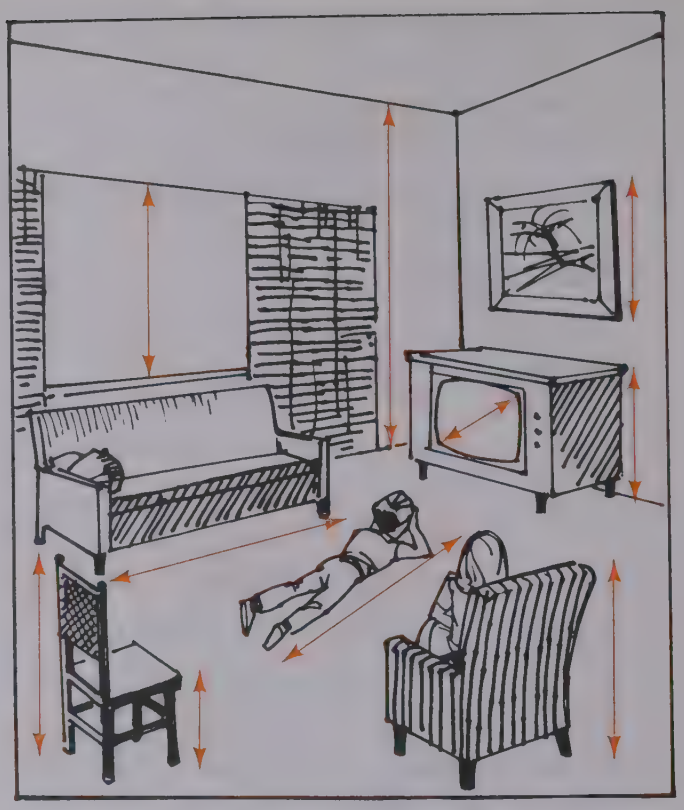
Example 1

Use your slide rule to multiply 50×3.

Solution

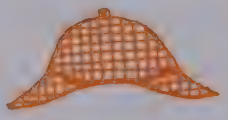
You knew the answer before you started, but can you do this problem on your slide rule? Do you need help?


(a)
START
PLACE 1 ON INNER SCALE AGAINST 50 ON OUTER SCALE.
(b)
TURN CURSOR SO DOTTED LINE IS ON 3 ON INNER CIRCLE.
(c)
READ ANSWER UNDER DOTTED LINE ON OUTER SCALE.
(d)
WRITE ANSWER. 50 × 3 = ?
STOP



Using metric units, give estimates of the measures indicated by the colored lines. Discuss your answers.

Did you have a problem in (c)? How did you interpret the number 1.5 under the dotted line? Obviously 50×3 is not 1.5!



Always estimate your answer before starting!

Problem 1

Use your slide rule to multiply 60×40. (How do you interpret the "24" under the dotted line?)

Problem 2

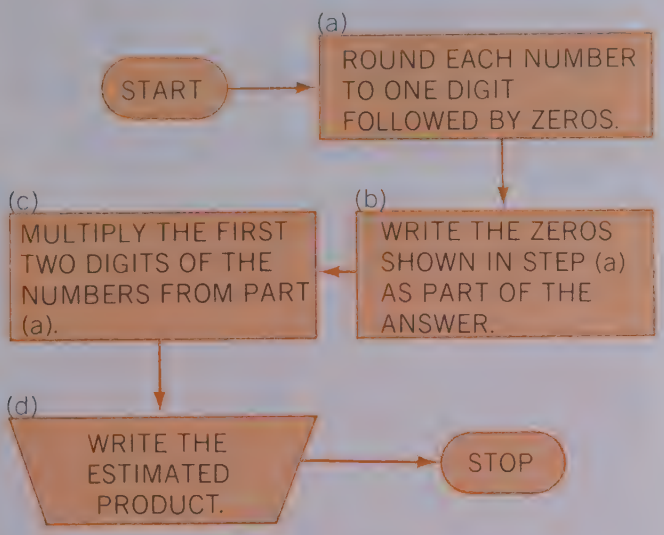
Can you do this one: 22×58? Did you estimate first?

Let's review a method of estimating the product.

Example 2

Estimate 36×72.

Solution


START
(a)
ROUND EACH NUMBER TO ONE DIGIT FOLLOWED BY ZEROS.
(b)
WRITE THE ZEROS SHOWN IN STEP (a) AS PART OF THE ANSWER.
(c)
MULTIPLY THE FIRST TWO DIGITS OF THE NUMBERS FROM PART (a).
(d)
WRITE THE ESTIMATED PRODUCT.
STOP

$36 \times 72 \simeq 40 \times 70$ (Step (a))

$\downarrow$

$\_\_00$ (Step (b))

$36 \times 72 \simeq 2800$ (Steps (c) and (d))

Do you remember how to round off numbers?

Can you make a rule for rounding off? Discuss your rule with your classmates. If you need more help, ask your teacher.

Do Questions 1, 2, 3, and 4 in Exercise 1·6.

Returning to our slide rule, we can now do any multiplication problem.

Example 3

Calculate 39×64.

Solution

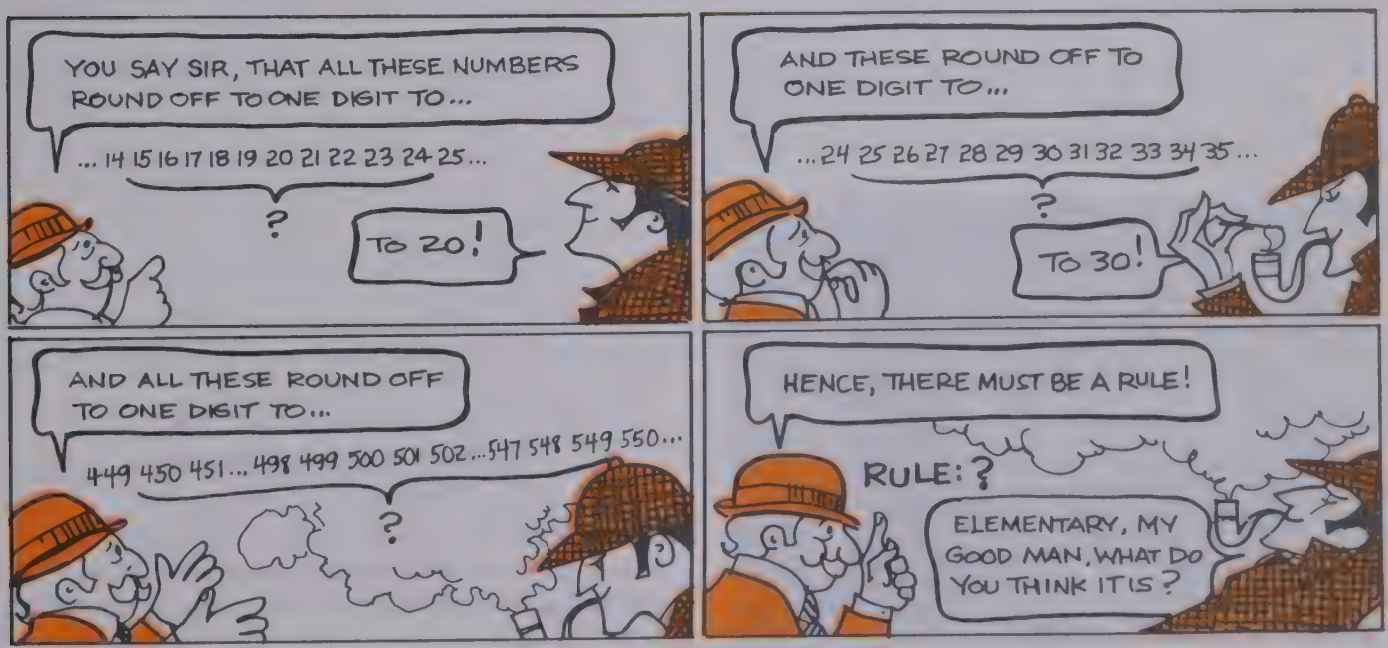
Estimate: $39 \times 64 \simeq 40 \times 60$

$\simeq 2400$

Using the slide rule, we get 2500. Check by multiplying using your pencil.

$$\begin{array}{r} 39 \\ \underline{64} \end{array}$$

Are we in error very much? Discuss.


YOU SAY SIR, THAT ALL THESE NUMBERS ROUND OFF TO ONE DIGIT TO...
...14 15 16 17 18 19 20 21 22 23 24 25...
?
TO 20!
AND THESE ROUND OFF TO ONE DIGIT TO...
...24 25 26 27 28 29 30 31 32 33 34 35...
?
TO 30!
AND ALL THESE ROUND OFF TO ONE DIGIT TO...
449 450 451... 498 499 500 501 502...547 548 549 550...
?
HENCE, THERE MUST BE A RULE!
RULE: ?
ELEMENTARY, MY GOOD MAN, WHAT DO YOU THINK IT IS?

EXERCISE 1·6

1. What does the symbol $\simeq$ mean?

2. Round each to one digit followed by one or more zeros.

   (a) 28 (b) 32 (c) 65 (d) 246
   (e) 287 (f) 315 (g) 15 (h) 45
   (i) 96 (j) 355 (k) 555 (l) 823

3. Estimate the product in each by following the steps in the flowchart in Example 2. A partial solution in Part (a) is given to help you.

   (a) $56 \times 82 \simeq \_\_\_0 \times \_\_\_0$
   $\simeq \_\_\_00$

   (b) 13×23 (c) 39×15 (d) 55×78
   (e) 14×35 (f) 16×87 (g) 99×44
   (h) 65×74 (i) 749×856 (j) 183×917

4. Challenge your friend! Flip a coin to see who starts. Give two two-digit numbers to your partner to estimate their product orally. If the answer is correct, give another. Continue until a wrong answer is given. Then your partner gives you two such numbers to estimate their product. Keep score to see who reaches ten (or some other total). The first to reach the total wins the game.

5. Between the years beginning 1946 to the end of 1969, 3 267 144 immigrants arrived in Canada. What is the average number of immigrants per year?

6. The peak years, in this period, were 1957 when 282 164 persons arrived and 1967 when 222 876 were admitted. How much is the average of these two years above the average in Question 5?

7. Use your slide rule to multiply each. Remember to estimate the answers first.

   (a) 17×43 (b) 28×16 (c) 45×67
   (d) 73×29 (e) 736×262 (f) 176×841

8. Use your slide rule to calculate the products for Question 3.

9. Play the game in Question 4, but this time, the answer must be found by using the slide rule.

1·7 More Estimations of Products and Quotients

Let's look at a few examples where estimations are helpful.

Example 1

Multiply 423×29.

Wages for Staff	
1 day	$423
29 days	$?

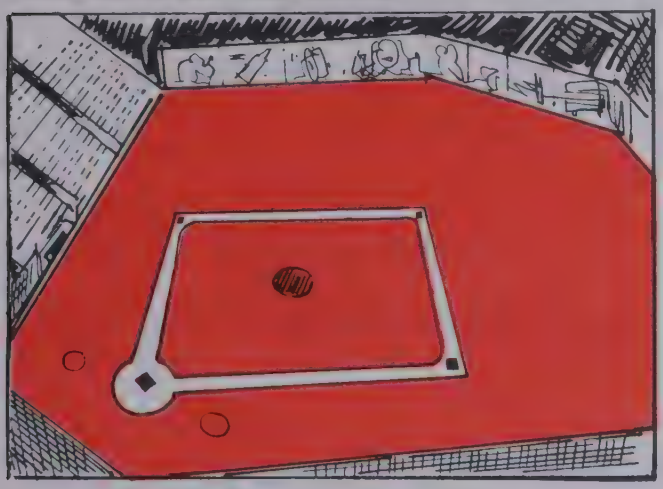
Solution

$$\begin{array}{r} 423 \\ \times\ 29 \\ \hline 3\,807 \\ 8\,46 \\ \hline 12\,267 \end{array}$$

An estimate of the answer can be worked out as follows:

423 can be approximated to 400;
29 can be approximated to 30.
$400 \times 30 = 12\,000$

The estimated answer of 12 000 indicates that the product of 12 267 is in the approximate range and serves as a check. Hence, \$12 267 appears to be correct.



Estimate the area, in metric units, of the various regions shown in the picture. Discuss your answers with your friends.

Example 2

Divide 37 494 by 18.

Solution

$$
\begin{array}{r}
2\,083 \\
18\overline{)\,37\,494} \\
\underline{36} \\
1\,49 \\
\underline{1\,44} \\
54 \\
\underline{54} \\
0
\end{array}
$$

Because 18 divides into 36 exactly 2 times, it is reasonable to mentally divide 18 into 36 000 to obtain an estimate for the answer. Since 18 into 36 000 yields 2000, then 18 into 37 494 will yield a quotient slightly greater than 2000. Therefore, 2083 appears to be in the range of the correct answer.

Example 3

What will be the approximate result when we compute

$$\frac{594 \times 72}{319}\,?$$

Solution

Rewrite the problem by rounding off the numbers to

$$\frac{600 \times 70}{300}$$

Then, by simplifying like this:

$$
\begin{aligned}
\frac{600 \times 70}{300} &= \frac{42\,000}{300} \\
&= \frac{420}{3} \\
&= 140
\end{aligned}
$$

The estimate of the answer is 140.

EXERCISE 1·7

Estimate the answer in each of the following; then choose the correct answer (from the choices offered) based on your estimate.

1. 49×72
   (a) 2968 (b) 4618 (c) 3528
2. 94×214
   (a) 20 116 (b) 15 886 (c) 25 396
3. 817×689
   (a) 562 913 (b) 631 253 (c) 460 853
4. $2464 \div 32$
   (a) 77 (b) 753 (c) 17
5. $148\,658 \div 311$
   (a) 4268 (b) 478 (c) 88
6. $3516 \div 12$
   (a) 293 (b) 393 (c) 3043
7. $\frac{86 \times 49}{301}$ (a) 26 (b) 43 (c) 14
8. $\frac{546 \times 76}{494}$ (a) 112 (b) 37 (c) 84

Estimate the answer to each.

9. 32×61
10. 57×510
11. 42×89
12. 78×33
13. 910×340
14. 72×88
15. $\frac{62 \times 39}{41}$
16. $\frac{76 \times 58}{32}$
17. $\frac{180 \times 520}{410}$

18. Use long multiplication and division to calculate the answer to Questions 9 through 17. Compare your answers with the estimates. (Or you may wish to use your slide rule.)

19. The three world-record giant tuna taken by rod and reel were caught off Prince Edward Island and off Nova Scotia. They were 453, 473, and 484 kg. What is the average of the tuna?

20. In this year's local fish derby, the largest fish caught was 12 kg 38 g. This topped last year's largest fish by 62 g. How heavy was last year's record fish?

1·8 Exponents

Exponents can be used to write certain number problems in a short form.

We can write $9 = 3 \times 3$
$= 3^2$

and $32 = 2 \times 2 \times 2 \times 2 \times 2$
$= 2^5$

3^2 and 2^5 are ways of writing 9 and 32 by using exponents. 3^2 and 2^5 are in exponential form. We read 3^2 as *three to the exponent two* or *three squared*. We read 2^5 as *two to the exponent five* or *two to the fifth*.

4^3 is *four cubed*. What number is it equal to? Sometimes we say 4^3 is the *third power of four*. In this case, 4 is the *base* of this power and 3 is the *exponent*.

Example 1

Find the numbers equivalent to:

(a) 9^2 (b) 5^4 (c) 10^5

Solution

(a) $9^2 = 9 \times 9 = 81$

(b) $5^4 = 5 \times 5 \times 5 \times 5 = 25 \times 25 = 625$

(c) $10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100\,000$

Example 2

Expand each of the following:

(a) x^3 (b) $6x^2$ (c) $(2x)^3$

Solution

(a) $x^3 = x \cdot x \cdot x$

(b) $6x^2 = 6 \cdot x \cdot x$

(c) $(2x)^3 = 2x \cdot 2x \cdot 2x$
$= 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$

(Have you seen the dot used this way?)

Pay particular heed to Parts (b) and (c) and note the difference. What is the difference?

Do Questions 1 and 2 in Exercise 1·8.

Example 3

Find the value of each expression for $x = 3$.

(a) $5x^2$ (b) $(2x)^4$

Solution

(a) $5x^2 = 5 \cdot 3^2 = 5 \cdot 3 \cdot 3 = 45$

(b) $(2x)^4 = (2 \cdot 3)^4 = 6^4 = 6 \cdot 6 \cdot 6 \cdot 6 = 1296$

Again, note the difference between (a) and (b).

Example 4

Express each as one power.

(a) $m^4 \cdot m^3$ (b) $\frac{n^5}{n^2}$

Solution

(a) $m^4 \cdot m^3 = m \cdot m \cdot m \cdot m \cdot m \cdot m \cdot m = m^7$

(b) $\frac{n^5}{n^2} = \frac{n \cdot n \cdot n \cdot n \cdot n}{n \cdot n} = n^3$

EXERCISE 1·8

1. Write in symbol form:
   (a) eleven cubed,
   (b) five to the seventh,
   (c) fourth power of nine,
   (d) ten to the exponent twelve,
   (e) x to the exponent five,
   (f) $3x$ to the exponent six,
   (g) the quantity $6 + x$ to the exponent three,
   (h) the quantity $2x + 5$ to the fourth multiplied by three.

2. Expand:
   (a) 3^8 (b) 12^5 (c) $(3a)^4$
   (d) $5x^3$ (e) $x^5 \cdot z^2$

3. Find the value of:
   (a) m^4 when $m = 3$,
   (b) $3x^5$ when $x = 6$,
   (c) $(5x)^2$ when $x = 8$,
   (d) $4x^2 - 3y^2$ when $x = 5$ and $y = 8$,
   (e) $x^2 - 13x + 7$ when $x = 20$,
   (f) $(2b)^3 - 6b^2$ when $b = 7$,
   (g) $a^2 + 3a - 7$ when $a = 8$,
   (h) $y^5 + y^3 + y$ when $y = 4$.

4. Express each as one power.
   (a) $x^2 \cdot x^3$ (b) $t \cdot t^2 \cdot t^3$ (c) $y^5 \cdot y^3$
   (d) $\frac{x^3}{x}$ (e) $\frac{m^7}{m^5}$ (f) $\frac{z^{12}}{z^8}$

5. Is $x^5 + x^3$ equal to x^8?

   Hint! Substitute $x = 2$ and work each part out; then compare.

6. When we subtract n^7 from n^{12}, should we get n^5? Test as in Question 5 above.

7. During the Yukon gold rush, a ship from the Klondike docked in Seattle with a reported two tonnes of gold on board at \$1 per gram. How many dollars worth of gold was on board? Express the amount in the form $\$2 \times 10^n$ where n is the exponent.

Yukon Gold Rush. Webster Canadiana Collection. New Brunswick Museum.

1·9 Equalities and Inequalities (with Whole Numbers)

The set of whole numbers (W) can be expressed as

$$W = \{0, 1, 2, 3, 4, 5, \ldots\}.$$

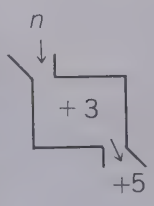
If n is any whole number ($n \in W$), then we can write equalities (which are called equations) such as the following:

(a) $n + 3 = 5$ (b) $n - 2 = 7$

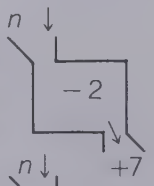
(c) $2n = 6$ (d) $4n - 8 = 4$

Let us think of a function machine.

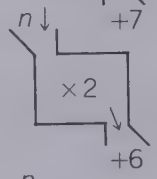
(a) For $n + 3 = 5$, we think of a number n being put into the machine. To this number, we add 3 and we know the answer is 5. What number is n?



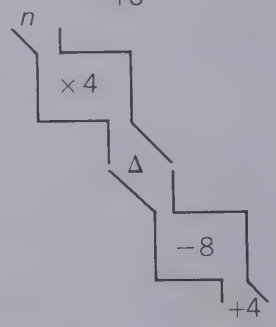
(b) For $n - 2 = 7$, we make this machine. Explain this computer.



(c) For $2n = 6$: Explain this machine.



(d) For $4n - 8 = 4$: Explain this double machine.



Similarly, we can write inequalities (which are called inequations) such as the following:

(e) $n > 4$ (f) $n + 3 < 10$

(g) $2n - 3 > 1$ (h) $5 + 3n < 7$

Are you familiar with the symbols $>$ and $<$?

$5 > 2$ is read, "5 is greater than 2."

$3 < 7$ is read, "3 is less than 7."

$a \leq b$ is read, "a is less than or equal to b."

$x \geq y$ is read, "x is greater than or equal to y."

How would you read (f) and (g) above?

Did you read them:

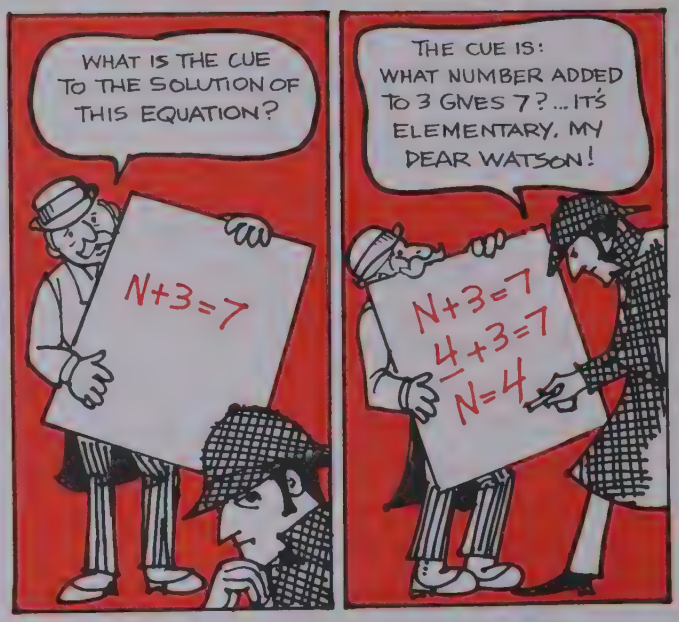
(f) $n + 3$ is less than 10?

and (g) $2n - 3$ is greater than 1?

Example 1

For $n \in W$, solve:

(a) $n + 5 = 8$ (b) $3n = 18$


WHAT IS THE CUE TO THE SOLUTION OF THIS EQUATION?
N+3=7
THE CUE IS: WHAT NUMBER ADDED TO 3 GIVES 7? ... IT'S ELEMENTARY, MY DEAR WATSON!
N+3=7
4+3=7
N=4

Solution

(a) 3 is the whole number n which, when added to 5, gives a sum of 8; i.e., $n = 3$.

(b) 6 is the whole number n which, when multiplied by 3, yields 18; i.e., $n = 6$.

Do Question 1 in Exercise 1·9.

Example 2

For $k \in W$, express the answer in set notation for each of the following:

(a) $k > 5$ (b) $k - 2 < 9$ (c) $3k \geq 12$

Solution

We need the set of whole numbers greater than 5.

(a) $k \in \{6, 7, 8, 9, 10, \ldots\}$ (This is read, "k is an element of the set of whole numbers greater than 5.")

(b) If $k - 2$ were equal to 9, then k would equal 11.
Because $k - 2$ is less than 9, then $k < 11$.
$\therefore k \in \{0, 1, 2, \ldots, 9, 10\}$.

(c) $3k$ is greater than or equal to 12.
For $3k = 12$, $k = 4$.
For $3k > 12$, $k > 4$.
$\therefore$ for $3k \geq 12$, $k \geq 4$,
and $k \in \{4, 5, 6, 7, \ldots\}$.

EXERCISE 1·9

1. For $n \in W$, solve:
   (a) $n + 7 = 12$ (b) $17 + n = 38$
   (c) $n + 48 = 142$ (d) $n + 2001 = 5555$
   (e) $n - 6 = 13$ (f) $n - 15 = 31$
   (g) $n - 64 = 100$ (h) $82 - n = 47$
   (i) $5n = 70$ (j) $8n = 144$
   (k) $3n - 11 = 10$ (l) $7n - 28 = 42$

2. For $x \in W$, express each answer in set notation (as done in Example 2 above).
   (a) $x > 7$ (b) $x \geq 3$
   (c) $x < 4$ (d) $x \leq 8$
   (e) $3x > 15$ (f) $7x \geq 28$
   (g) $12x < 84$ (h) $6x \leq 54$
   (i) $x + 6 > 11$ (j) $9 + x \geq 17$
   (k) $2 + x < 7$ (l) $13 + x \leq 15$
   (m) $x + 7 \geq 13$ (n) $x - 6 \leq 21$

3. If $k \in \{4, 5, 6, 7, 8, 9\}$, find the value of k in:
   (a) $4k + 7 \geq 28$

   (b) $5k - 8 \leq 17$ (c) $14 - 2k > 2$
   (d) $8 + 6k < 38$

4. If $y \in \{2, 4, 6, 8, 10, 12\}$, find the values of y in:
   (a) $2y > 15$ (b) $3y < 13$
   (c) $y + 7 > 10$ (d) $y - 2 < 8$
   (e) $3 + 2y < 20$ (f) $10 - y > 7$
   (g) $13 - 2y > 4$ (h) $32 - 5y > 1$

5. For $x \in W$, solve:
   (a) $5 + x = 4$ (b) $x + 6 = 3$

   To what set of numbers does x belong? Do the solutions to (a) and (b) yield the required number for x? Since $x \in W$, how can we express the solutions to (a) and (b)? Discuss.

REVIEW EXERCISE 1

1. Arrange from largest to smallest:
   (a) three hundred thousand.
   (b) fourteen thousand seven hundred and two.
   (c) eighty-one thousand and forty-six.
   (d) thirty five million eight thousand.
   (e) nine thousand eight hundred and seventy-five.
   (f) six hundred and ninety-three thousand four hundred and eight.

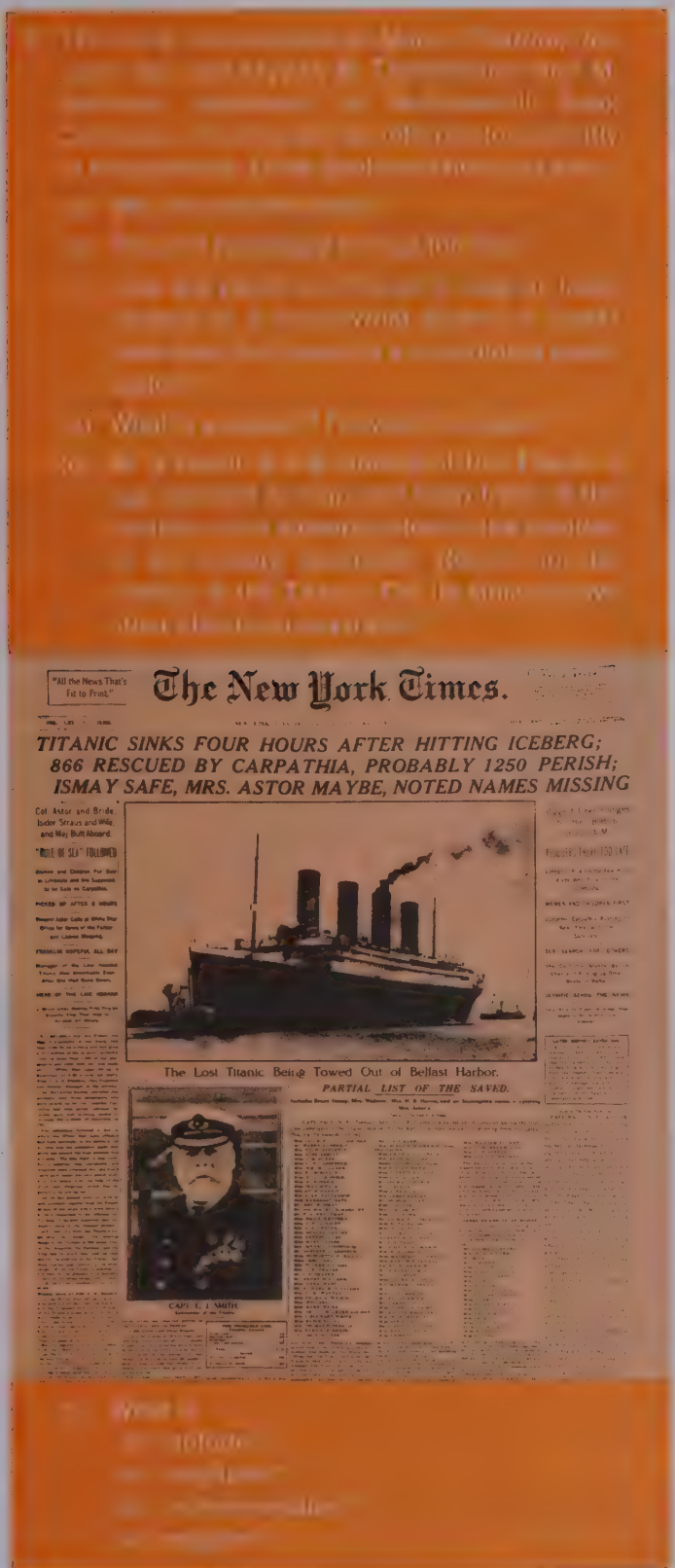
2. Which of these is an example of the:
   (a) Distributive property?
   $4(a-3)=4a-12$, $8\times 11=11\times 8$, $5\times 7=35$
   (b) Identity element?
   $3+8=11$, $9+8=8+9$, $4+0=4$
   (c) Associative property?
   $7\times 1=7$, $3\times(4\times 5)=(3\times 4)\times 5$,
   $6\times 2=2\times 6$
   (d) Commutative property?
   $4+3+2=9$, $12+5=5+12$,
   $3(x+8)=3x+24$

3. Evaluate the following expressions for $m=3$, $n=8$:
   (a) $14n-2mn$ (b) $mn-(5m+n)$
   (c) $\dfrac{7(n-2m)+(m+n)}{n-m}$

4. Find the value of:
   (a) $2x^3$ when $x=4$
   (b) $3x^2-7x+5$ when $x=5$
   (c) y^3-2 when $y=8$

5. For $n\in W$, solve:
   (a) $6-n=2$ (b) $3n+5=26$
   (c) $7+n\geq 12$ (d) $11+n>17$
   (e) $n-12<3$ (f) $n-8\geq 5$

Using the Library


"All the News That's Fit to Print."

The New York Times.

TITANIC SINKS FOUR HOURS AFTER HITTING ICEBERG; 866 RESCUED BY CARPATHIA, PROBABLY 1250 PERISH; ISMAY SAFE, MRS. ASTOR MAYBE, NOTED NAMES MISSING

The Lost Titanic Being Towed Out of Belfast Harbor.

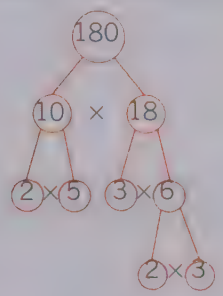
PARTIAL LIST OF THE SAVED.

CAPT. E. J. SMITH

Do You Remember . . .

. . . how to find the H.C.F. (Highest Common Factor) of any two or three whole numbers? Suppose the numbers are 180, 630, and 2310.

First, find the prime factors of each number. Have you seen this method of finding prime factors?



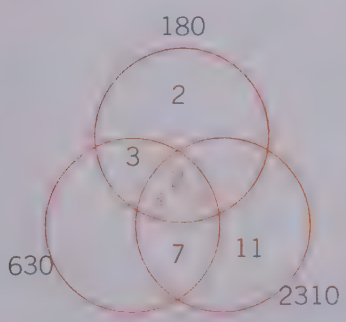
$\therefore 180 = 2 \times 5 \times 3 \times 2 \times 3$
$= 2 \times 2 \times 3 \times 3 \times 5$

Find the prime factors of 630 and 2310 by the above method. After you have done that, arrange them as follows:

$180 = 2 \times 2 \times 3 \times 3 \times 5$
$630 = 2 \times 3 \times 3 \times 5 \times 7$
$2310 = 2 \times 3 \times 5 \times 7 \times 11$

To find the H.C.F., choose all the factors that are common to the three numbers (in color). Those factors are 2, 3, and 5. The product of these factors, $2 \times 3 \times 5 = 30$, is then the H.C.F.

We can also show this by Venn diagrams.



Find the H.C.F. of:

(a) 42, 210, 660 (b) 168, 252, 564

Something to Think About

Numbers can be arranged so that they form patterns. We call these patterns *sequences*. The natural (counting) numbers is the simplest pattern of numbers to follow; e.g., 1, 2, 3, 4, ... Then even numbers in the ascending order is another example of a sequence; e.g., 2, 4, 6, 8, ...

Give the next three numbers in each of these sequences:

(a) 1, 4, 7, 10, 13, 16, . . .

(b) 2, 5, 8, 11, . . .

(c) 3, 7, 11, 15, . . .

(d) 1, 2, 4, 8, . . .

(e) 2, 6, 18, 54, . . .

(f) 3, 11, 19, 27, . . .

(g) 2, 12, 22, 32, . . .

(h) 8, 17, 26, 35, 44, . . .

(i) 1, 10, 100, 1000, . . .

(j) 11, 101, 1001, 10 001, . . .

Have You Discovered that . . .

. . . the early Greeks discovered a set of numbers called *perfect numbers*? A perfect number is equal to the sum of its own factors (1 is considered a factor here). One example of a perfect number is 6. The factors of 6 are 1, 2, 3, and their sum is 6. (*Note*: We do not consider 6 a factor of itself.)

Find another such perfect number (consider all factors, not just prime factors).

Can a prime number ever be a perfect number? Why or why not?

Try the number 496 to see if it is a perfect number.

—— . —— . —— . —— . ——

. . . the smallest whole number that is evenly divisible by all these numbers (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) is a four-digit number? Find what that number is.

Chapter Two
THE SEARCH CONTINUES

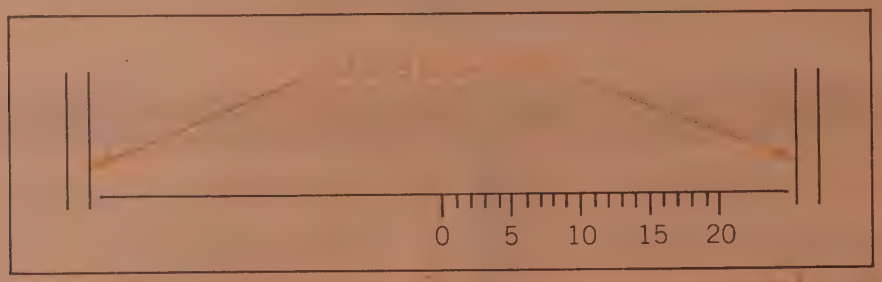
Integers

... WHAT FURTHER INFERENCES MAY WE DRAW?
DO NONE SUGGEST THEMSELVES? YOU KNOW MY METHODS. APPLY THEM!
I CAN THINK ONLY OF THE OBVIOUS CONCLUSION...
I THINK THAT WE MIGHT VENTURE A LITTLE FURTHER THAN THAT. LOOK AT IT IN THIS LIGHT...

Project

Project 1

Using two pieces of stiff cardboard, make the following strips. Draw the scales as shown.



Place the two together like this:

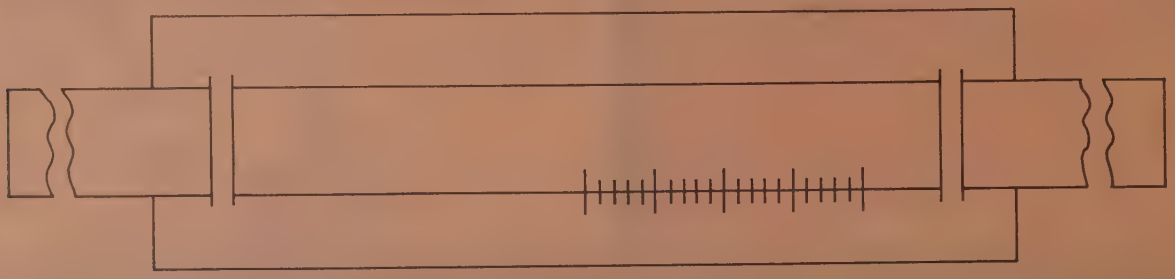


Figure 2-1

Notice how we use the slide rule to add 3 + 12.

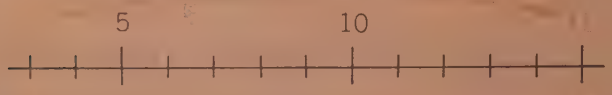
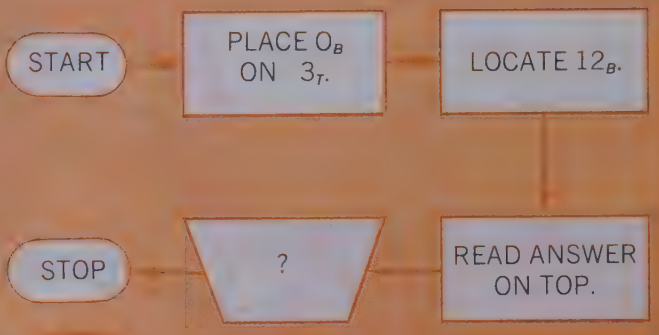


Figure 2-2

$3 + 12 = 15$

0_B means zero on bottom scale.

What does 3_T mean?



Do Question 1 in Exercise 2·1.

2·1 Using Your Slide Rule in the Search

Let us subtract 15 – 12.

Using the slide rule, we set 12 in the bottom slide below 15 of the upper slide.

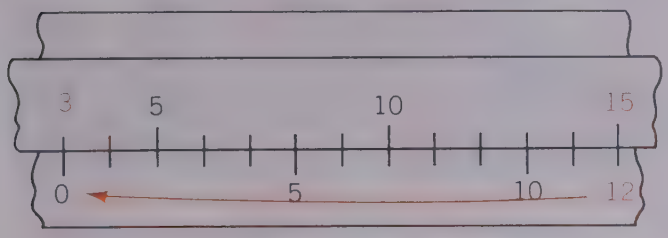


Figure 2-3

15 – 12 = 3

Compare Figure 2-2 and Figure 2-3.

Notice that subtraction is the *undoing* of addition.

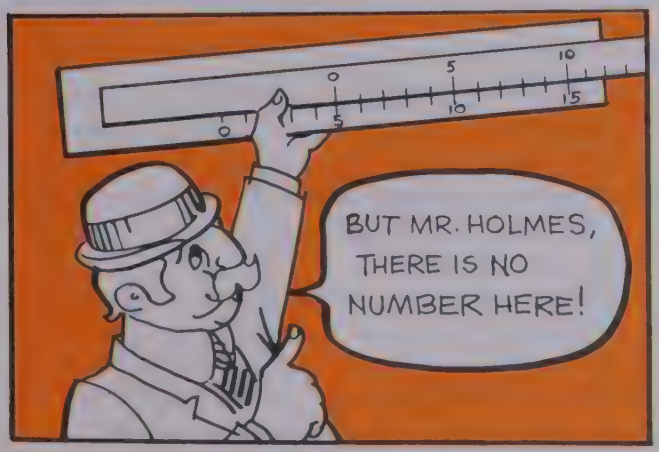
Do Questions 2 and 3 in Exercise 2·1.

Use your slide rule to subtract 7 – 12.

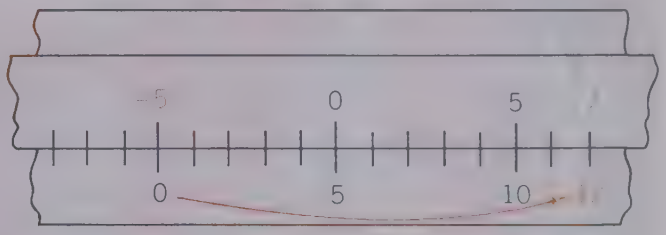
Discuss what you find.

Do you have a suggestion as to what we should do to solve this problem?

Is this what you got when you subtracted 7 – 12?



It seems we need to extend the scale on the top slide. Let's do both scales while we are at it! Since the new numbers are in the opposite direction of those already on the scale, let's label them with this sign in front of each: –. We call this sign *negative*.

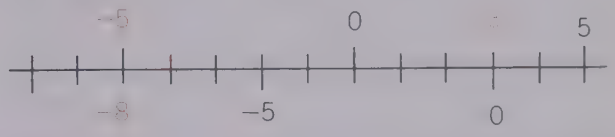


Solution: 7 – 12 = –5

Do Question 4 in Exercise 2·1.

Use your slide rule to add 3 + (–8).

Did you get this?



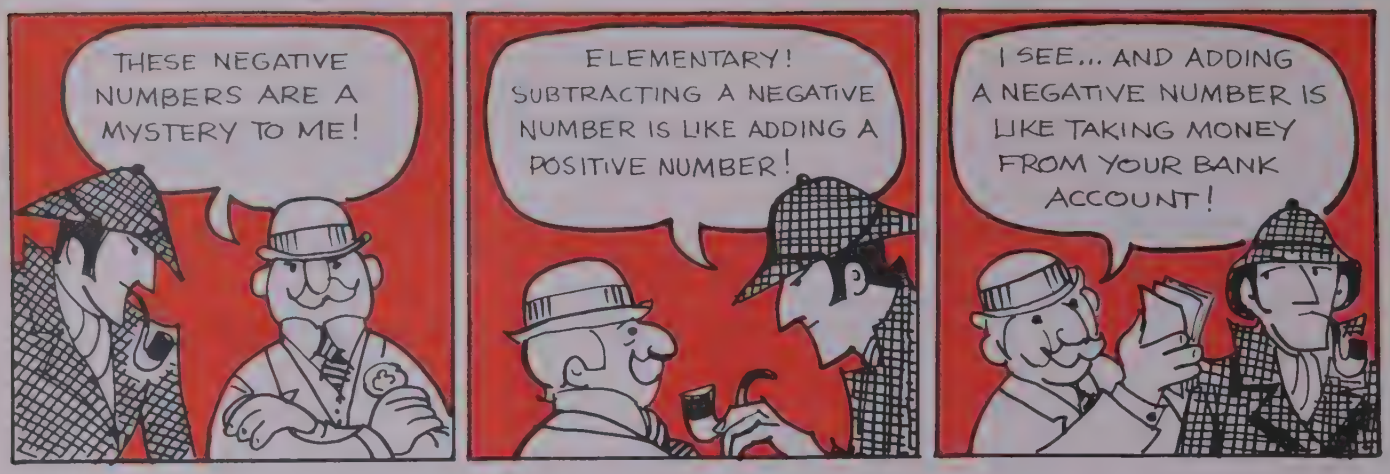
3 + (–8) = –5

Do Questions 5 and 6 in Exercise 2·1.

Try this one with your slide rule before reading further.

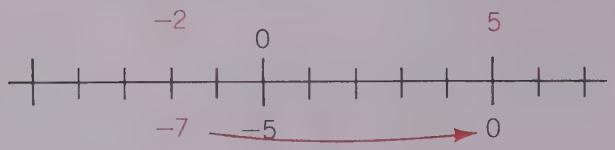
(–3) + (–4)

(–3) + (–4) = –7


THESE NEGATIVE NUMBERS ARE A MYSTERY TO ME!
ELEMENTARY! SUBTRACTING A NEGATIVE NUMBER IS LIKE ADDING A POSITIVE NUMBER!
I SEE... AND ADDING A NEGATIVE NUMBER IS LIKE TAKING MONEY FROM YOUR BANK ACCOUNT!

Do this one. $(-2)-(-7)$.

Is this what you get? $(-2)-(-7)=5$



$(-2)-(-7)=5$

EXERCISE 2·1

Use the slide rule to do Questions 1 to 8.

1. Add:
   (a) $6+5$ (b) $8+3$ (c) $8+7$
   (d) $4+5$ (e) $2+13$ (f) $4+12$

2. Subtract:
   (a) $11-5$ (b) $13-6$ (c) $17-8$
   (d) $14-8$ (e) $18-5$ (f) $15-9$

3. Write a flowchart for subtracting.

4. Subtract:
   (a) $9-14$ (b) $8-17$ (c) $5-18$
   (d) $3-16$ (e) $1-16$ (f) $4-13$

5. Add:
   (a) $5+(-7)$ (b) $13+(-6)$ (c) $4+(-19)$
   (d) $15+(-3)$ (e) $3+(-8)$ (f) $16+(-9)$

6. Write a flowchart for adding numbers like these in Question 5.

7. Add:
   (a) $(-3)+(-7)$ (b) $(-7)+(-6)$
   (c) $(-11)+(-12)$ (d) $(-12)+(-5)$
   (e) $(-8)+(-9)$ (f) $(-4)+(-17)$

8. Subtract:
   (a) $(-3)-(-7)$ (b) $(-4)-(-19)$
   (c) $(-17)-(-9)$ (d) $(-8)-(-3)$
   (e) $(-19)-(-8)$ (f) $0-(-12)$
   (g) $(-12)-(-9)$ (h) $(+9)-(-17)$

9. Calculate (without your slide rule if possible):
   (a) $(+3)+(+9)$ (b) $(-45)-(+30)$
   (c) $(+92)+(+44)$ (d) $(-22)-(-11)$
   (e) $(+59)+(-44)$ (f) $(-63)+(+41)$

10. Write each of the following in a simpler way. The first one is done for you.
    (a) $0-(+3)=-3$ (b) $0-(-46)$
    (c) $0-(-4)$ (d) $0-(-13)$
    (e) $0-(-9)$ (f) $0-(+88)$
    (g) $0+(+4)$ (h) $0+(+18)$
    (i) $0-(-79)$ (j) $0+(-17)$

2·2 All About Integers

Tim saved $18 so he could buy a portable radio he saw in the store. Because $18 was not enough, he borrowed $6 from his sister to purchase this radio. How much did the radio cost Tim? Now Tim had the radio but he was $6 in debt.

Tim kept a record of his financial standing. His record looked like this:

April 12 – Savings		$12
April 21 – Errand for Dad	($1)	$13
May 7 – Worked afternoon at Sam's	($5)	$18
May 15 – Bought radio	($24)	?

Figure 2-4

How do you suppose Tim filled in the blank in his record after he bought the radio?

* * *

One who lives in Canada cannot escape experiences of extreme temperatures. Yvonne, who lives in Winnipeg, enjoys temperatures in the thirties during some summer days and suffers those of forty below zero for some cold winter days.

The Celsius thermometer is the instrument for measuring temperatures. It gives the temperature of pure boiling water under proper pressure conditions at 100 degrees, and the temperature of freezing (pure) water at zero degrees.

In mathematics, man has found a way of designating *being in debt and readings below zero.* We graph the whole number arrangement like this:

Figure 2-5

Relative to point zero, the equally-spaced points to the right are the points of the natural numbers. How far to the right would the numbered points extend? All *natural numbers* belong to the set of positive numbers.

Consider Figure 2-5. Extend the equally-spaced points to the left of zero. We produce a situation similar to the thermometer that shows temperatures *below zero.*

Figure 2-6

The numbers represented by the points to the left of zero are members of the set called negative numbers. Notice in Figure 2-6 how the negative numbers are reflections of the positive numbers about the point zero. What do we mean by *reflection?* Discuss.

Figure 2-6 is a representation of the set of numbers called **integers**. In set notation, the following set of numbers represents the integers:

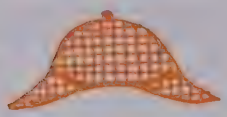
$$I = \{\ldots, -4, -3, -2, -1, 0, +1, +2, +3, \ldots\}$$

Note! In some books, the negative and positive numbers are written in symbols like this: $^{-}3$, $^{+}7$, $^{+}56$, $^{-}29$. In this book, the positive and negative signs will appear lower, like this: $+5$, -8, -33, $+112$. Sometimes we do not place the positive sign in front of the positive numerals. Hence, $+3$ is sometimes written 3.

EXERCISE 2·2

1. What do the two sets of three dots inside the braces mean? $I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
2. Why do we not say, "The whole numbers belong to the set of positive numbers"?
3. In Figure 2-6, each number to the right is always greater than the number to the left. For example:

1 is greater than −4
6 is greater than 5
7 is greater than −1
−2 is greater than −5



Numbers to the right on the number line are greater than those to the left.

Which number of each pair is greater:

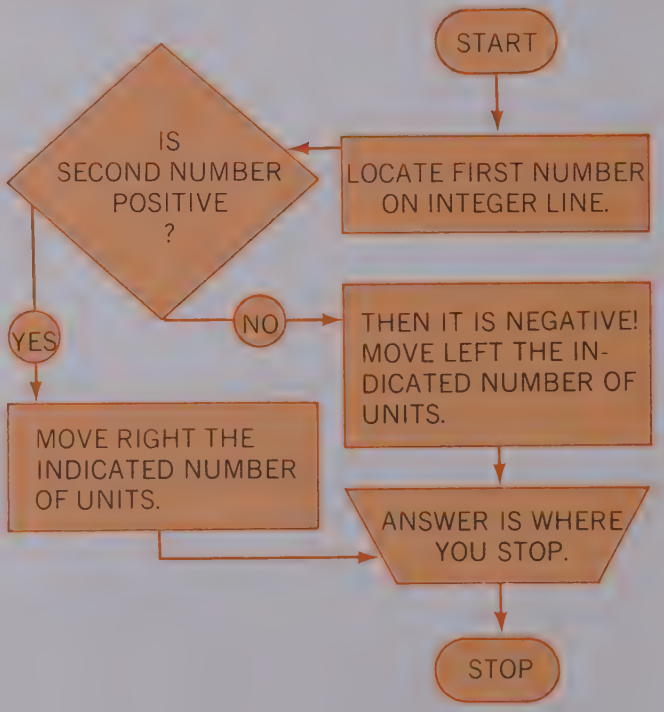
(a) 2 or 0? (b) −3 or +5?
(c) −4 or 1? (d) 0 or −2?
(e) −2 or −1? (f) −12 or −15?
(g) −376 or −2329? (h) −20 or −15?

4. Write each set of numbers in order from smallest to largest.
   (a) {12, −4, 0, 7, −9}
   (b) {−18, −5, −12, −7, −13}
   (c) {150, −70, −140, 20, −200}
5. Describe the numbers missing from W that we require for solving problems such as: $x + 5 = 4$, $7 + x = 3$.
6. Write three number sentences using n that require the missing numbers to solve. Give them to your classmates to solve.
7. How could Tim have filled the blank in his record of Figure 2-4? (Use an integer.)
8. Give examples, other than those given in this section, which require the use of signed numbers (integers).

2·3 Operations in the Set of Integers

Try to work Questions 1 and 2 in Exercise 2·3. If you need help, study the two sections *Addition* and *Subtraction.*

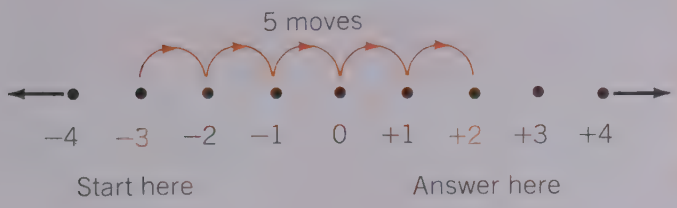
Addition


START
IS SECOND NUMBER POSITIVE ?
LOCATE FIRST NUMBER ON INTEGER LINE.
NO
YES
THEN IT IS NEGATIVE! MOVE LEFT THE INDICATED NUMBER OF UNITS.
MOVE RIGHT THE INDICATED NUMBER OF UNITS.
ANSWER IS WHERE YOU STOP.
STOP

Add:

(a) $(-3)+(+5)$

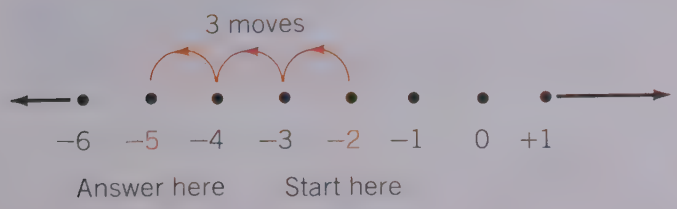
> Adding a positive number is just simple addition.



Therefore, $(-3)+(+5)=+2$.

(b) $(-2)+(-3)$

> Adding a negative number is just like subtracting a positive number.



Therefore, $(-2)+(-3)=-5$.

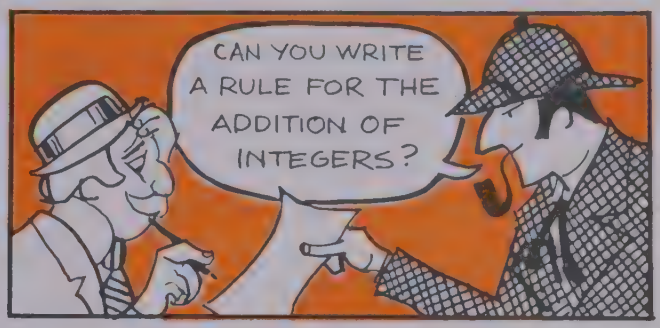
Now add:

(a) $-6+(+4)$ (b) $(+4)+(-7)$ (c) $-5+(-3)$

(d) $+8+(+4)$ (e) $+7+(-2)$ (f) $-2+(-9)$

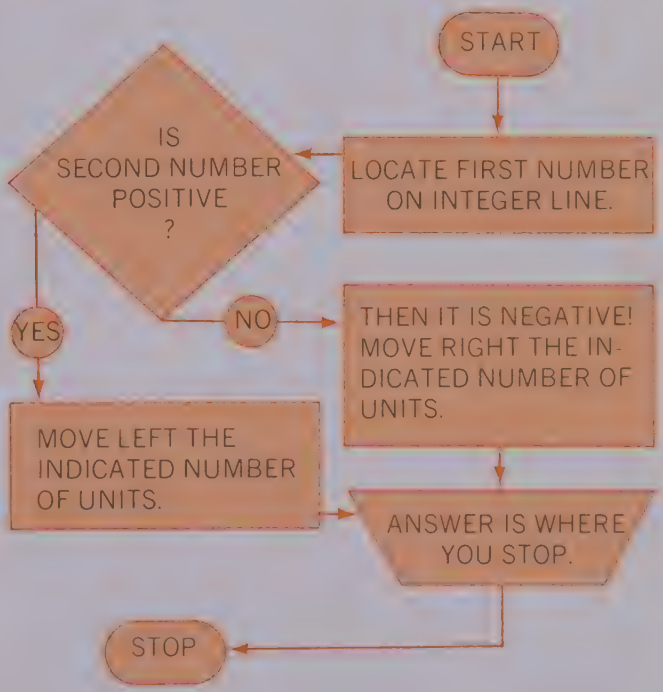
Check your method of adding integers by using these pattern problems. Do you see a pattern?

+2	−2	+2	−2
+5	−5	−5	+5
+9	−9	+9	−9
+4	−4	−4	+4
+13	−13	+13	−13
+26	−26	−26	+26
+67	−67	+67	−67
+23	−23	−23	+23



Do Question 1 in Exercise 2·3.

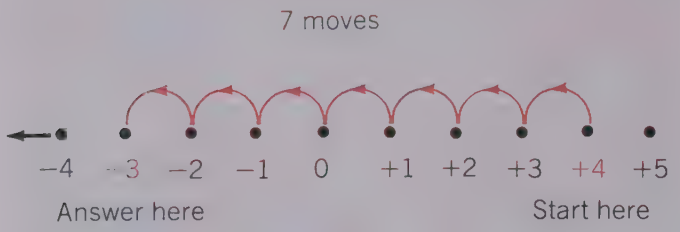
Subtraction


START
LOCATE FIRST NUMBER ON INTEGER LINE.
IS SECOND NUMBER POSITIVE ?
YES
NO
MOVE LEFT THE INDICATED NUMBER OF UNITS.
THEN IT IS NEGATIVE! MOVE RIGHT THE INDICATED NUMBER OF UNITS.
ANSWER IS WHERE YOU STOP.
STOP

Subtract:

(a) $(+4) - (+7)$

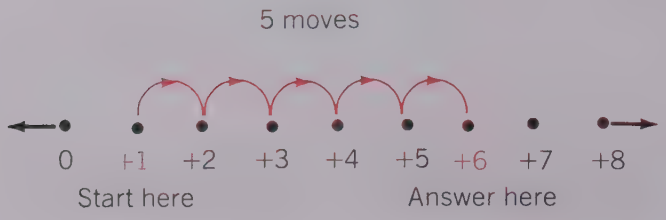
Subtracting a positive number is just like simple subtraction.



Therefore, $(+4) - (+7) = -3$.

(b) $(+1) - (-5)$

Subtracting a negative number is like adding a positive number.



Therefore, $(+1) - (-5) = +6$.

Now subtract:

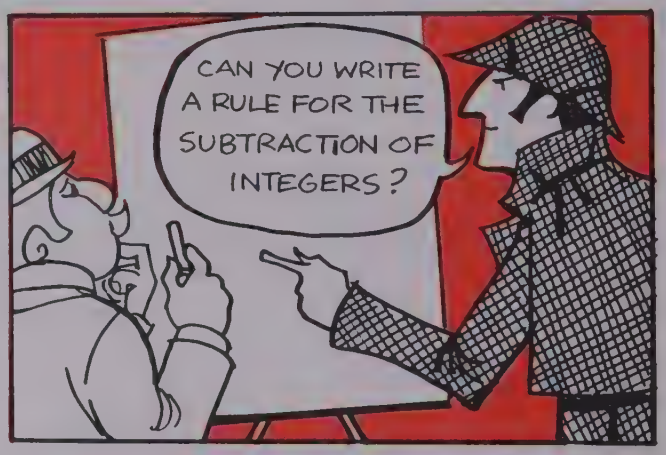
(a) $(-1) - (+6)$ (b) $(-2) - (-6)$ (c) $+7 - (+3)$

(d) $+4 - (-8)$ (e) $-7 - (-4)$ (f) $+9 - (+5)$

What do you observe about the operations of addition and subtraction on the I-line? Would you agree that these operations are inverses of each other; i.e., subtraction is the inverse operation of addition?

Check your method of subtracting integers by using these pattern problems. Do you see a pattern?

$+4$	-4	$+4$	-4
$+3$	-3	-3	$+3$
$+8$	-8	$+8$	-8
$+5$	-5	-5	$+5$
$+18$	-18	$+18$	-18
$+26$	-26	-26	$+26$
$+51$	-51	$+51$	-51
$+38$	-38	-38	$+38$

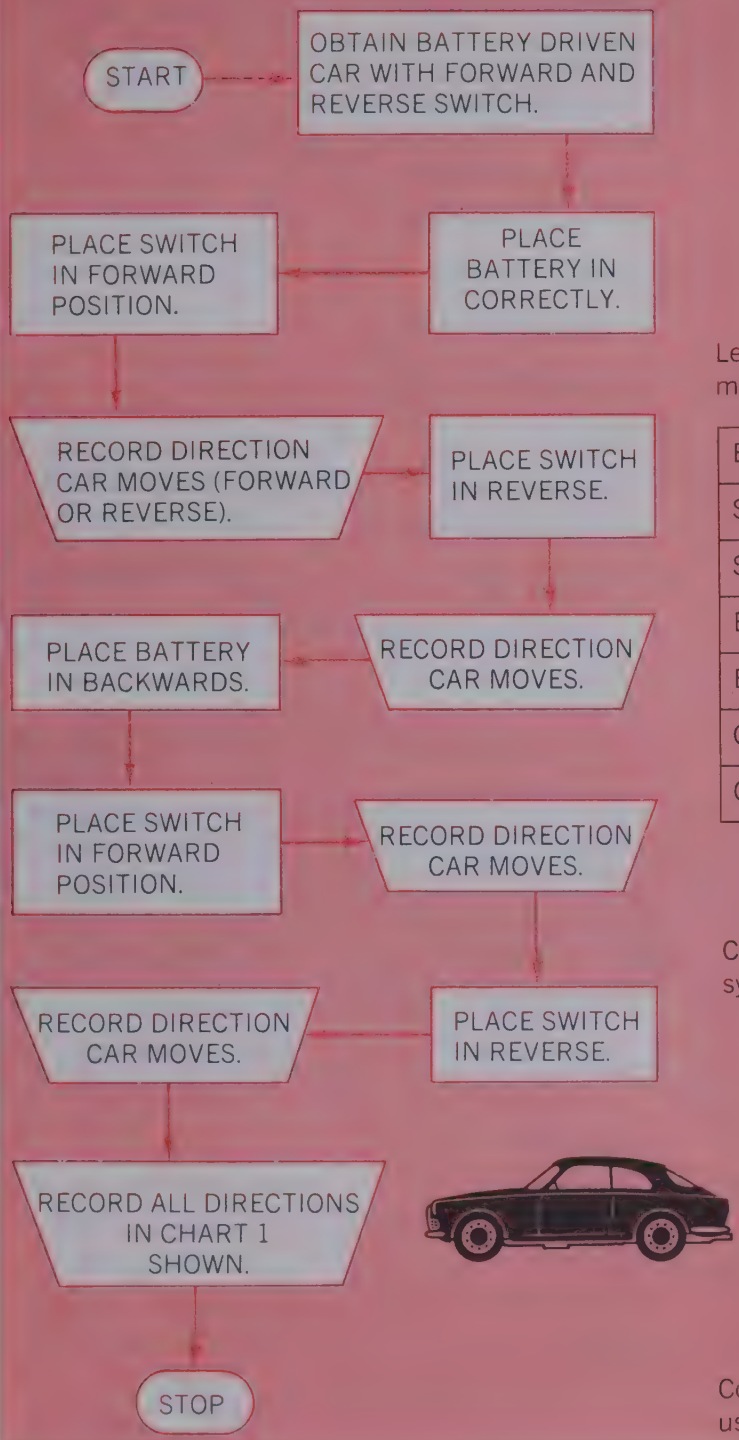


Do Question 2 in Exercise 2·3.

There are about half a million apple trees on 22 000 ha of orchard in the Annapolis Valley of Nova Scotia. How many trees per hectare is this?

Project

Project 2


START
OBTAIN BATTERY DRIVEN CAR WITH FORWARD AND REVERSE SWITCH.
PLACE BATTERY IN CORRECTLY.
PLACE SWITCH IN FORWARD POSITION.
PLACE SWITCH IN REVERSE.
RECORD DIRECTION CAR MOVES (FORWARD OR REVERSE).
RECORD DIRECTION CAR MOVES.
PLACE BATTERY IN BACKWARDS.
PLACE SWITCH IN FORWARD POSITION.
RECORD DIRECTION CAR MOVES.
PLACE SWITCH IN REVERSE.
RECORD DIRECTION CAR MOVES.
RECORD ALL DIRECTIONS IN CHART 1 SHOWN.
STOP

	Battery placed	
Switch in	correctly	backwards
forward		
reverse		

Chart 1

Let us translate the information in the chart into mathematical symbolism.

Experimental data in words		Symbol
Switch in forward position	can be represented by	+
Switch in reverse position		
Battery in correctly		
Battery in backward		
Car goes forward		
Car goes backward		

Chart 2

Copy Chart 2 in your notebook and complete the symbol column.

	+	

Chart 3

Copy Chart 3. It is a replica of Chart 1. Complete it using symbols.

Copy the charts below. Use the information from Chart 3 to complete each.

×	+5	−4
+8		
−9		

×	−7	+6
+12		
−7		

×	+9	−15	−26	−8	+12
−9					
−12					
−17					
+49					

Make up rules for multiplying integers.

Can you devise another project that will illustrate multiplication of "signs"?

(a) $+8 \times n = +40$ (b) $-7 \times n = -42$
(c) $+8 \times n = -32$ (d) $-9 \times n = +36$

Find n in each case.

Use the above information to answer these questions:

(a) $(+40) \div (+8)$ (b) $(-42) \div (-7)$
(c) $(-32) \div (+8)$ (d) $(+36) \div (-9)$

Can you make up a rule for dividing integers?

Summary: Multiplication and Division Rules

To multiply:	Result is:	To divide:	Result is:
$(+8) \times (+5)$	$(+40)$	$(+18) \div (+3)$	$(+6)$
$(+8) \times (-3)$	(-24)	$(-18) \div (+9)$	(-2)
$(-5) \times (-4)$	$(+20)$	$(-33) \div (-3)$	$(+11)$
$(-5) \times (+6)$	(-30)	$(+32) \div (-8)$	(-4)

EXERCISE 2·3

+7	+12	+3	+6	−2	−5	−13	−4
+4	−5	−10	−6	+9	−5	+8	−7

1. Add the above.
2. Subtract the above.
3. Multiply the above.
4. Divide:
   (a) $(+42)$ by $(+7)$ (b) (-25) by $(+5)$
   (c) (-28) by (-7) (d) $(+12)$ by (-4)
   (e) (-18) by (-6) (f) $(+72)$ by (-8)
5. Perform the indicated operations.
   (a) $(+4) + (-17)$ (b) $(+6) \times (-8)$
   (c) $(+12) - (+5)$ (d) $(-42) \div (+6)$
   (e) $(-36) \div (-4)$ (f) $(+8) - (-5)$
   (g) $(+5) \times (+9)$ (h) $(+8) + (+15)$
   (i) $(-14) - (-6)$ (j) $(-12) \times (-3)$
   (k) $(-3) + (-3)$ (l) $(+54) \div (+9)$
   (m) $(+48) \div (-8)$ (n) $(-6) + (+3)$
   (o) $(-7) \times (+5)$ (p) $(-9) - (+12)$
   (q) $(+12) + (-13)$ (r) $(+6) \times (+7)$
   (s) $(-24) \div (-3)$ (t) $(-8) - (-10)$
   (u) $(-7) + (-5)$ (v) $(+4) - (+9)$
6. Subtract:
   (a) $+3 - (+8)$ (b) $+19 - (+47)$
   (c) $+193 - (+497)$ (d) $+3987 - (+875)$
   (e) $+3 - (-9)$ (f) $+27 - (-81)$
   (g) $+111 - (-214)$ (h) $+3894 - (-618)$
   (i) $-8 - (-7)$ (j) $-31 - (-19)$
   (k) $-139 - (-456)$ (l) $-1321 - (-45)$
   (m) $-2 - (+6)$ (n) $-81 - (+61)$

7. Add:

(a) $+9+(+7)$ (b) $+23+(+91)$

(c) $+536+(+117)$ (d) $+1496+(+6311)$

(e) $+7+(-6)$ (f) $+78+(-63)$

(g) $+811+(-914)$ (h) $+1716+(-7641)$

(i) $-1+(-9)$ (j) $-39+(-61)$

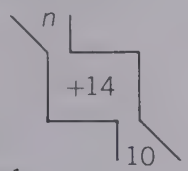
(k) $-391+(-591)$ (l) $-2139+(-7191)$

(m) $-3+(+7)$ (n) $-42+(+38)$

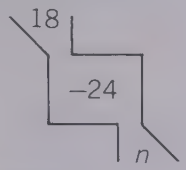
(o) $-158+(+719)$ (p) $-3156+(+2911)$

8. Solve for n using the set I only.

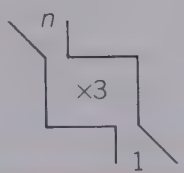
(a) $14+n=10$



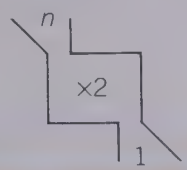
(b) $18-24=n$



(c) $3n=1$



(d) $2n=1$



2·4 Properties of Integers

$I = \{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$

Closure Property

In Section 1·2, we concluded that the whole numbers are closed under two operations and are not closed under the other two operations. For what two operations are whole numbers not closed? Illustrate with two examples.

Now apply the closure property to the system of integers under the operations of addition, subtraction, multiplication, and division. Use many examples (from the integers) to test it. What conclusion do you reach?

Do not read further until you have arrived at the conclusions for the four operations.

Now, do your conclusions agree with the following?

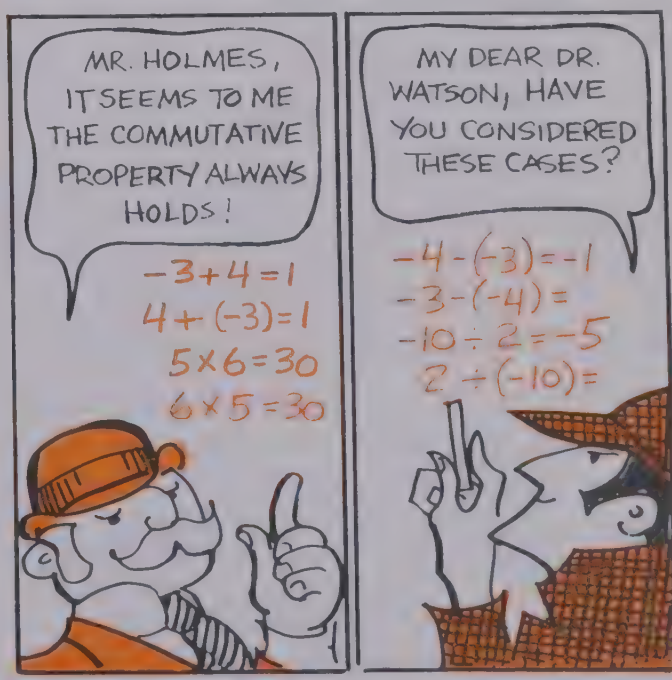
(a) The set of integers is closed under the operations of addition, subtraction, and multiplication.

(b) It is not closed under division.

If you do not agree with these conclusions, discuss them in class.

Commutative Property

The commutative property holds for the set of whole numbers under addition and multiplication only. Does the commutative property in the set of integers also hold true under addition and multiplication? Give examples to illustrate your answer.



Does the commutative property hold under subtraction in the set of integers? Can you think of an example for some integers where the commutative property under subtraction holds? Where it does not hold?

Check the commutative property under division in the set of integers. Does it hold? Are there any integers for which it holds? Are there any where it does not hold? Discuss.

Do Question 1 in Exercise 2·4.

Associative Property

As in Section 1·2, the associative property under either addition or multiplication also holds for the set of integers. If we choose three integers such as +5, –3, and –6, then

$$[(+5)+(-3)]+(-6)=(+2)+(-6)=-4$$

and $$(+5)+[(-3)+(-6)]=(+5)+(-9)=-4.$$

Here we commute and associate differently. Does this change the answer? Try each.

Do the operation in the brackets [ ] first.

$[(-6)+(+5)]+(-3)=?$ Is the result different?

Now observe:

$$[(+5)\times(-3)]\times(-6)=(-15)\times(-6)=+90$$

and $$(+5)\times[(-3)\times(-6)]=(+5)\times(+18)=+90.$$

Again, we commute and associate differently. Does this change the answer?

$$[(-6)\times(+5)]\times(-3)=?$$

How does this result compare?

Choose any three integers. Add the three in four different ways. Are the results always the same?

Multiply the three in four different ways. Are the results always the same?

Use [ ] and ( ) to show the association in each case.

Choose any three integers. Subtract the three in four different ways similar to the way you added them above.

What do you notice?

Try to divide the three in four different ways similar to the way you multiplied them above. What happens?

Identity Element

Refer to Section 1·3. What is the additive identity element for whole numbers? Is this same number the additive identity for the set of integers? How will you test it?

$$(-3)+\underline{?}=-3$$

Similarly, what is the multiplicative identity element for whole numbers? Is this same number the multiplicative identity for the set of integers? Discuss.

$$(-5)\times\underline{?}=-5$$

Using metric units, estimate the heights jumped, the mass of the shot, the distance jumped.

Distributive Property

Refer to Section 1·3 if necessary. Choose any three integers for a, b, and c. Use these integers to make replacements in:

A. $a(b+c)=ab+ac$

B. $a(b-c)=ab-ac$

Do both sentences remain true after the replacements? Choose another set of three integers for a, b, and c. Make replacements in sentences A and B. Are the sentences still true?

Try another set of three replacements. What is your conclusion? Discuss with your classmates and your teacher.

EXERCISE 2·4

1. What property is illustrated by each example?
   (a) $3+12=12+3$
   (b) $4(a-7)=4a-28$
   (c) $(-5)+(+2)=-3$
   (d) $(2+9)+6=2+(9+6)$
   (e) $(-11)+0=-11$
   (f) $-7(x-3)=-7x+21$
   (g) $(4\times 3)\times(-6)=4\times[3\times(-6)]$
   (h) $5\times 1=5$
   (i) $(-7)\times(+5)=(+5)\times(-7)$
   (j) $9\times(-3)=-27$
   (k) $15+0=15$
   (l) $(3+5)+7=3+(5+7)$
   (m) $2(n-1)=2n-2$
   (n) $(-7)\times 1=-7$
   (o) $3+(-8)=(-8)+3$

2. Multiply:
   (a) $3\times(-5)$ (b) $42\times(-3)$
   (c) $56\times(-27)$ (d) $133\times(-112)$
   (e) $(-7)(-9)$ (f) $(-83)(-7)$
   (g) $(-95)(-36)$ (h) $(-279)(-123)$
   (i) $(-6)(+4)$ (j) -76×8

What are the metric units of land area? What are the sizes of a small farm, a large prairie wheat farm, and a large ranch?

Refer to the Appendix if you are not familiar with or are having difficulty with metric units.

3. Divide:
   (a) $42\div(-7)$ (b) $112\div(-8)$
   (c) $276\div(-12)$ (d) $1440\div(-32)$
   (e) $-36\div 4$ (f) $-378\div 9$
   (g) $-616\div 11$ (h) $-2752\div 43$
   (i) $-64\div(-8)$ (j) $-168\div(-6)$
   (k) $-420\div(-15)$ (l) $-2211\div(-33)$

4. Make replacements from *I* as indicated in each sentence. Which sentences are true? Write correctly those sentences which are not true.
   (a) $a(b+c)=ab+ac$
   $a=-3, b=+4, c=-5$
   (b) $m(n+p)=mn+mp$
   $m=+9, n=-7, p=-13$
   (c) $r(s-t)=rs+rt$
   $r=+12, s=-8, t=-9$
   (d) $x(y+z)=xy+xz$
   $x=23, y=14, z=36$
   (e) $k(m-n)=km+mn$
   $k=-17, m=-49, n=-56$

For Questions 5, 6, and 7, consider the set *I*.

5. For what two operations does the commutative property not hold?

6. For how many operations does the associative property hold? Which ones are they?

7. (a) What is the additive identity element?

   (b) What is the multiplicative identity element?

8. What two integers yield a sum of zero? Find another two which add to zero; and another two. How many such pairs of integers are there? When two integers add up to give the identity element, these integers are called additive inverses of each other. How would you describe two numbers whose sum is zero?

9. Can two integers multiplied together give a product of 1? How many such pairs of integers are there?

10. Use the distributive property to expand each. The first is done for you.

    (a) $2(3x - 5) = 6x - 10$

    (b) $4(-4x + 2y)$

    (c) $-3(2x - 3y)$

    (d) $-8(-6x - 6y)$

    (e) $-7(3y - 4x - 8z)$

    (f) $-9(16y - 12z)$

11. Evaluate each when $x = -7, y = +8$.

    (a) $-3(4x - 2y)$

    (b) $6(7y - 2x) - 3y$

    (c) $-5(2x - 3y) + 4x$

    (d) $-9(-3x + 4y) + 5x$

    (e) $-11(-4x - 5y) + 10x$

Project

Project 3

In your notebook, draw the pattern shown. How many different routes are there from *A* to *B*? Remember you can only go the direction indicated by the arrows.

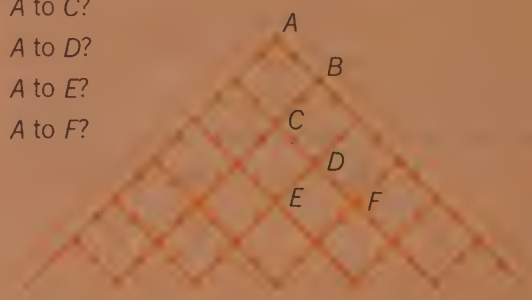
How many different routes are there from:

(a) *A* to *C*?

(b) *A* to *D*?

(c) *A* to *E*?

(d) *A* to *F*?

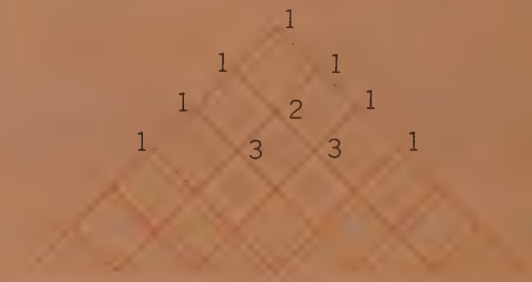


Draw this second chart in your book.

Record the number of routes from *A* to each intersection. Intersections *B*, *C*, and *G* have been done for you.

What number patterns do you see? Discuss each pattern presented by classmates. Which do you like best? Why?

Copy this large diagram. Using the same rules and the pattern you like best from those discussed earlier, complete this chart.



2·5 Evaluating Algebraic Expressions (in I)

Often we are required to find the value of an expression or formula when certain variables are given definite values.

Example 1

Find the value of:

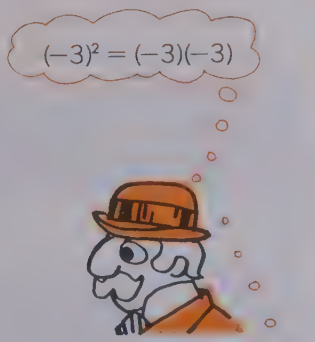
(a) $-2a^2$ (b) $5a^3$

when $a = -3$.

Solution

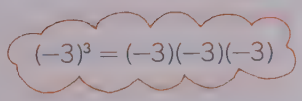
(a) Replacing a with (-3):

$$\begin{aligned} -2a^2 &= -2(-3)^2 \\ &= -2(-3)(-3) \\ &= -2(+9) \\ &= -18 \end{aligned}$$



(b) Replacing a with (-3):

$$\begin{aligned} 5a^3 &= 5(-3)^3 \\ &= 5(-3)(-3)(-3) \\ &= 5(-3)(+9) \\ &= 5(-27) \\ &= -135 \end{aligned}$$



Example 2

Evaluate $7x - 3y$ when:

(a) $x = +2$ and $y = +5$;

(b) $x = -4$ and $y = -10$.

Solution

(a) $$\begin{aligned} 7x - 3y &= 7(+2) - 3(+5) \\ &= 14 - 15 \\ &= -1 \end{aligned}$$

(b) $$\begin{aligned} 7x - 3y &= 7(-4) - 3(-10) \\ &= -28 + 30 \\ &= +2 \end{aligned}$$

Example 3

The price (p) in cents per plate to cater a small banquet is given by the caterers as

$$p = 550 - 5x$$

where x is the number of guests over 50.

Calculate the per plate price if there are:

(a) 75 guests (b) 35 guests

Solution

(a) If there are 75 guests, then x is 25.

$$\begin{aligned} p &= 550 - 5x \\ p &= 550 - 5(25) \qquad \text{(Put 25 in place of } x\text{.)} \\ p &= 550 - 125 \\ p &= 425 \end{aligned}$$

The price per plate is \$4.25.

(b) If there are 35 guests, then $x = -15$. (Follow?)

$$\begin{aligned} p &= 550 - 5x \\ p &= 550 - 5(-15) \qquad \text{(Put } -15 \text{ in place of } x\text{.)} \\ p &= 550 + 75 \\ p &= 625 \end{aligned}$$

The price per plate is \$6.25.

EXERCISE 2·5

1. For $m = +3$, find the values of:
   (a) $7m$ (b) m^2
   (c) $2m^3$ (d) $-6m + 4m^2$
   (e) $5m - 6m^2$ (f) $8m + m^3$
   (g) $m^2 - 4m$ (h) $m^3 + 2m$
   (i) $-2m^2 - 7m$ (j) $3m^3 - 4m^2$

2. For $s = -5$, find the values of:
   (a) $9s$ (b) $3s^2$
   (c) $s^2 - s + 1$ (d) $4(s^2 - 3s)$
   (e) $3(s - s^2)$ (f) $4s^2 - 2s$
   (g) $-5s^3$ (h) $-5s^2$
   (i) $3s - 5s^2$ (j) $-4s + 1$
   (k) $4 - 6s^2$ (l) $-15 - 6s^2$

3. For $x = +4$ and $y = -7$, evaluate each of the following:
   (a) $x + y$ (b) $2y - 3x$ (c) $2x - y$
   (d) $-3y - 2x$ (e) $x^2 - y^2$ (f) $y^2 - x^2$
   (g) $3x + 2y$ (h) $3(x - y)$ (i) $\dfrac{5x - 4y}{3x}$
   (j) $x^2 + y^2$ (k) $x - y$ (l) $2x - 4y$
   (m) $5x + 3y$ (n) $8x - 9y$ (o) $2x^2 + 4y$

4. For $t = -4$, evaluate:
   (a) t^3 (b) t^2 (c) $2t^2 - t$
   (d) $3t^2 - 2t^3$ (e) $t^2 - 2t$
   (f) $3t - 4t^2$ (g) $3t^2 - 5t + 7$
   (h) $t^2 - 2t(7 + t)$ (i) $t^3 + t(t^2 - 3)$

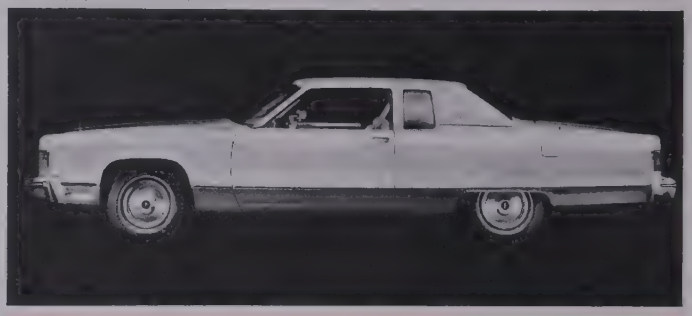
5. For $a = -2$ and $b = 5$, evaluate:
   (a) $\dfrac{7a - 2b}{-8}$ (b) $\dfrac{5a^2 + 3b}{7b}$ (c) $\dfrac{2a(b^2 - 5a)}{ab}$

6. The price (p) in cents to serve a banquet to n guests is given by $p = 625 + 375n$.
   How much would it cost to serve:
   (a) 40 guests? (b) 100 guests?

7. The formula for the perimeter of a rectangle is $p = 2\ell + 2w$. Find the perimeter of the rectangles when all measures are in metres.
   (a) $\ell = 5, w = 4$
   (b) $\ell = 12, w = 8$
   (c) $\ell = 13, w = 9$
   (d) $\ell = 23, w = 14$
   (e) $\ell = 36, w = 24$

8. The formula for the area of a rectangle is $A = \ell w$. Find the area of the rectangle when all measures are in metres.
   (a) $\ell = 9, w = 7$
   (b) $\ell = 26, w = 11$
   (c) $\ell = 39, w = 4$
   (d) $\ell = 25, w = 25$
   (e) $\ell = 50, w = 50$

9. The formula for the distance (d) travelled by a car when the speed (s) is constant is $d = st$ when t is the time. In this question, d is in kilometres, s is in kilometres per hour, and t is in hours. Find d when:
   (a) $s = 90, t = 3$
   (b) $s = 75, t = 8$
   (c) $s = 115, t = 9$
   (d) $s = 85, t = 11$
   (e) $s = 65, t = 15$

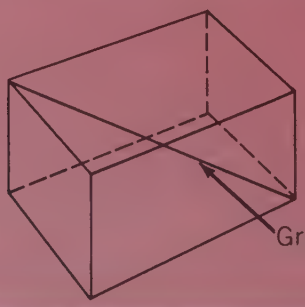


Project

Project 4

Obtain a closed box or rectangular block about 10 cm × 15 cm × 20 cm and a ruler.

Working on a table, find the measure of the *great diagonal* of the box as shown in the diagram. Explain your solution.



2·6 Equalities and Inequalities (with Integers)

The set of integers (I) is sometimes expressed as:

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

If n is an integer, we write $n \in I$. (Read: "n is an element of the set of integers.")

Example 1

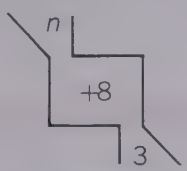
For $n \in I$, solve:

(a) $n + 8 = 3$ (b) $4n = -28$

Solution

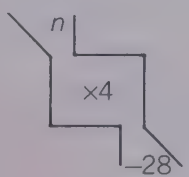
(a) Ask ourselves: What number added to 8 gives 3?

Answer: -5; i.e., $-5 + 8 = 3$. Therefore, $n = -5$.



(b) Ask ourselves: What number multiplied by 4 yields -28?

Answer: -7; i.e., $4(-7) = -28$. Therefore, $n = -7$.



Example 2

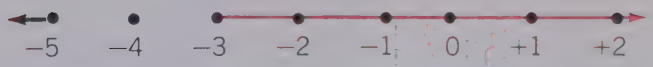
For $x \in I$, express the answer in set notation for each of the following inequalities:

(a) $x \leq 2$ (b) $x + 5 > 1$ (c) $5x > 19$

Solution

(a) $x \in \{\ldots, -3, -2, -1, 0, +1, +2\}$

(b) If $x + 5 = 1$,
then $x = -4$.
Because $x + 5 > 5$,
then $x > -4$.
Therefore, $x \in \{-3, -2, -1, 0, +1, \ldots\}$.



(c) If $x = 3$, then $5x = 15$.
Because $5x > 19$, x cannot be 3.
If $x = 4$, then $5x = 20$.
This satisfies the inequality, $5x > 19$.
Does $x = 5$ satisfy the inequality?
Does $x = 6$? $x = 7$?
$x \in \{4, 5, 6, 7, \ldots\}$

EXERCISE 2·6

1. Complete each sentence by rewriting each as shown in (a).
   (a) $5 - 2 = -2 + 5$ (b) $-5 + 3 =$ ■
   (c) $4 - (-3) =$ ■ (d) $6 + (-4) =$ ■
   (e) $3 + 2n =$ ■ (f) $3 - 2n =$ ■
   (g) $8 + (-5n) =$ ■ (h) $9 - (-3n) =$ ■

2. For $n \in I$, solve:
   (a) $n + 12 = 9$ (b) $10 + n = 4$
   (c) $n - 7 = 7$ (d) $n - 12 = -6$
   (e) $5 - n = 2$ (f) $13 - n = 7$
   (g) $9 - n = 12$ (h) $15 - n = 16$

3. For $x \in I$, solve:
   (a) $3x = 33$ (b) $12x = 60$
   (c) $-4x = 28$ (d) $-7x = -35$
   (e) $x - 2 = 18$ (f) $x + 2 = -7$
   (g) $5x - 0 = -15$ (h) $6 + x = 1$
   (i) $2x + 9 = 9$ (j) $8 + 3x = 8$

4. For $y \in I$, solve:

(a) $2y \geq 0$ (b) $-3y > -3$

(c) $5y < 10$ (d) $2y \leq 4$

(e) $2y \geq -4$ (f) $7y \geq -7$

(g) $y + 1 < 5$ (h) $y - 1 > 2$

(i) $2y - 3 > 5$ (j) $2y + 3 < 7$

(k) $4 - y > 0$ (l) $5 - y \geq 3$

(m) $6 - y < -1$

5. If $x \in \{-3, -2, -1, 0, 1, 2, 3\}$, find the values of x in each of:

(a) $3x - 2 \geq -5$ (b) $2x \leq 6$

(c) $1 - x > 0$ (d) $3 - x < 1$

(e) $5 + x < -1$ (f) $4x + 1 \leq 9$

6. In the years between the beginning of 1946 and the end of 1970, Canada admitted immigrants from all parts of the world. The average of these two numbers represents approximately the number of immigrants;

$$2^7 \times 3^5 \times 5^3 \text{ and } 2^5 \times 3^6 \times 5^3.$$

Remember! $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

(a) Calculate the approximate number of immigrants.

(b) Calculate the average number of immigrants per year.

7. The peak years of this period were 1957 when 282 164 persons arrived and in 1967 when 222 876 persons were admitted. How much is the average of these two years above the average in Question 6?

8. For $t \in I$, can there be a solution to:

(a) $3t = 10$? (b) $2t - 5 = 4$?

Discuss.

2·7 Graphing of Solution Sets

Let us take the set $\{-2, -1, 0, 1\}$ and show that set on the integer line. We will represent each integer point by an open dot. To represent the above set on the I-line, we fill in each corresponding open dot.

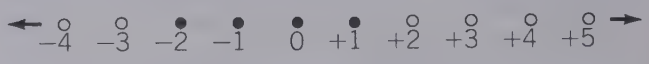
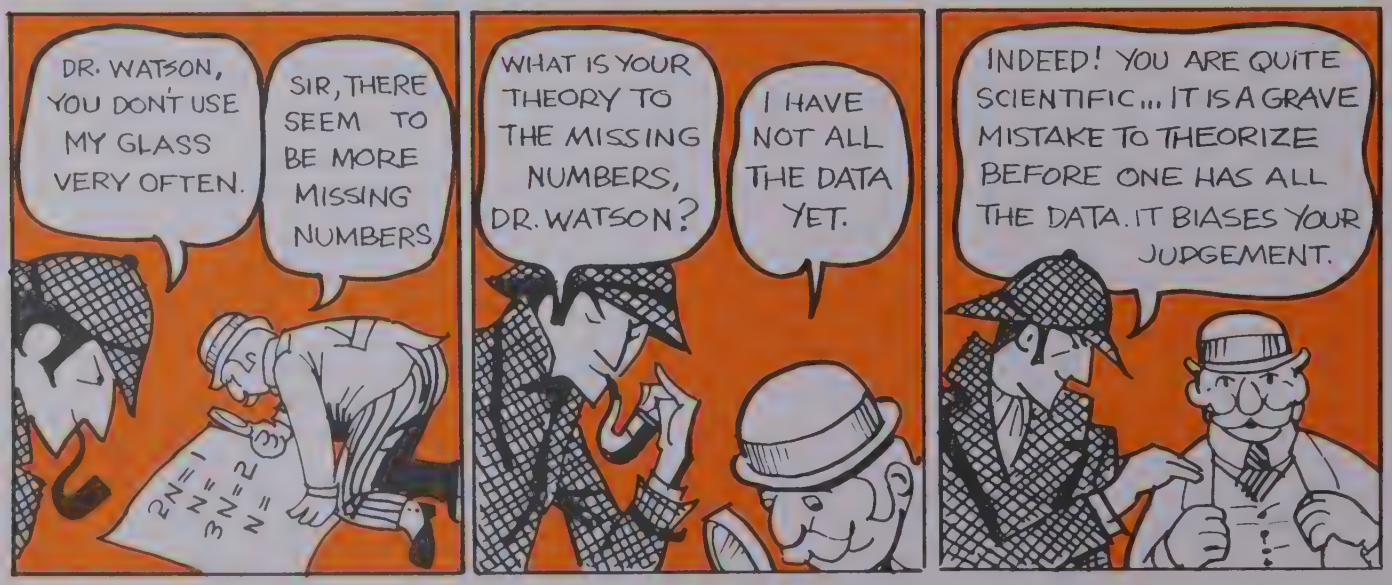


Figure 2-7

Figure 2-7 is a graphical representation of the set $\{-2, -1, 0, 1\}$. (Notice that the dots are evenly spaced.)


DR. WATSON, YOU DON'T USE MY GLASS VERY OFTEN.
SIR, THERE SEEM TO BE MORE MISSING NUMBERS
WHAT IS YOUR THEORY TO THE MISSING NUMBERS, DR. WATSON?
I HAVE NOT ALL THE DATA YET.
INDEED! YOU ARE QUITE SCIENTIFIC... IT IS A GRAVE MISTAKE TO THEORIZE BEFORE ONE HAS ALL THE DATA. IT BIASES YOUR JUDGEMENT.

Example 1

Graph the solution set for $2n - 1 > 5$, where $n \in I$.

Solution

If $2n - 1 = 5$

then $2n = 6$

and $n = 3$

Now test for $n < 3$ and $n > 3$ in

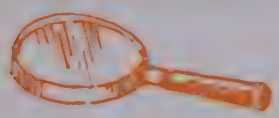
$$2n - 1 > 5.$$

Replace n with a number less than 3 – use 2.

$$2(2) - 1 < 5$$

Therefore $n \not< 3$.

$\not<$ means "is not less than."



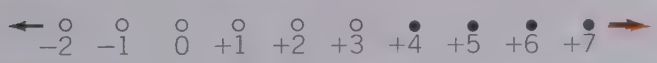
$$2n - 1 > 5$$

Now replace n with a number greater than 3 – use 4.

$$2(4) - 1 > 5$$

Therefore, $n > 3$.

For $2n - 1 > 5$, the graph of the solution set is:



Example 2

Graph the solution set for $2 + 3x \leq 11$, where $x \in I$.

Solution

When $2 + 3x = 11$

$3x = +9$

and $x = +3$

We know that $x = 3$ satisfies the condition

$$2 + 3x = 11.$$

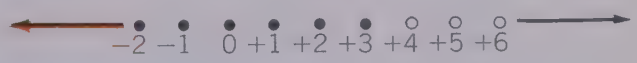
But what values of x satisfy $2 + 3x < 11$?

Mountain or rock climbing is an exciting hobby. Rappelling is one of the aspects of mountain climbing. How high do you think a climber might be when rappelling?

Let's try the integers greater than 3; e.g., $x = 4$. Does this work?

Let's try the numbers less than 3; e.g., $x = 2$. Does this work? Which integers do we want – those greater than or those less than +3?

For $2 + 3x \leq 11$, $x \leq 3$.



EXERCISE 2·7

Graph the solution set for each of the following:

(a) $3x > -15$ $x \in I$.

(b) $2m - 3 < 3$ $m \in I$.

(c) $4 - y \geq 1$ $y \in I$.

(d) $8t - 5 \leq -5$ $t \in I$.

(e) $3 + 4x < 15$ $x \in I$.

If you need more practice, graph the solution sets of Question 4 in Exercise 2·6.

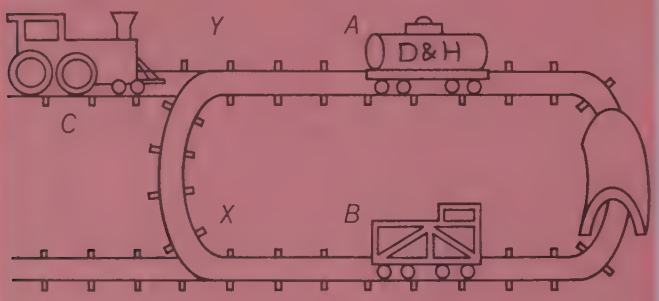
Project

Project 5

The map below shows a section of a railroad. The tunnel is not wide enough for the boxcars but will take the engine.

Each section of track from *X* and *Y* to the tunnel is long enough to accommodate the two cars and the engine.

Each end of the engine and cars can be used for pushing or pulling.



Challenge:

(1) Switch car *A* to position *B* and car *B* to position *A*.

(2) Switch car *A* to position *B* and car *B* to position *A* *and* return engine to its original place *C*.

Copy the plan on a sheet of paper—the size shown. Place a penny at *C* to represent the engine and nickels at *A* and *B* to represent the cars.

Devise a scheme requiring the fewest operations. No flying switches!

2·8 Solving Verbal Problems

Often we are faced with solving a word problem where no equation or inequation is given. It is left to us to formulate the number sentence for each problem. This is usually far more critical than one expects. Therefore, a good training in translating verbal sentences into number sentences is basic.

Let us study the following examples.

Example 1

When 13 is added to a certain number, the sum is 20. Find the number.

Solution

Let n represent the required number.

For, "When 13 is added to a certain number", we then write:

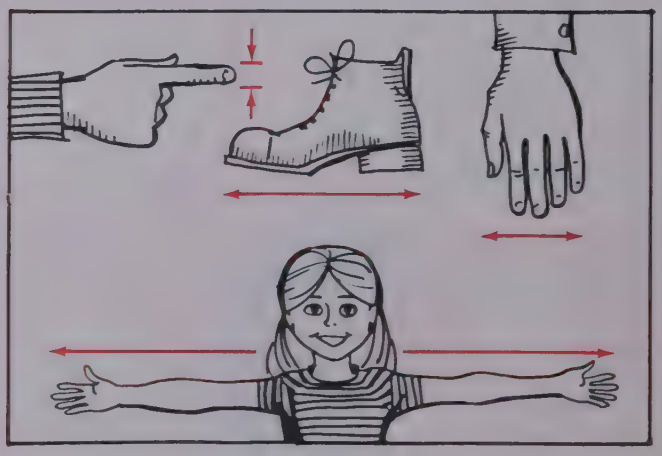
$$n + 13.$$

Then the translated number sentence is:

When 13 is added to a certain number, the sum is 20.

$$\begin{array}{r} \downarrow\ \downarrow \\ n + 13 = 20 \\ n = 7. \end{array}$$

The required number is 7.

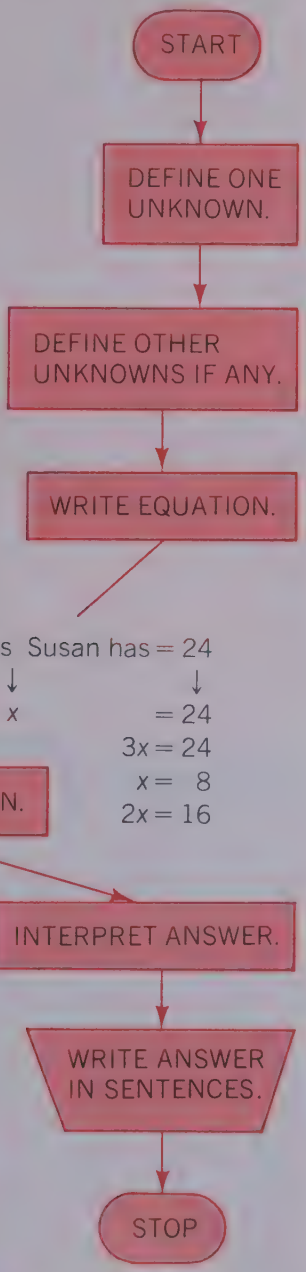


Approximately what are the metric measures for you?

Example 2

Sandy has twice as many marbles as Susan. Together they have 24 marbles. How many marbles does each have?

Solution



Let x represent the number of marbles Susan has.

Then Sandy has $2x$ marbles since she has twice as many as Susan.

The translated number sentence is then written:

marbles Sandy has + marbles Susan has = 24

$2x + x = 24$

$3x = 24$

$x = 8$

$2x = 16$

Susan has 8 marbles.

Sandy has 16 marbles.

EXERCISE 2·8

1. Write each of the following as a number sentence. Use n for the required number.
   (a) If 4 is added to a number, the sum is 7.
   (b) If a number is tripled, it becomes 18.
   (c) 14 is 5 less than a certain number.
   (d) Four times a given number is greater than 10.
   (e) When 3 is added to a certain number, the sum is less than 10.
   (f) When 5 is subtracted from a certain number, the result is 6.
   (g) When 3 is subtracted from a certain number, the result is equal to, or less than, 8.
   (h) 17 exceeds a certain number by 10.
   (i) A number decreased by 4 results in 2.
   (j) The perimeter of an equilateral triangle is 15.
   (k) In a rectangle, the length is 2 units longer than the width. Express the perimeter P. Express the area A. *Hint!* Sketch diagram and label it.
   (l) Jim and Jack have the same amount of money. Bill has 4 dollars. The three boys together have 10 dollars.

For each of the following word problems, write out a number sentence and then solve it.

2. If 8 is added to a certain number, the number is 15. Find the number.
3. 17 is 9 less than a certain number. Find the number.
4. When 12 is subtracted from a given number, the result is 7. Find the number.
5. If 17 exceeds a number by 10, find the number.
6. A number decreased by 5 is 9. Find the number.
7. The sides of a triangle are $3x$, $4x$, and $6x$. The perimeter is 39 cm. Calculate the length of each side.

8. Billy is 3 years older than his sister Mary. If their ages total 25, find their ages.

9. A 160 ha farm has a portion sown to wheat, another to rye, and the remaining portion to flax. The wheat portion is 9 times as large as the flax and the rye portion is 6 times as large as the flax. How large are the portions?

10. The sum of two numbers is 22. One number is 4 less than the other. Find the two numbers.

11. A rope is 16 m long. It is cut into two pieces so that one piece is 7 times as long as the other. What are the lengths of the pieces?

12. When a certain number is tripled and 7 is added, the result is 22. Find the number.

REVIEW EXERCISE 2

1. Arrange the numbers in these sets from smallest to largest:

   (a) $\{7, -1, 5, -3, 2, 0, -7\}$.

   (b) $\{-12, -17, -9, -14, -6, 2, -3\}$.

2. Perform the indicated operations.

   (a) $(+3) - (+5)$ (b) $(-2) - (+7)$

   (c) $(-7) + (+4)$ (d) $(+8) + (-4)$

   (e) $(+5) + (-8)$ (f) $(-6) - (-5)$

   (g) $(-12) - (+5)$ (h) $(+7) - (-4)$

   (i) $(+13) - (+9)$ (j) $(+3) - (+8)$

   (k) $(-6) + (+15)$ (l) $(-7) - (-7)$

3. If $2m = 7r - 5$, find m when $r = 3$.

4. Find the value of each when $n = -7$.

   (a) $n^2 - 3n - 25$ (b) $2n(9 - n)$

   (c) $\frac{3n - 5}{8}$ (d) $\frac{4n^2 - 292}{3 - 3n}$

5. For $x \in I$, solve:

   (a) $13 - x = 2$ (b) $1 - x = 6$

   (c) $5x - 4 = 16$ (d) $3x + 7 = 13$

6. For $y \in I$, solve and represent by a graph:

   (a) $2y - 3 \leq 3$ (b) $8 + 3y > -4$

7. Translate into number sentences and solve:

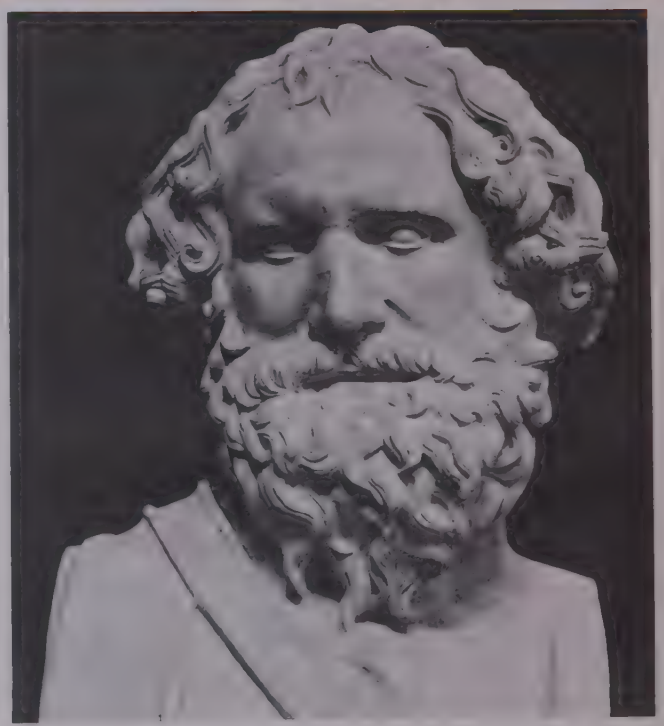
   (a) May has 7 books less than Jean. If the girls have a total of 17 books, how many has each?

   (b) 28 is 9 less than a certain number. Find the number.

8. For each of the following integers, give its additive inverse.

   (a) +5 (b) +23 (c) −1 (d) −12 (e) −57 (f) +261

Using the Library



Do You Remember . . .

. . . What the L.C.M. (lowest common multiple) of two numbers means? Let's take the numbers 4 and 6.

What is the smallest number that both 4 and 6 will divide into evenly? To find it, follow these steps:

$4 = 2 \times 2$
$6 = 2 \times 3$

(Factor each into its prime factors and place as shown.)

$2 \times 2 \times 3$ (Choose one factor from each column.)

$= 12$ (Find their product.)

12 is a multiple of 4 ($4 \times 3 = 12$).

12 is a multiple of 6 ($6 \times 2 = 12$).

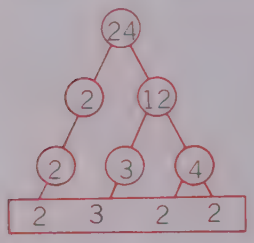
12 is the L.C.M. of 4 and 6.

Use this method to find the L.C.M. of:

(a) 6, 10 (b) 6, 9 (c) 12, 16
(d) 18, 24 (e) 18, 45 (f) 36, 60
(g) 45, 36, 60 (h) 40, 24, 60 (i) 42, 210, 660

Do You Remember . . .

. . . what a prime number is? How many prime numbers are there between 1 and 100?



24 is not prime since it has factors.
19 is prime since it has no factors.

Compute a factor tree for each of the following. Which are prime numbers?

(a) 119 (b) 169 (c) 173 (d) 221 (e) 347 (f) 401

Something to Think About

The object of this puzzle is to use the same digit, four times, with any of the four operations to express a result. Division is to be shown as a fraction. Observe these two examples.

17 can be expressed by using four fours: $4 \times 4 + \frac{4}{4}$.

98 can be expressed by using four nines: $99 - \frac{9}{9}$.

How would you use:

1. four twos to give 4?
2. four threes to give 5?
3. four fours to give 7?
4. four fives to give 11?
5. four sixes to give 24?
6. four sevens to give 1?
7. four eights to give 16?
8. four nines to give 81?
9. four twos to give 10?
10. four threes to give 34?
11. four fours to give 44?
12. four fives to give 6?
13. four sixes to give 102?
14. four sevens to give 63?
15. four eights to give 88?
16. four nines to give 1?

How many of the counting numbers less than 50 can you make using four 4's?

Have You Discovered that . . .

. . . there is a pyramid of numbers called Pascal's Triangle? It looks like this:

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5   10  10  5   1
```

Take a close look at this pyramid of numbers and try to find the pattern. Do you think you can write the next three lines?

Note:

$$\begin{aligned} 1 &= 1 \\ 1 + 1 &= 2 \\ 1 + 2 + 1 &= 4 \\ 1 + 3 + 3 + 1 &= 8 \\ 1 + 4 + 6 + 4 + 1 &= 16 \end{aligned}$$

Obviously, the sums of the lines form a pattern. What would the next five numbers of this sequence of sums be? Test them out on the sum of the lines in the triangle that you extended.

Chapter Three
THE PLOT THICKENS

Rational Numbers

YOUR SUGGESTION, SIR, IS A MOST ABSURD ONE!
I SPOKE WITHOUT KNOWING ALL THE FACTS.
YOU MAY BE SURE SIR, THAT I HAVE VERY GOOD GROUNDS BEFORE I COME TO AN OPINION.

3·1 Some Missing Numbers are Found

Sally is making a bookcase for her bedroom. She has a plank 4 m long which she wants to saw into three sections of equal length. How long should she make each section? Working in the set *I*, Sally thinks:

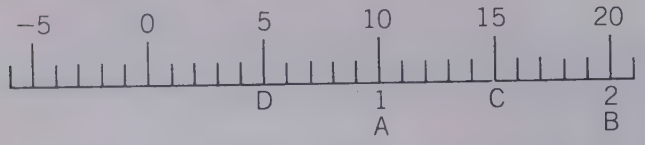
$3 \times x = 4$

$3 \times 1 = 3$ and $3 \times 2 = 6$

x must be a number between 1 and 2.

"There must be a number missing!"

Let us examine the addition-subtraction slide rule you made in Chapter 2.



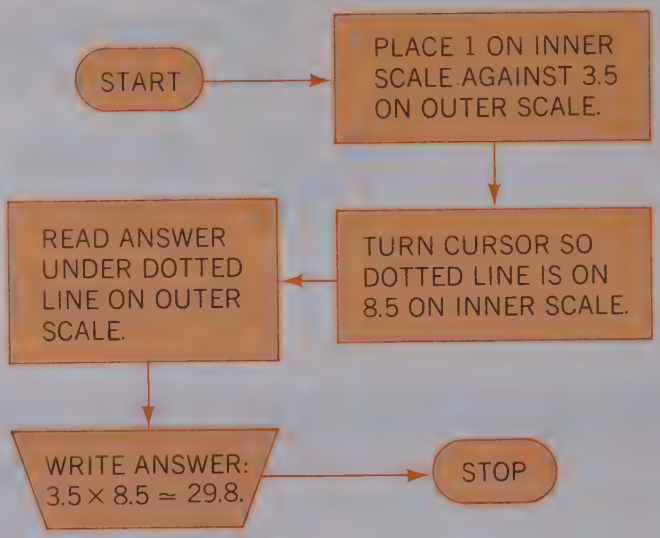
We can consider point *A* as 1, 10, 100, etc., and point *B*, accordingly, would be 2, 20, 200, etc.

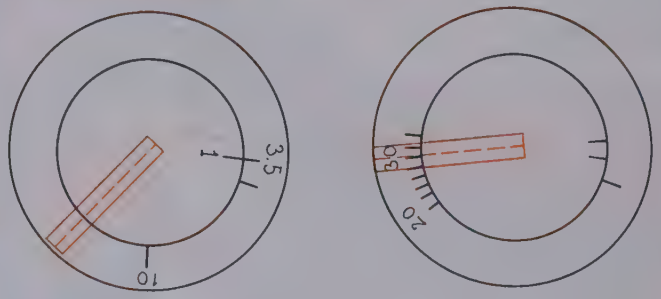
If *A* were 1 and *B* were 2, what would points *D* and *C* represent?

If *A* were 100 and *B* were 200, what would points *D* and *C* represent? Discuss.

Do Question 1 in Exercise 3·1.

Use your circular slide rule to multiply 3.5×8.5.


START
PLACE 1 ON INNER SCALE AGAINST 3.5 ON OUTER SCALE.
TURN CURSOR SO DOTTED LINE IS ON 8.5 ON INNER SCALE.
READ ANSWER UNDER DOTTED LINE ON OUTER SCALE.
WRITE ANSWER: 3.5 × 8.5 ≈ 29.8.
STOP

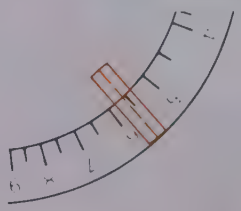


EXERCISE 3·1

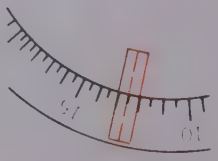
1. Work in pairs. Use your addition slide rule to solve the following. Compare your answers.
   (a) $1.5 + 10.6$ (b) $0.8 + 1.3$ (c) $40 + 10$
   (d) $35 + 125$ (e) $170 + 95$ (f) $190 - 65$

2. Multiply using your circular slide rule: 7.2×9.7.

3. If you had difficulty reading the scales for Question 2, practise reading these.
   (a)

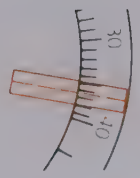


   (b)

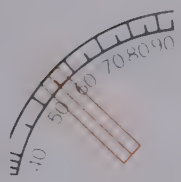


(c)

(d)

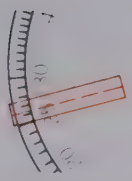


(e)



(f)

(g)



(h)

4. *Work in pairs.* Each person places the cursor of his slide rule on a number which he records on his paper. Exchange slide rules and read the number. Check each person's work. Repeat several times.

5. Use the cursor to locate these numbers on your slide rule.

(a) 1.7	(b) 1.5	(c) 2.5	(d) 11.5
(e) 22	(f) 47	(g) 87.5	(h) 15.4
(i) 26.8	(j) 43.2	(k) 73.4	(l) 82.8

Did you have difficulty with parts (g) to (l)? Discuss.

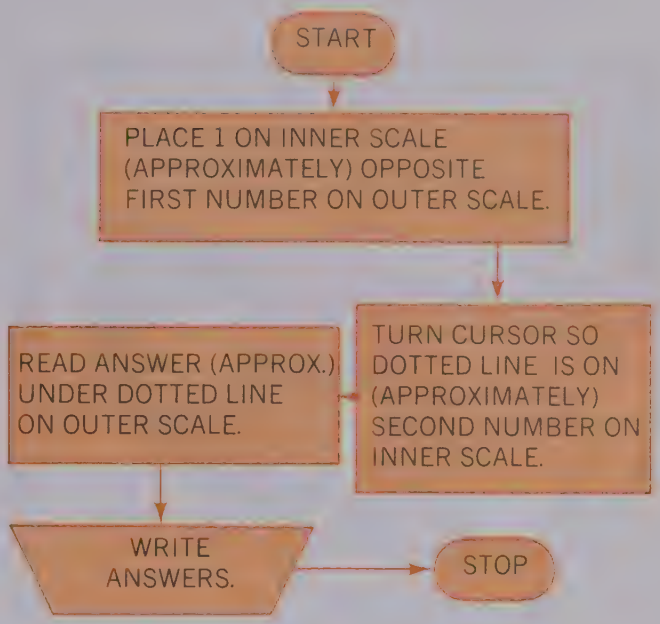
What will you do when you encounter numbers such as those in (g) to (l)?

What will you do when you encounter numbers such as 2.34, 8.95, 11.3, and 28.3?

Name five other numbers with which you would have similar problems. Discuss these with your partner.

6. Each person writes a number between 1 and 100 on a sheet of paper. His partner locates the number on his slide rule using the cursor. Check each other's work. Repeat several times.

7. Use your slide rule to multiply 3.72×21.2. If you need help, use this flowchart.


START
PLACE 1 ON INNER SCALE (APPROXIMATELY) OPPOSITE FIRST NUMBER ON OUTER SCALE.
TURN CURSOR SO DOTTED LINE IS ON (APPROXIMATELY) SECOND NUMBER ON INNER SCALE.
READ ANSWER (APPROX.) UNDER DOTTED LINE ON OUTER SCALE.
WRITE ANSWERS.
STOP

Did you get 78? If not, ask your partner or teacher to help you.

Try these now:

(a) 9×6.4	(b) 8.3×6	(c) 7.8×5
(d) 12.6×4	(e) 16.8×4	(f) 8×3.6
(g) 4.6×12.3	(h) 14.6×6.4	(i) 18.7×3.7

8. Make up ten multiplication problems (that can be done on your slide rule) and give them to your partner to do. Then check his answers when he has finished.

Aircraft maintenance is an ever-growing field of work in both the Canadian Armed Forces and in commercial aviation.

3·2 A Review of the Case So Far . . .

SIR, WOULD IT ASSIST US IF WE WERE TO REVIEW THE FACTS OF THE CASE OF THE MISSING NUMBERS?
DR. WATSON, YOUR SUGGESTION IS MOST APPROPRIATE. LET US BEGIN...

Our number system has been built by starting with the single set of counting numbers.

N 1 2 3 4 5 →

And adding other numbers to this set:

W 0 1 2 3 4 5 →

Needed to solve: $3 - 3 = n$

Still more numbers are added to make the set of integers:

I ← −3 −2 −1 0 1 2 3 4 →

Needed to solve: $5 - 8 = n$

What set of numbers shall we add next?

Let us consider closure.

In each of these, is n an integer?

Addition:	$4 + (-4) = n$	$-7 + (-9) = n$
Subtraction:	$4 - (-3) = n$	$7 - 9 = n$
Multiplication:	$3 \times (-4) = n$	$-5 \times (-8) = n$
Division:	$3 \div 4 = n$	$-5 \div 2 = n$

It appears that the numbers we must include are the fractions such as $\frac{3}{4}$ and $-\frac{5}{2}$.

I ← −4 −3 $-\frac{5}{2}$ −2 −1 0 $\frac{3}{4}$ 1 2 3 4 5 6 →

From your experiences with the slide rule, you have learned that between the integers on a number line there are other points that represent fractions. On the slide rule, the fractions were decimal fractions.

Let us study Figure 3-1. The dots divide the space between the integer points into three equal parts. And so we find new fraction points between the integer points. How would you read the numbers represented by points *A*, *B*, and *C* in Figure 3-1? Do not proceed further until you have decided what they are.

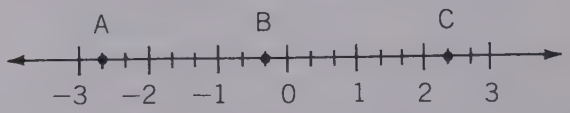


Figure 3-1

Did you decide that *A*, *B*, and *C* represent $-2\frac{2}{3}$, $-\frac{1}{3}$, and $+2\frac{1}{3}$ respectively?

EXERCISE 3·2

1. Solve each of these using the set of fractions.
   (a) $n = 6 \div 4$ (b) $3n = 5$
   (c) $2n = -3$ (d) $-2n = 5$
2. Write three more equations that have as their solutions numbers from the set of fractions.

3·3 Operations on the Set of Rational Numbers

In previous study, you may have acquired sufficient capability to deal with this section. Try the following

questions. If you have no difficulties with these, proceed to the next section. On the other hand, if you do have difficulty, then proceed with the remainder of this section.

Perform the indicated operations:

1. $\frac{3}{7}+\frac{4}{9}$
2. $\frac{5}{8}+\frac{3}{4}+\frac{7}{12}$
3. $\frac{5}{6}-\frac{7}{9}$
4. $\frac{2}{3}-\frac{7}{15}-\frac{5}{6}$
5. $\frac{4}{9}\times\frac{15}{28}$
6. $\frac{3}{8}\times\frac{12}{25}\times\frac{5}{9}$
7. $\frac{5}{8}\div\frac{3}{4}$
8. $\frac{12}{35}\div\frac{4}{5}$
9. $4.07+16.9-3.826-9.1+5.008$
10. $\dfrac{7.8\times 0.05}{0.65}$

How well did you make out with the above questions?

To find out, check your results with the following answers:

1. $\frac{55}{63}$
2. $\frac{47}{24}$ or $1\frac{23}{24}$
3. $\frac{1}{18}$
4. $-\frac{19}{30}$
5. $\frac{5}{21}$
6. $\frac{1}{10}$
7. $\frac{5}{6}$
8. $\frac{3}{7}$
9. 13.052
10. 0.6

If you did not get the correct answers to most of them, the following examples should be of help.

Example 1

Add: (a) $\frac{1}{2}+\frac{1}{4}$ (b) $\frac{1}{3}+\frac{1}{4}$

Solution

(a) $\frac{1}{2}+\frac{1}{4}=\frac{?}{4}+\frac{?}{4}$ (Use common denominator.)

$=\frac{2+1}{4}$ (Change each fraction to a new one with the common denominator.)

$=\frac{3}{4}$

(b) $\frac{1}{3}+\frac{1}{4}=\frac{?}{12}+\frac{?}{12}$ (LCM of 3 and 4 is 12.)

$=\frac{4+3}{12}$

$=\frac{7}{12}$

Example 2

Perform the indicated operations of Questions 2, 4, 6, 8, 9, and 10 above.

Solutions

2. $\frac{5}{8}+\frac{3}{4}+\frac{7}{12}=\frac{15}{24}+\frac{18}{24}+\frac{14}{24}$

$=\frac{47}{24}$

$=1\frac{23}{24}$

4. $\frac{2}{3}-\frac{7}{15}-\frac{5}{6}=\frac{20}{30}-\frac{14}{30}-\frac{25}{30}$

$=-\frac{19}{30}$

6. $\dfrac{3}{8}\times\dfrac{12}{25}\times\dfrac{5}{9}=\dfrac{3}{4\times 2}\times\dfrac{3\times 4}{5\times 5}\times\dfrac{5}{3\times 3}$

$=\dfrac{3\times 3\times 4\times 5}{3\times 3\times 4\times 5\times 2\times 5}$

$=\dfrac{3\times 3\times 4\times 5}{3\times 3\times 4\times 5\times 2\times 5}$

$=1\times 1\times 1\times 1\times\dfrac{1}{2}\times\dfrac{1}{5}$

$=\dfrac{1}{10}$

8. $\dfrac{12}{35}\div\dfrac{4}{5}=\dfrac{3\times 4}{5\times 7}\times\dfrac{5}{4}$ (Why $\frac{5}{4}$?)

$=\dfrac{3\times 4\times 5}{7\times 4\times 5}$

$=\dfrac{3}{7}$

When dividing, invert the divisor and multiply.

9. $4.07+16.9-3.826-9.1+5.008$

$=(4.07+16.9+5.008)-3.826-9.1$

(Can you do this another way?)

$=25.978-3.826-9.1$

$=22.152-9.1$

$=13.052$

10. $\dfrac{7.8\times 0.05}{0.65}=\dfrac{0.390}{0.65}=0.6$

7.8	1 decimal place
0.05	2 decimal places
0.390	3 decimal places
3	

In the following exercise do those questions which relate to your difficulties.

EXERCISE 3·3

Perform the indicated operations.

1. $\frac{2}{5} - \frac{7}{15}$
2. $\frac{3}{4} - \frac{5}{8}$
3. $\frac{1}{2} - \frac{1}{3}$
4. $\frac{2}{3} \div \frac{1}{4}$
5. $\frac{1}{3} - \frac{1}{5}$
6. $\frac{3}{4} - \frac{3}{2}$
7. $\frac{4}{3} - \frac{3}{4}$
8. $\frac{1}{2} - \frac{1}{4}$
9. $\frac{5}{6} - \frac{3}{8}$
10. $\frac{3}{4} - \frac{1}{2}$
11. $\frac{5}{6} - \frac{2}{3}$
12. $\frac{4}{5} - \frac{1}{2}$
13. $\frac{3}{4} - \frac{1}{3}$
14. $\frac{6}{7} - \frac{11}{21}$
15. $\frac{2}{3} - \frac{5}{6} - \frac{7}{12}$
16. $\frac{8}{9} - \frac{7}{12} - \frac{7}{5}$
17. $1 - \frac{2}{5} - \frac{3}{14}$
18. $2\frac{1}{2} - 4\frac{1}{3} - 3\frac{7}{8}$
19. $5\frac{1}{3} - 1\frac{3}{5} - 2\frac{1}{15}$
20. $5\frac{2}{9} - 3\frac{12}{17}$
21. $\frac{7}{11} - \frac{5}{9}$
22. $2\frac{6}{13} - 5\frac{3}{5}$
23. $1\frac{1}{4} - \frac{1}{3}$
24. $2\frac{3}{4} - 1\frac{7}{9}$
25. $3.1 - 4.25 - 8.02$
26. $12.01 - 8.946$
27. $276.5 - 87.29 + 24$
28. 16.15×6.4
29. $3.1 \cdot 4.12$
30. $15.6 \cdot 3.8$
31. 18.9×0.02
32. $44.02 \div 3.1$
33. $1.792 \div 5.6$
34. $11.584 \div 36.2$
35. $34.2 \div 8.75$
36. $\dfrac{32.8 \times 1.75}{40}$
37. $\dfrac{1.68 \cdot 0.55}{8.8 \cdot 0.021}$
38. $\dfrac{1 \cdot 0.1 \cdot 0.01}{0.001 \cdot 0.0001}$

For extra practice in multiplying and dividing.

39. $\frac{1}{3} \cdot \frac{1}{2}$
40. $\frac{3}{2} \cdot \frac{1}{2}$
41. $\frac{1}{4} \cdot \frac{1}{3}$
42. $\frac{1}{2} \times \frac{2}{3}$
43. $\frac{6}{8} \times \frac{3}{4}$
44. $\frac{7}{8} \times \frac{4}{5}$
45. $1\frac{1}{2} \times 1\frac{1}{2}$
46. $1\frac{1}{4} \times 2\frac{1}{2}$
47. $2\frac{1}{2} \times 3\frac{1}{2}$
48. $1\frac{3}{4} \times 1\frac{1}{2}$
49. $3\frac{4}{5} \times 1\frac{5}{6}$
50. $2\frac{7}{8} \cdot 1\frac{1}{8}$
51. $2 \div \frac{1}{2}$
52. $3 \div \frac{1}{3}$
53. $3 \div \frac{2}{3}$
54. $\frac{3}{2} \div \frac{3}{4}$
55. $\frac{1}{2} \div \frac{1}{4}$
56. $\frac{1}{4} \div \frac{1}{2}$
57. $\frac{2}{3} \div \frac{1}{2}$
58. $\frac{3}{4} \div \frac{2}{3}$

The chart below shows the Indian population and the number of hectares of reserves in each province in Canada in 1968.

	Population	Hectares
British Columbia	45 152	341 233
Alberta	26 440	651 088
Saskatchewan	32 579	509 151
Manitoba	32 227	219 368
Ontario	51 731	624 202
Quebec	25 650	76 220
New Brunswick	4 039	15 222
Nova Scotia	4 287	10 356
Prince Edward Island	409	1 110
Northwest Territories	2 471	1 975
Yukon	5 911	872

(a) On the average, what is the number of hectares per person on the reserves in each area? (Round each answer to one decimal place.)

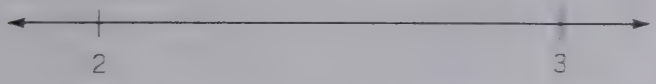
(b) What is the average for the whole of Canada?

(c) Which provinces or territories are above the average in (b)?

(d) In which province or territory do you think the Indians are most affluent? Is the degree of affluence related to the answers in part (a)? Discuss.

3·4 Fractions Between Fractions

Truly, there are many points between any two points. Let's illustrate:



One number between 2 and 3 is the average of these two numbers.

Add the two numbers: $2 + 3 = 5$

Divide the sum by 2: $5 \div 2 = 5 \times \frac{1}{2}$

$= \frac{5}{2}$

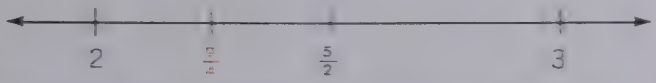
Now we can find the average of 2 and $\frac{5}{2}$.

Add the two numbers: $2 + \frac{5}{2} = \frac{4 + 5}{2}$

$= \frac{9}{2}$

Divide the sum by 2: $\frac{9}{2} \div 2 = \frac{9}{2} \times \frac{1}{2}$

$= \frac{9}{4}$

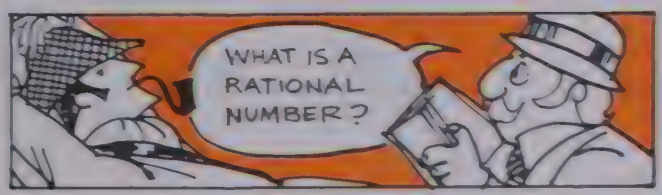


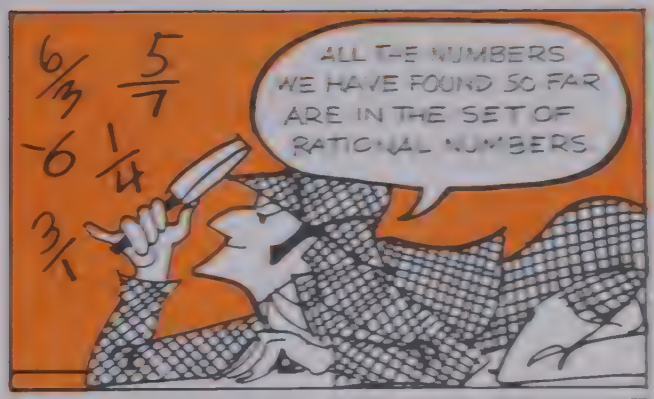
We could continue to find the averages in a similar way. There are many numbers between any two integers. The set of all these in-between fraction numbers, along with the set of integers, is called the set of **rational numbers**. The symbol for the set of rational numbers is Q.

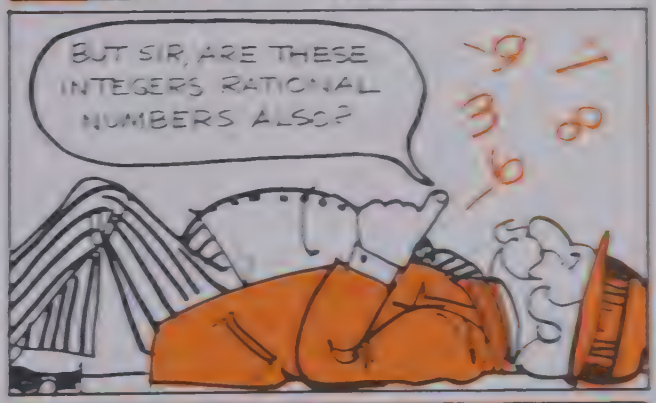
Each integer can be written in a fraction form:

$3 = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = \frac{12}{4}$, etc.

How would you write -5 as a fraction?

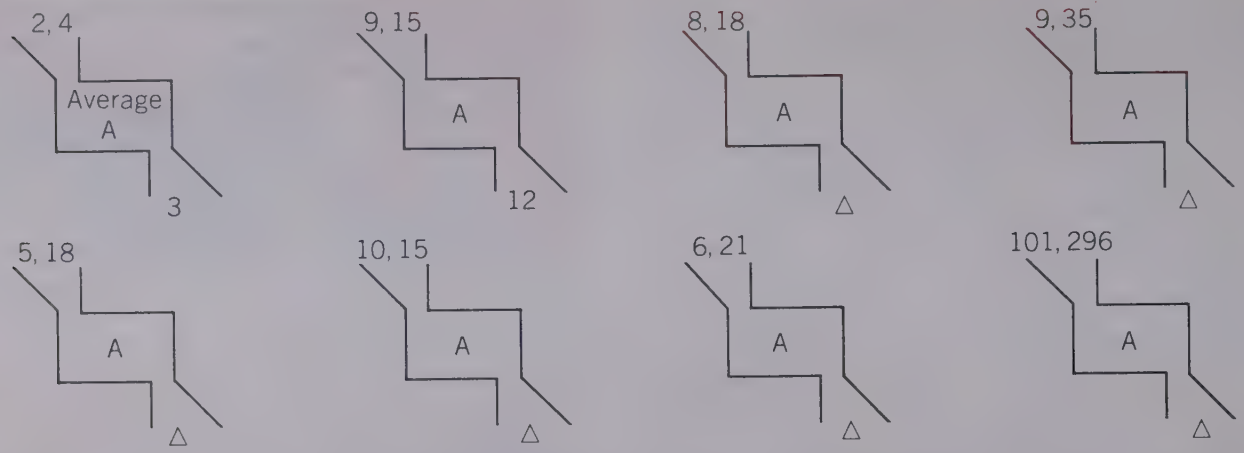






Using the Function Machine: The Average of Two Numbers

1. Find the output for each machine.

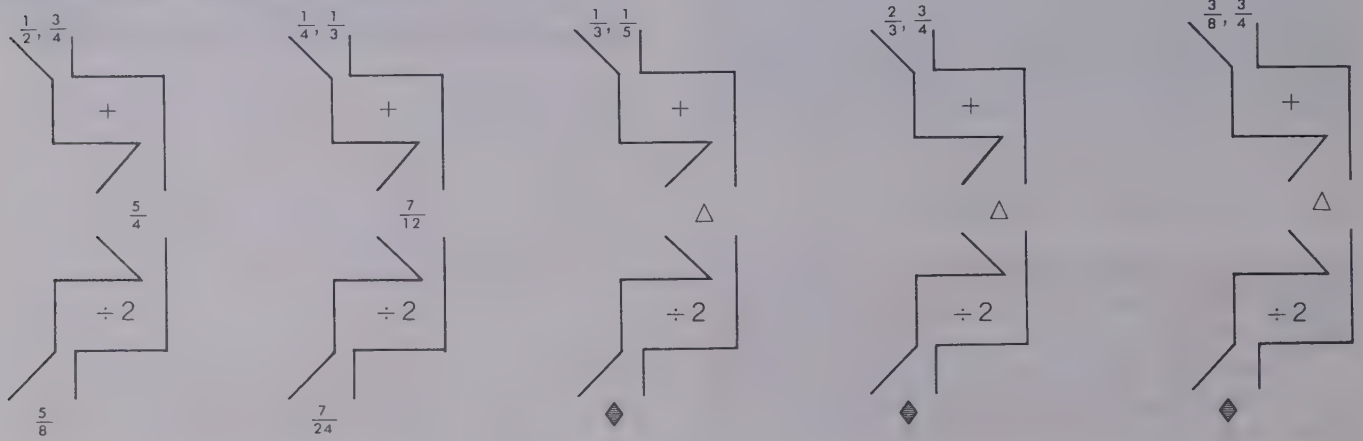


2. More inputs and outputs were recorded in these charts. Complete the charts.

Input		Output
▽	□	△
1	2	
6	11	
5	6	
7.4	7.6	
9.3	9.8	

Input		Output
▽	□	△
5.71	5.72	
11.33	11.34	
15.675	15.676	
101.90	101.91	
1 000 000	1 000 001	

3. Find the output of each set of machines.



4. More inputs and final outputs of the machines in Question 3 are recorded in this chart. Copy and complete the chart.

Input		Output
▽	□	◆
$\frac{1}{2}$	$\frac{1}{3}$	
$\frac{1}{2}$	$\frac{2}{3}$	
$\frac{1}{4}$	$\frac{1}{5}$	
$\frac{3}{4}$	$\frac{5}{6}$	
$\frac{3}{7}$	$\frac{4}{7}$	

Input		Output
▽	□	◆
$2\frac{1}{2}$	$3\frac{1}{2}$	
$2\frac{5}{6}$	$2\frac{9}{12}$	
$7\frac{3}{4}$	$7\frac{1}{8}$	
$6\frac{1}{2}$	$6\frac{1}{4}$	
$9\frac{1}{8}$	$9\frac{1}{3}$	

Example 1

What number does point A represent if it lies halfway between points $1\frac{1}{3}$ and $1\frac{1}{2}$ on the rational number line (Q-line)?

Solution

The average of the two numbers will give us the solution.

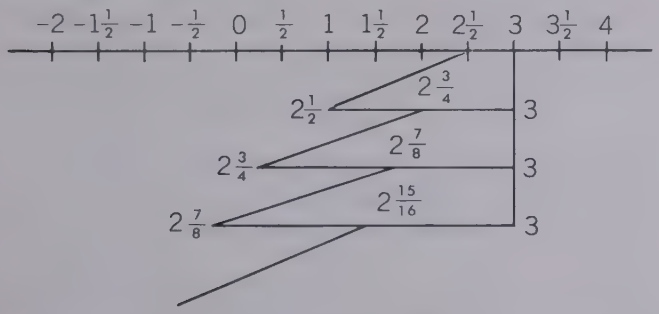
Add: $1\frac{1}{3} + 1\frac{1}{2} = 1\frac{2}{6} + 1\frac{3}{6}$
$= 2\frac{5}{6}$

Divide by 2: $2\frac{5}{6} \div 2 = \frac{17}{6} \times \frac{1}{2}$
$= \frac{17}{12}$
$= 1\frac{5}{12}$

Point A represents the number $1\frac{5}{12}$.

Example 2

Follow this graphic representation of number points on the Q-line.



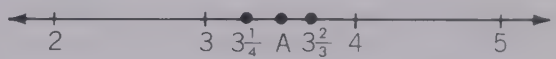
The space between $2\frac{1}{2}$ and 3 is magnified and halved. Then a part of that is magnified again and halved; and again the procedure is repeated.

This procedure can be extended indefinitely. Once we project our thinking to include this process, we come to a very profound realization. On the Q-line for any two neighbouring number points we wish to choose, we can always locate and identify number points between them.

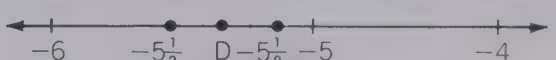
There is a rational number between every pair of distinct rational numbers.

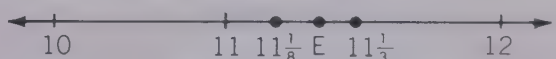
EXERCISE 3·4

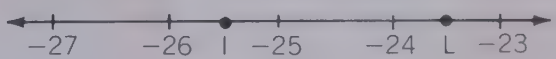
1. In each case, what number does the lettered point represent if it lies halfway between the neighbouring numbered points on the Q-line?

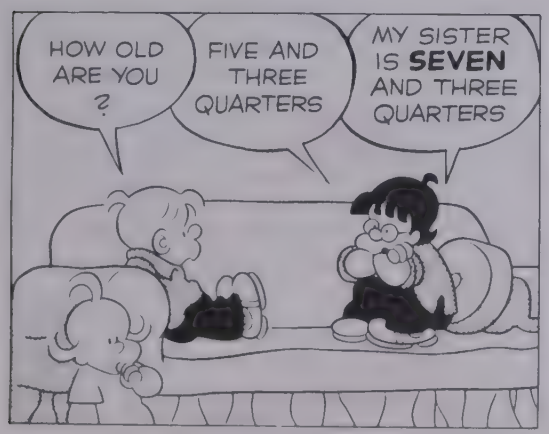
(a) 

(b)

(c) 

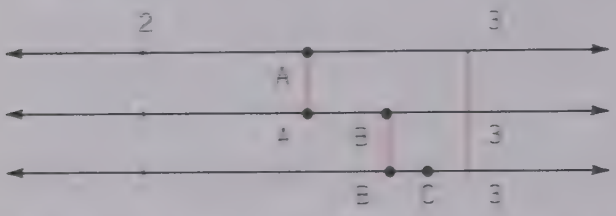
(d) 

(e) 



2. How many points are there between these two points? Explain.

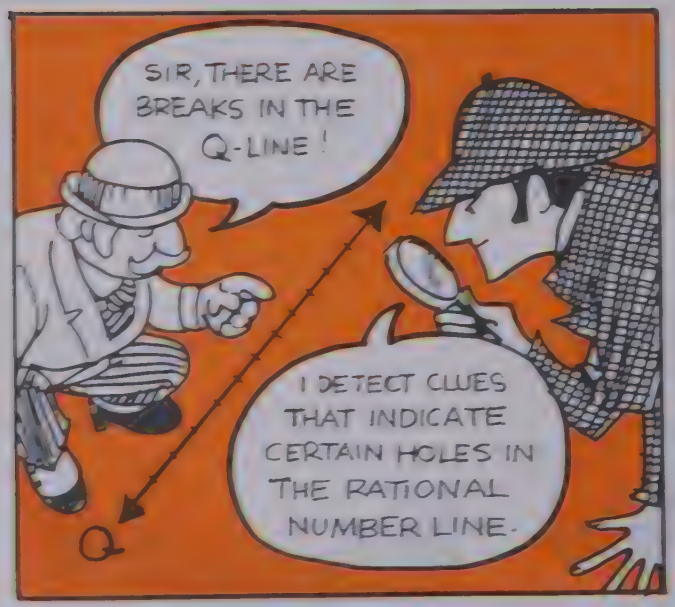
3. Identify each point in this graphic representation. The midpoint in each case is the average of the two numbers.



4. The answer to an addition problem is the sum; to a multiplication problem, it is the product. What is the name of the answer to a division question? Why do you think $\mathbb{Q}$ is used as the symbol to represent the set of rational numbers?

What Do You Think

If we consider all the possible number points on the Q-line, would the number line be a solid line or would there be breaks (unnumbered points) in it? Discuss.



3·5 Decimals as the Set of Rational Numbers

We have just learned that all rational numbers can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Now let us try to write rational numbers in a decimal form.

Example 1

Express the following in decimal form:

(a) $\frac{3}{5}$ (b) $\frac{7}{8}$ (c) $\frac{6}{125}$

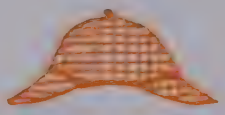
Solution

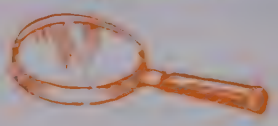
We divide the denominator into the numerator.

(a)
$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \\ \underline{3\,0} \\ 0 \end{array}$$
$\frac{3}{5} = 0.6$

(b)
$$\begin{array}{r} 0.875 \\ 8\overline{)7.000} \\ \underline{6\,4} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$
$\frac{7}{8} = 0.875$

(c)
$$\begin{array}{r} 0.048 \\ 125\overline{)6.000} \\ \underline{5\,00} \\ 1\,000 \\ \underline{1\,000} \\ 0 \end{array}$$
$\frac{6}{125} = 0.048$





Example 2

Express the following in decimal form.

(a) $\frac{2}{3}$ (b) $\frac{5}{12}$ (c) $\frac{4}{7}$

Solution

(a) We divide the denominator into the numerator. We must keep in mind that:

$$2 = 2.0 = 2.00 = 2.000 = 2.0000 \text{ etc.}$$

Therefore, for $\frac{2}{3}$,

$$\begin{array}{r} 0.666\ldots \\ 3\overline{)2.0000} \\ 18 \\ \hline 20 \\ 18 \\ \hline 20 \\ 18 \\ \hline 2 \end{array}$$

Hence, $\frac{2}{3} = 0.666\ldots$ with the six repeating.

We show a repeating decimal by a dot or dots above the repeating numbers. Thus,

$\frac{2}{3} = 0.666\ldots = 0.\dot{6}$

Sometimes we write:

$\frac{2}{3} = 0.7$

or $\frac{2}{3} = 0.67$

or $\frac{2}{3} = 0.667$ and so on.

We mean that $\frac{2}{3}$ is equal to 0.7 (or 0.67 or 0.667) when rounded to the appropriate place value. To be absolutely correct we should write:

$\frac{2}{3} \simeq 0.7$

or $\frac{2}{3} \simeq 0.67$.

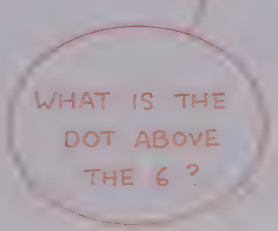
Just remember, when we write $\frac{2}{3} = 0.7$, we mean the decimal is rounded off.

(b) Similarly, for $\frac{5}{12}$:

$$\begin{array}{r} 0.416\,6\ldots \\ 12\overline{)5.000\,00} \\ 48 \\ \hline 20 \\ 12 \\ \hline 80 \\ 72 \\ \hline 80 \\ 72 \\ \hline 8 \end{array}$$

Here again the six is repeating.

$\therefore \frac{5}{12} = 0.416\dot{6}$



(c) And for $\frac{4}{7}$.

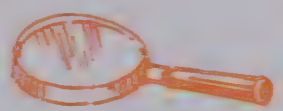
$$\begin{array}{r} 0.571\,428\,5\ldots \\ 7\overline{)4.000\,000\,0} \\ 35 \\ \hline 50 \\ 49 \\ \hline 10 \\ 7 \\ \hline 30 \\ 28 \\ \hline 20 \\ 14 \\ \hline 60 \\ 56 \\ \hline 40 \\ 35 \\ \hline 5 \end{array}$$

Here we notice that 571 428 is the repeating part.

$\therefore \frac{4}{7} = 0.\dot{5}71\,428\dot{}$

(Note the use of two dots to indicate the repeating part.)

Have you noticed in these divisions that the remainder is never zero and that sooner or later the digits in the quotient start to repeat? Decimals of this type are called **repeating decimals**.



Example 3

Which of the following are terminating decimals and which are repeating decimals?

(a) $2.\dot{1}$ (b) -8.007 (c) 5.0000

(d) $12.62\dot{8}$ (e) $-3.\dot{5}04\dot{7}$ (f) -4.545

Solution

Terminating decimals are -8.007, 5.0000, and -4.545.

Repeating decimals are $2.\dot{1}$, $12.62\dot{8}$, and $-3.\dot{5}04\dot{7}$.

Is it possible to have a nonterminating, nonrepeating decimal? Yes, one such example is the value of π. Two other examples are: 0.505 005 000 500 005... and 0.121 231 234 123 45....

EXERCISE 3·5

1. Express each of the following in decimal form. Then state whether each is a terminating or repeating decimal.

   (a) $\frac{5}{6}$ (b) $-\frac{7}{8}$ (c) $\frac{4}{5}$

   (d) $\frac{5}{9}$ (e) $-\frac{8}{15}$ (f) $-4\frac{3}{7}$

   (g) $-8\frac{24}{125}$ (h) $12\frac{1}{40}$ (i) $6\frac{3}{200}$

   (j) $33\frac{1}{3}$ (k) $-125\frac{1}{8}$ (l) $16\frac{1}{6}$

2. Which of the following paired numbers are exactly equal? Which are approximately equal?

   (a) $(\frac{1}{4}, 0.25)$ (b) $(\frac{3}{20}, 0.15)$

   (c) $(\frac{1}{7}, 0.143)$ (d) $(\frac{5}{9}, 0.556)$

   (e) $(\frac{88}{200}, 0.44)$ (f) $(1\frac{1}{6}, 1.1\dot{6})$

   (g) $(8\frac{6}{7}, 8.\dot{8}57\dot{1})$ (h) $(3\frac{15}{40}, 3.37)$

3. If you try to use a calculator to add numbers such as $133\frac{3}{7} + 165\frac{1}{6}$, you will have difficulty. A calculator does not have fractions. You will need to change each number to decimal notation rounded to the appropriate number of decimals. Here are two examples of numbers rounded to three decimal places.

   $$133\tfrac{3}{7} = 133.429$$
   $$165\tfrac{1}{6} = 165.167$$

   A. Prepare these numbers below for use on a calculator (round to two decimal places).

   If you have a calculator obtain the product, quotient, sum, or difference as indicated.

   (a) $42\frac{1}{3} + 56\frac{3}{4} = ?$ (b) $73\frac{4}{5} + 29\frac{3}{8} = ?$

   (c) $62\frac{1}{9} - 43\frac{3}{8} = ?$ (d) $6\frac{1}{8} \times 4\frac{7}{15} = ?$

   (e) $9\frac{2}{11} \times 3\frac{5}{21} = ?$ (f) $175\frac{1}{8} \div 25\frac{1}{3} = ?$

   B. Using fractions, calculate the sums in (a) and (b).

   C. Using decimal notation, calculate the sums in (a) and (b).

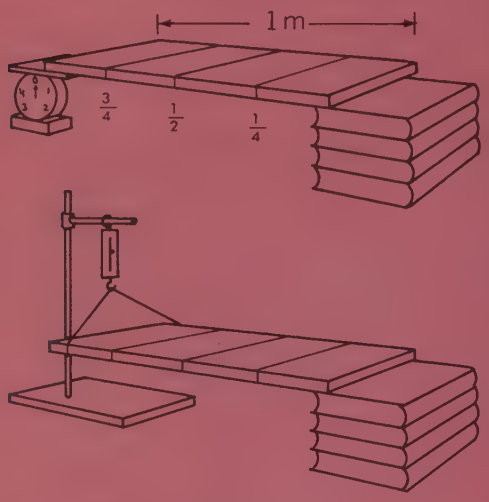
   D. Would you think decimals are always easier to work with? Explain.

4. Use your library to find the value of π to more than ten decimal places. Does the decimal repeat?

Projects

Project 1

A. Set up the apparatus as shown. Divide the board into four equal parts and label them $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$. Use either method or devise your own.



1. Adjust the scale to read zero when set up. This is most important!

2. Mark the strong wood slat, which should be 1 m long, as shown.

3. Place a two-kilogram mass at the one-half mark. Read the scale. Write an equation using the numbers, two, one half, and the reading on the scale. How many different equations can you write? Discuss the equations.

4. Repeat Step 3 for these:

   (a) 2 kg at $\frac{3}{4}$ (b) 4 kg at $\frac{3}{4}$

   (c) $1\frac{1}{2}$ kg at $\frac{1}{2}$ (d) $3\frac{1}{2}$ kg at $\frac{1}{4}$

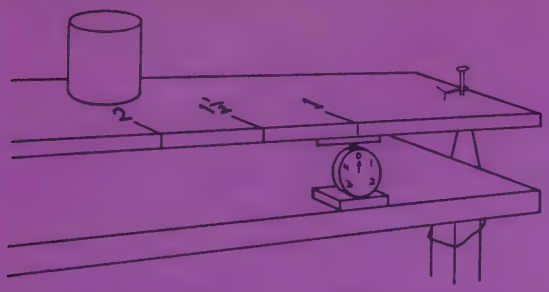
5. Use the scale to solve these:

   (a) $2 \times \frac{1}{4}$ (b) $2\frac{1}{2} \times \frac{2}{4}$

   (c) $1\frac{1}{2} \times \frac{1}{4}$ (d) $3 \times \frac{3}{4}$

6. Make up ten problems that can be answered using your apparatus. Calculate the answer to each before challenging your partner to answer them using the apparatus.

B. Set up the apparatus as shown here.



1. Adjust the scale to zero. This is essential!
2. Place a three-kilogram mass at the 2 mark. Read the scale. Write as many equations as you can.
3. Repeat for these:
   (a) $1\frac{1}{2}$ kg at 2 (b) 2 kg at $1\frac{1}{2}$
   (c) $1\frac{1}{2}$ kg at $1\frac{1}{2}$ (d) $2\frac{1}{2}$ kg at $1\frac{1}{2}$
4. Make up ten multiplication problems involving fractions. Solve each by using your apparatus. Challenge your partner to answer your problems using the apparatus. Does your apparatus always give the correct answer exactly? Discuss.

Project 2

Survey your class to determine what make of car, if any, is owned by each student or each student's parents. Record your data in a chart like this:

Make of Car	Number	Fraction of Total	Decimal
None	3		
Ford	4		
Chevrolet			
Pontiac			
⋮			
Total			

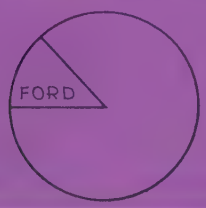
(a) Show the data in a bar graph.

(b) Determine the fraction of the total that own Ford, Chevrolet, etc.

(c) Express each fraction in (b) as a decimal.

(d) Show the data in (b) as a circle graph.
Hint! If $\frac{4}{30}$ own Fords, then

$$\frac{4}{30} \times 360 = 48 \rightarrow 48°.$$



3·6 Changing Terminating Decimals to Fractions

There are two procedures used in changing decimals to fractions. One procedure is used with terminating decimals and the other is used with repeating decimals. Repeating decimals will be dealt with in **Math Probe 2.**

Example 1

Express each of the following decimals as fractions or mixed numbers. Simplify all fractions.

(a) 0.75 (b) 0.008 (c) 3.12 (d) 12.375 (e) 8.0625

Solution

(a) $0.75 = \frac{75}{100} = \frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

(b) $0.008 = \frac{8}{1000} = \frac{8 \div 8}{1000 \div 8} = \frac{1}{125}$

(c) $3.12 = 3\frac{12}{100} = 3\frac{12 \div 4}{100 \div 4} = 3\frac{3}{25}$

(d) $12.375 = 12\frac{375}{1000} = 12\frac{375 \div 125}{1000 \div 125} = 12\frac{3}{8}$

(e) $8.0625 = 8\frac{625}{10\,000} = 8\frac{625 \div 125}{10\,000 \div 125} = 8\frac{5}{80} = 8\frac{1}{16}$

EXERCISE 3·6

1. Express each decimal in fraction form. Simplify.

(a) 0.55	(b) 0.64	(c) 0.075
(d) 0.0004	(e) 2.7	(f) 8.45
(g) 10.005	(h) 9.14	(i) 6.048
(j) 12.728	(k) 4.0625	(l) 11.11

2. Trapping of wild animals for their fur pelts is still practised in Canada. Some data for the 1970-71 winter season are shown.

Kind of Pelts	Number	Value of Each Pelt
Black bear	2 007	$19.82
Beaver	355 379	12.55
Fox, cross or red	34 744	11.79
Mink	67 378	7.83
Muskrat	1 572 885	1.34
Seals, hair	185 144	9.36
Squirrel	475 373	0.32
Wildcat	3 408	12.43

(a) What is the total value of each type of pelt?

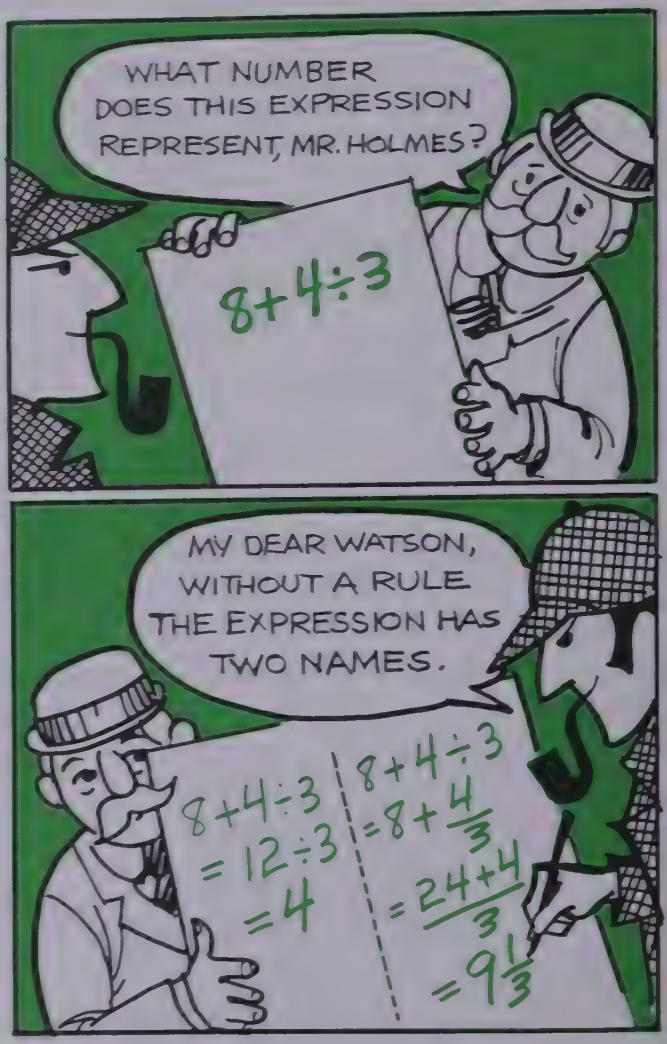
(b) Which kind of pelt produced the most income?

(c) There is a great deal of controversy about *humane* trapping techniques. Debate in class the statement "All trappers should be compelled to quit using the old-fashioned traps in favour of the new humane traps."

(d) Discuss whether the current method of killing seals is inhumane. Do you think seal killing should be banned? Discuss.

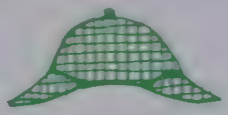
3·7 Working With the Set of Rational Numbers

What number does the expression $3 - 7 \times 4$ name? Did you get 40 or did you get 31? If we add the 3 and 7 before multiplying, we have $10 \times 4 = 40$. If we multiply the 7 and 4 before adding, we have $3 - 28 = 31$. Which is correct?



STOP Do Question 1 in Exercise 3·7 before proceeding.

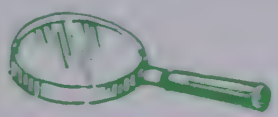
Since we want each expression to name just one number, we will need to agree on the order in which we perform the operations.



Rule 1 In an expression of two or more different operations,

– first – perform the multiplication and division in the order in which they appear.

– second – perform the addition and subtraction in the order in which they appear.



Example 1

Simplify:

(a) $14 - 8 \div 4 + 2$ (b) $31 - 16 \times \frac{5}{4} \div 4 + 3\frac{2}{3}$

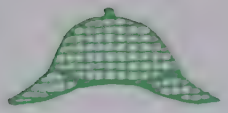
Solution

(a) $14 - 8 \div 4 + 2$ (1) Perform division first.
$= 14 - 2 + 2$ (2) Perform addition and subtraction in order they appear.
$= 14$

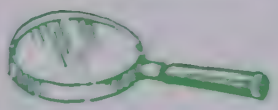
(b) $31 - 16 \times \frac{5}{4} \div 4 + 3\frac{2}{3}$ (1) Multiply first.
$= 31 - \frac{80}{4} \div 4 + 3\frac{2}{3}$
$= 31 - 20 \div 4 + 3\frac{2}{3}$ (2) Simplify.
$= 31 - 5 + 3\frac{2}{3}$ (3) Divide.
$= 26 + 3\frac{2}{3}$ (4) Add and subtract in order each occurs.
$= 29\frac{2}{3}$

Do Questions 2 and 3 in Exercise 3·7 before proceeding.

Sometimes brackets are used to indicate the order in which the operations are to be done.



Rule 2 [illegible]



Example 2

Simplify:

(a) $(2 + 6) \div (2 - 7)$ (b) $(4 + 6 \div 3) \cdot (18 - 6)$

Solution

(a) $(2 + 6) \div (2 - 7)$
$= (8) \div (-5)$ (1) Rule 2.
$= \frac{8}{-5} = -\frac{8}{5}$ (2) Divide – Rule 2.

(b) $(4 + 6 \div 3) \times (18 - 6)$ (1) Rule 2 using Rule 1. Divide first.
$= (4 + 2) \times (18 + 6)$
$= (6) \times (24)$ (2) Add – Rule 2.
$= 144$ (3) Multiply.

Do Question 4 in Exercise 3·7 before proceeding.

Division problems can be written in more than one way: 4 divided by 3 can be written as $4 \div 3$ or as $\frac{4}{3}$. Therefore, when we see any one form, we can write it in any one of the other forms. This is very useful.

Example 3

Simplify: (a) Divide $\frac{2}{3}$ by $\frac{4}{5}$ (b) $\dfrac{\frac{5}{6}}{\frac{2}{3}}$

Solution

(a) Divide $\frac{2}{3}$ by $\frac{4}{5}$ can be written as:

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{2 \times 5}{3 \times 4} = \frac{10}{12} = \frac{5}{6}$$

(b) $\dfrac{\frac{5}{6}}{\frac{2}{3}}$ can be written as:

$$\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{5 \times 3}{6 \times 2} = \frac{15}{12} = 1\frac{1}{4}$$

EXERCISE 3·7

1. By changing the order in which you perform the operations, what different numbers can you get for each expression?

   (a) $3+4\times5$ (b) $7\times4-8$

   (c) $4\div5+6$ (d) $8+4\div6$

   (e) $6+8\times2\div4$ (f) $14-6\div3\times2$

   (g) $3+\frac{1}{2}\times4-3$ (h) $8\div\frac{3}{4}-6\times\frac{4}{3}$

2. Use Rule 1 to calculate the correct value of each expression in Question 1.

3. Calculate the correct value of each expression. Use Rule 1.

   (a) $\frac{6}{8}\div\frac{1}{2}+\frac{3}{4}\times\frac{1}{2}$ (b) $\frac{2}{3}+\frac{1}{4}\times2-\frac{1}{4}$

   (c) $\frac{3}{4}+\frac{2}{3}\times\frac{1}{2}+\frac{1}{4}$ (d) $\frac{2}{3}\div\frac{1}{2}-\frac{1}{3}\times\frac{4}{3}$

   (e) $2\frac{1}{2}\times4+2\frac{1}{2}\div2$ (f) $6\frac{3}{4}-2\frac{1}{4}\times3-4$

   (g) $2.3\times4-3\times1.3$ (h) $6.5\div0.5-3\div1.5$

In metric units, estimate the temperatures indicated in these scenes. Compare your estimates with your friends.

4. Simplify. Use Rule 2 and Rule 1.

   (a) $(3+4)\div(15-8)$

   (b) $(6-9)\div(4+5)$

   (c) $(5-3)\times(4\div3)$

   (d) $(7-13)\times(15-38)$

   (e) $(2\div4+\frac{3}{2})\times(\frac{3}{4}+1\frac{1}{2})$

   (f) $(4-6\div2)\times(8-3\times9)$

   (g) $(5\times5+2.5)\times(3\times9\div2.7)$

   (h) $(18\times3\div9)\div(21-14\div7)$

   (i) $(2.7\div9+6)\times(1.4\times6-18)$

   (j) $(7.2\div6-6)\times(6.4\div8-2)$

5. Simplify:

   (a) $\frac{2}{3}\div\frac{1}{3}$ (b) $\frac{3}{4}\div\frac{3}{4}$ (c) $\frac{5}{6}\div\frac{1}{2}$

   (d) $\frac{5}{8}\div2$ (e) $\frac{5}{8}\div\frac{1}{3}$ (f) $\frac{3}{8}\div\frac{3}{4}$

   (g) $\frac{3}{8}\div\frac{4}{3}$ (h) $\frac{3}{4}\div\frac{1}{2}$ (i) $\dfrac{\frac{3}{4}}{\frac{2}{3}}$

   (j) $\dfrac{\frac{1}{2}}{\frac{1}{4}}$ (k) $\dfrac{-\frac{3}{4}}{\frac{5}{8}}$ (l) $\dfrac{\frac{2}{3}}{\frac{1}{2}}$

   (m) $\dfrac{\frac{1}{3}}{\frac{3}{4}}$ (n) $\dfrac{\frac{1}{2}}{-\frac{5}{8}}$ (o) $\dfrac{\frac{3}{8}}{2}$

   (p) $\dfrac{3}{\frac{2}{3}}$ (q) $\dfrac{-4}{\frac{2}{3}}$ (r) $\dfrac{\frac{5}{8}}{\frac{1}{2}}$

   (s) $\dfrac{\frac{3}{8}}{\frac{5}{8}}$ (t) $\dfrac{\frac{6}{8}}{-\frac{1}{2}}$ (u) $\dfrac{2\frac{1}{2}}{1\frac{1}{4}}$

   (v) $\dfrac{3\frac{3}{4}}{3}$ (w) $\dfrac{-4\frac{1}{2}}{\frac{3}{4}}$ (x) $\dfrac{\frac{1}{2}}{4\frac{1}{4}}$

   (y) $\dfrac{\frac{5}{6}-\frac{1}{2}}{\frac{2}{3}}$ (z) $\dfrac{\frac{1}{2}+\frac{1}{3}}{-\frac{1}{6}}$

3·8 Evaluation of Algebraic Expressions

In Section 2·5, we covered this topic with integers as the replacement set. In this section, we shall pursue the topic except that the replacement set will be the set of rational numbers (Q).

EXERCISE 3·8

1. Find the value of each expression for $n = \frac{2}{3}$. (Simplify if necessary.)

   (a) $9n - 4$ (b) $n^2 + \frac{1}{3}n$

   (c) $3n(2 + 9n)$ (d) $n^{-2} + 2n - \frac{5}{3}$

2. Evaluate each of the following for $x = \frac{5}{4}$ and $y = \frac{3}{8}$. (Simplify if necessary.)

   (a) $x - 2y$ (b) $\frac{2}{3}xy$ (c) $\frac{x - y}{3x}$

3. The perimeter of a rectangle (in metres) is given by $P = 2l - 2w$ where l and w are the length and width in metres. Calculate the perimeter in each case.

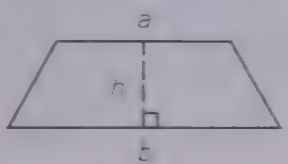
	l	w
(a)	3.2	1.6
(b)	4.5	2.7
(c)	1.7	0.8
(d)	14.5	8.5
(e)	13.8	7.4

4. The area of a triangle in square centimetres is given by $A = \frac{1}{2}bh$ where b is the measure of the base in centimetres and h is the measure of the height in centimetres.

   Calculate A for the following values of b and h.

	b	h
(a)	8	6
(b)	3	9
(c)	6	7
(d)	5.2	9.1
(e)	11.5	13.6
(f)	15.4	11.4
(g)	23	12

5. The area of a trapezoid in square centimetres is given by $A = \frac{1}{2}h(a + b)$ where h, a, and b are in centimetres and represent the lengths in the diagram.



   Calculate A.

	a	b	h
(a)	4	6	3
(b)	6	8	4
(c)	3	4	3
(d)	2.1	5.3	3.2
(e)	5.1	4.2	4.8
(f)	6.1	3.3	5.8
(g)	7.3	4.4	3.3

6. Find the value of each expression for $a = 1.02$, $b = 0.5$.

   (a) $3a - 7b$ (b) $\frac{a - b}{ab}$ (c) $7a + b^2$

7. Simplify each expression for $m = 3.2$, $n = 0.8$.

   (a) $\frac{4mn}{m + n}$ (b) $2m^2 - 7n^2$ (c) $\frac{3m}{5n} + \frac{8n}{5m}$

8. Evaluate and simplify:

   (a) $\frac{3x - 2y}{x + y}$ for $x = \frac{1}{7}$ and $y = \frac{1}{2}$

   (b) $\frac{5m}{3n} - \frac{3n}{5m}$ for $m = \frac{1}{4}$ and $n = \frac{1}{2}$

   (c) $4a^2 - 3b^2$ for $a = \frac{3}{5}$ and $b = \frac{7}{10}$

9. Evaluate and simplify. (Use $\pi = 3\frac{1}{7}$ or 3.14.)

   (a) $19n^2 - 42$ $(n = 0.6)$

   (b) $n^2 - \frac{n}{5} - 1.2$ $(n = 2.5)$

   (c) $n^3 - n^2 + n - \frac{1}{4}$ $(n = \frac{1}{2})$

   (d) $0.1\frac{a + b}{0.3c}$ $(a = 0.3,\ b = 0.8,\ c = 0.4)$

   (e) $4a - 5a^2$ $(a = \frac{1}{2})$

   (f) $\frac{3m^2 - 3n^2}{m - n}$ $(m = \frac{1}{2},\ n = \frac{1}{4})$

   (g) $V = \frac{4}{3}\pi r^3$ $(r = 3\frac{1}{2})$

   (h) $V = \frac{1}{3}\pi r^3$ $(r = 1\frac{1}{2})$

   (i) $A = 4\pi r^2$ $(r = 2.5)$

   (j) $V = e^3$ $(e = 3.5)$

   (k) $V = \frac{4}{3}\pi r^3$ $(r = 1.2)$

   (l) $V = \frac{1}{3}\pi r^2$ $(r = 4.5)$

   (m) $A = 4\pi r^2$ $(r = \frac{3}{4})$

   (n) $V = e^3$ $(e = \frac{1}{5})$

3·9 Exponential Notation and Scientific (Standard) Notation

We have had some experience with exponential notation. In Chapter 1 we expressed a power as some base to an exponent. Now we will concern ourselves with powers of base 10. Notice the following:

$10^2 = 100,\ 10^3 = 1000,\ 10^4 = 10\,000,\ 10^5 = 100\,000$

Do you see a pattern in this?

How many zeros will 10^8 have?

Write out with zeros: 10^{12}, 10^{20}.

Take the above examples and write them in a column like this.

$$\begin{aligned} 10^5 &= 100\,000 \\ 10^4 &= 10\,000 \\ 10^3 &= 1\,000 \\ 10^2 &= 100 \end{aligned}$$

Proceed with this column. Complete the right-hand side.

$$\begin{aligned} 10^1 &= ? \\ 10^0 &= ? \\ 10^{-1} &= ? \\ 10^{-2} &= ? \end{aligned}$$

Now check your answers to the above blanks.

$$\begin{aligned} 10^1 &= 10 \\ 10^0 &= 1 \\ 10^{-1} &= \tfrac{1}{10} \text{ or } 0.1 \\ 10^{-2} &= \tfrac{1}{100} \text{ or } 0.01 \end{aligned}$$

Complete this column:

$$\begin{aligned} 10^{-3} &= ? \\ 10^{-4} &= ? \\ 10^{-5} &= ? \\ 10^{-6} &= ? \end{aligned}$$

Do you get $10^{-6} = \dfrac{1}{1\,000\,000}$? What decimal number do you get for 10^{-6}? Which is it, 0.000 001 or 0.000 000 1?

The above pattern is the basis for *scientific notation;* sometimes it is called *standard notation*. Scientific notation is used by mathematicians, scientists, engineers, and others all around the world.

You can now learn how to write a number in scientific notation from the following examples.

Number		Number in Scientific Notation
$425 = 4.25 \times 100$		$= 4.25 \times 10^2$
$25 = 2.5 \times 10$		$= 2.5 \times 10^1$
$8.3 = 8.3 \times 1$		$= 8.3 \times 10^0$
$0.673 = 6.73 \times 0.1$		$= 6.73 \times 10^{-1}$
$0.079 = 7.9 \times 0.01$		$= 7.9 \times 10^{-2}$

Can you make a general statement on how a number is written in scientific notation? Where is the decimal always placed? By what is the decimal form always multiplied?

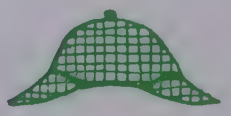
Do Questions 1 and 2 in Exercise 3·9.

Scientific notation is useful in computing. Here are examples in computing products.

Remember!

$$\begin{aligned} 10 \times 100 &= 10^1 \times 10^2 \\ &= 10^{1+2} \\ &= 10^3 \quad \text{agrees} \end{aligned} \qquad 10 \times 100 = 1000 = 10^3$$

$$\begin{aligned} 100 \times 1000 &= 10^2 \times 10^3 \\ &= 10^{2+3} \\ &= 10^5 \quad \text{agrees} \end{aligned} \qquad 100 \times 1000 = 100\,000 = 10^5$$



$10^n \times 10^m = 10^{n+m}$

Example 1

With the aid of scientific notation, compute the following:

(a) $7000 \times 1\,000\,000$ (b) 4200×500

(c) 0.75×0.004

Solution

(a) Using standard notation:

$7000 = 7.0 \times 10^3$ and $1\,000\,000 = 1.0 \times 10^6$

$\therefore \quad 7000 \times 1\,000\,000$

$= (7.0 \times 10^3) \times (1.0 \times 10^6)$

$= 7.0 \times 10^3 \times 1.0 \times 10^6$

$= (7.0 \times 1.0) \times (10^3 \times 10^6)$

$= 7.0 \times 10^9$. Did you follow this?

Check: You know that by long multiplication:

$7000 \times 1\,000\,000 = 7\,000\,000\,000 = 7.0 \times 10^9$.

(b) $4200 \times 500 = (4.2 \times 10^3) \times (5.0 \times 10^2)$

$= (4.2 \times 5.0) \times (10^3 \times 10^2)$

$= 21.0 \times 10^5$

$= (2.1 \times 10^1) \times 10^5$

$= 2.1 \times (10^1 \times 10^5)$

$= 2.1 \times 10^6$

When you learn this full procedure you may skip some of the above steps.

(c) $0.75 \times 0.004 = (7.5 \times 10^{-1}) \times (4.0 \times 10^{-3})$

$= (7.5 \times 4.0) \times (10^{-1} \times 10^{-3})$

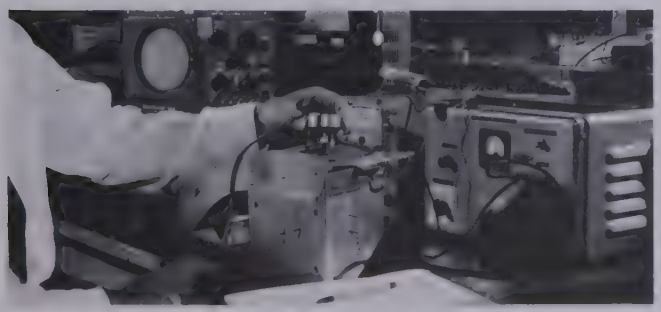
$= 30.0 \times 10^{-4}$

$= (3.0 \times 10^1) \times 10^{-4}$

$= 3.0 \times (10^1 \times 10^{-4})$

$= 3.0 \times 10^{-3}$

Did you follow every step?



Science is baffling at times.
Mathematics helps to explain science.

Example 2

Compute using scientific notation:

(a) $5\,000\,000 \div 1000$

(b) $36\,720 \div 120$

(c) $0.006\,25 \div 25$

Remember!

$1000 \div 100 = 10^3 \div 10^2 \qquad 1000 \div 100 = \dfrac{1000}{100}$

$= 10^{3-2} \qquad = 10^1$

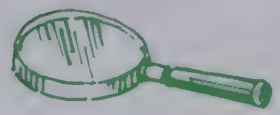
$= 10^1$ agrees

$10\,000 \div 100 = 10^4 \div 10^2 \qquad 10\,000 \div 100 = \dfrac{10\,000}{100}$

$= 10^{4-2} \qquad = 100$

$= 10^2$ agrees $\qquad = 10^2$

$10^n \div 10^m = 10^{n-m}$



Solution

(a) Using scientific notation:

$5\,000\,000 = 5.0 \times 10^6$ and $1000 = 1.0 \times 10^3$

$\therefore \dfrac{5\,000\,000}{1000} = \dfrac{5.0 \times 10^6}{1.0 \times 10^3} = 5.0 \times 10^3$.

Check:

$5\,000\,000 \div 1000$ can be written $\dfrac{5\,000\,000}{1000}$

$\dfrac{5\,000\,000}{1000} = 5000 = 5.0 \times 10^3$.

(b) $\dfrac{36\,720}{120} = \dfrac{3.672 \times 10^4}{1.2 \times 10^2}$

$= \dfrac{3.672}{1.2} \times 10^{4-2} \qquad \dfrac{10^4}{10^2} = \dfrac{10\,000}{100}$

$= 3.06 \times 10^2 \qquad = 100 = 10^2$

(c) $\dfrac{0.006\,25}{25} = \dfrac{6.25 \times 10^{-3}}{2.5 \times 10^1}$

$= \dfrac{6.25}{2.5} \times 10^{-3-1}$

$= 2.5 \times 10^{-4}$

EXERCISE 3·9

1. Write each number in scientific notation.

   (a) 620 000 (b) 4698 (c) 0.0099

   (d) 0.000 526 (e) 84 670 000 (f) 65

   (g) 0.444 (h) 0.000 000 12 (i) 71 124

2. Express the following data on the size of Canada in scientific notation.

   (a) From northern tip to southern tip — 4620 km

   (b) From eastern tip to western tip — 5190 km

   (c) Ocean coastline of mainland — 28 700 km

   (d) Ocean coastline of islands — 67 300 km

   (e) Area — 10 000 000 km^2

   (f) Population — 22 100 000

   (g) Area of sanctuaries for wildlife — 404 000 km^2

   (h) Amount of railway tracks — 96 500 km

   (i) Amount of roads — 772 000 km

   (j) Length of Trans-Canada Highway — 7850 km

   (k) Length of St. Lawrence Seaway — 3660 km

3. Compute with the aid of scientific (standard) notation.

   (a) 320×2000 (b) 345×15

   (c) 0.4×0.18 (d) 6400×0.05

   (e) $0.012 \times 0.000\,06$ (f) $640 \div 20$

   (g) $640 \div 80$ (h) $48\,000 \div 240$

   (i) $9.8 \div 0.007$ (j) $0.000\,84 \div 21$

   (k) $0.0096 \div 0.003$ (l) $\dfrac{25 \times 4000}{0.005}$

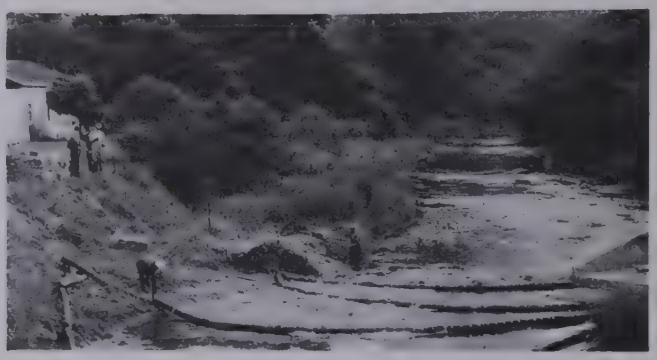
   (m) $\dfrac{6000 \times 35\,700}{420\,000}$

4. To burn one litre of fuel oil requires 1.5×10^1 kg of air. If a homeowner uses an average of 32.6 ℓ of oil per day, how many kilograms of air will be used in heating his home for 3.0×10^1 days? (Express the answer in scientific notation.)

5. It is estimated that it would cost about 2.7×10^9 dollars yearly to clean up our polluted waters over a five-year period.

   (a) How much would such a program cost over the five years?

   (b) If our population in those five years averages 2.25×10^7, what would be the cost per person in those five years?



6. Refer to Question 2. What is the average population per square kilometre in Canada?

7. Round all the data in Question 2 to two digits followed by zeros (we say "rounded to two significant figures", e.g., 2875 → 2900).

8. Use the data in Question 7. What is the average population per square kilometre in Canada? Compare this answer with that of Question 6. Discuss.

9. Use the data in Question 2.

   (a) If Canada were a true rectangle, what would be its area?

   (b) The number of kilometres of roads is how many times that of railroads?

   (c) What is the total perimeter of Canada's ocean coastline?

   (d) What is the total distance of roads and railroads in Canada?

10. If the mass of the earth is about 5.4×10^{21} t and the mass of the sun is about 2.2×10^{25} t, express the mass of the earth as a fraction of the mass of the sun.

11. The speed of light is about 2.98×10^5 km/s. How far does light travel in one year?

12. The nearest known star beyond our sun is about 4×10^{13} km from the earth. How long does it take light from the star to reach earth?

13. The number of molecules in 1 g of hydrogen gas is 600 000 000 000 000 000 000 000. There are 30 000 000 000 000 000 000 000 000 molecules in 1 g of water. Write each of these numbers using scientific notation. Compare the number of molecules in 1 g of hydrogen with the number in 1 g of water.

14. The radius of the nucleus of an atom is 0.000 000 000 000 3 cm. Rename this number in scientific notation.

15. The moon is about 403 000 km from the earth. The blastoff velocity of Apollo 10 was 38 700 km/h. Its slowest speed was 3260 km/h. What would have been the number of hours for the one-way trip if:
    (a) the blastoff speed had been maintained?
    (b) the slowest speed had been the average?

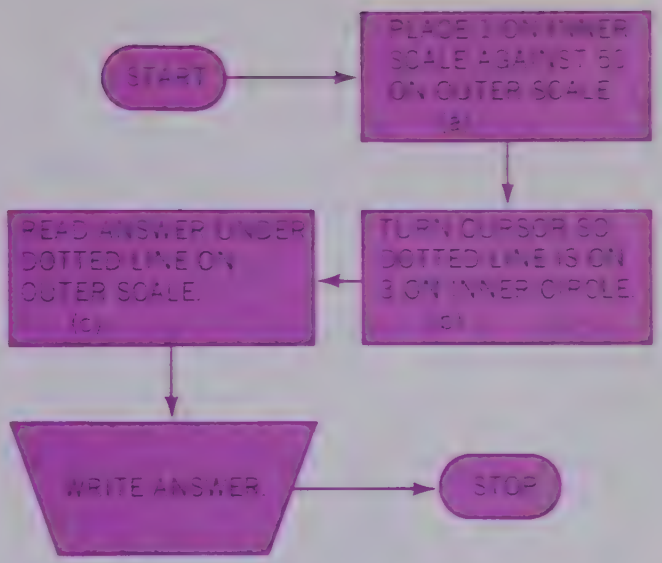
3·10 Using Your Slide Rule With Large Numbers

Example 1

Use your slide rule to multiply: 50 × 3.

Solution

You know the answer before you start, but can you do this problem on your slide rule? Here is some help.


START
PLACE 1 ON INNER SCALE AGAINST 50 ON OUTER SCALE
(a)
TURN CURSOR SO DOTTED LINE IS ON 3 ON INNER CIRCLE
(b)
READ ANSWER UNDER DOTTED LINE ON OUTER SCALE.
(c)
WRITE ANSWER.
STOP

Did you have problems in step (b)? How did you interpret the number "1.5" under the dotted line?

Always estimate your answer before starting.

Let us use scientific notation to help us employ the slide rule when multiplying large numbers.

Example 2

Multiply 62 × 46.

Solution

Estimate first: 60 × 50 = 3000.

The answer should be near 3000.

Write the problem using scientific notation:

$$\begin{aligned} 62 \times 46 &= 6.2 \times 10^1 \times 4.6 \times 10^1 \\ &= 6.2 \times 4.6 \times 10^1 \times 10^1 \\ &= 6.2 \times 4.6 \times 10^2 \end{aligned}$$

Using the slide rule, we multiply 6.2 × 4.6.

The answer is approximately 28.5.

Therefore, $62 \times 46 = 28.5 \times 100$
$= 2850.$

EXERCISE 3·10

1. What does the symbol $\simeq$ mean?

2. Estimate the product in each, then use the slide rule to calculate each of the following.

   (a) 13×23 (b) 39×15
   (c) 55×78 (d) 14×35
   (e) 16×87 (f) 99×44
   (g) 655×740 (h) 749×856
   (i) 183×917

3. Play this game with your partner. Flip a coin to see who starts. Give two 2-digit numbers to your partner who is to estimate their product orally. If the answer is correct give another two until a wrong answer is given. Then it is your partner's turn to give you products to estimate. Keep score to see who reaches 20 correct answers first to win the game. Variations: Give three 2-digit numbers.

   Give two or three 3-digit numbers.

4. Play the game in Question 3 using your slide rules instead of estimating the products.

5. You should remember the squares of the whole numbers 1 to 15.

   Complete this chart:

$$1 \times 1 = ?$$
$$2 \times 2 = ?$$
$$\vdots$$
$$15 \times 15 = ?$$

6. Using your slide rule, what number multiplied by itself yields:

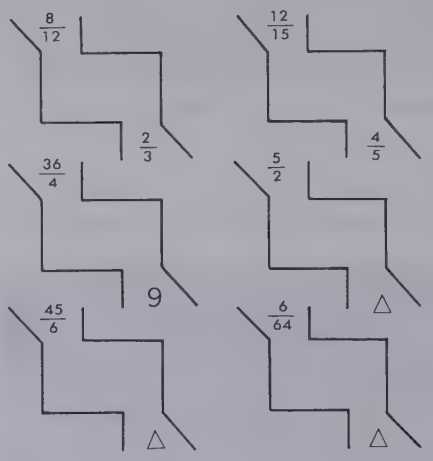
   (a) 49 (b) 64 (c) 81
   (d) 121 (e) 144 (f) 225

7. What number multiplied by itself yields:

   (a) 50 (b) 10 (c) 75

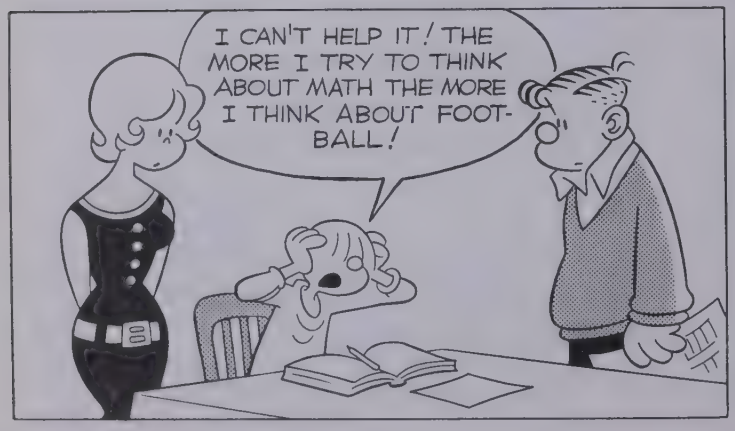
3·11 Percentage Function Machine

1. Complete the output for each:

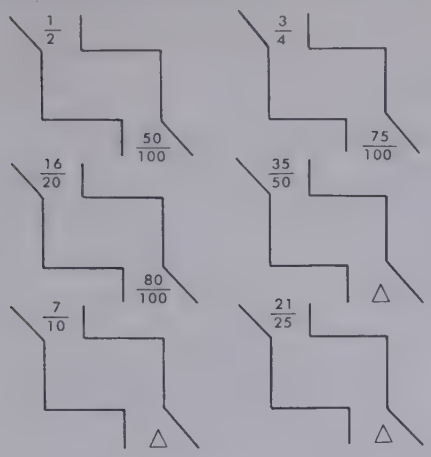


Input □	Output △
$\frac{6}{8}$	
$\frac{12}{30}$	
$\frac{7}{35}$	
$\frac{8}{81}$	
$\frac{15}{100}$	

Tell in your own words what the operation of this Function Machine is. Explain how you obtained each answer.

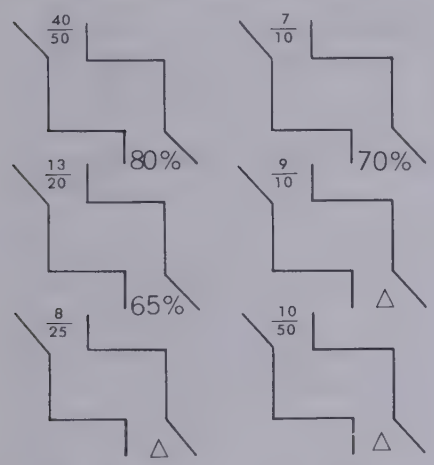


2. Complete the output for each:



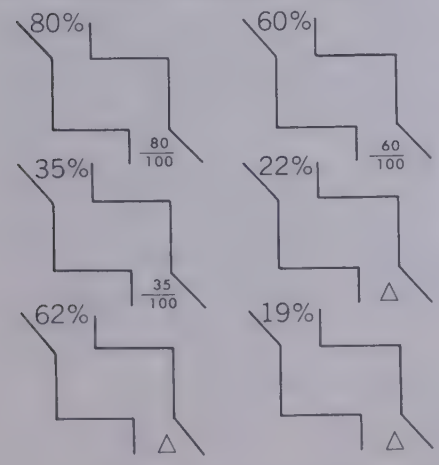
Input □	Output △
$\frac{1}{4}$	
$\frac{19}{20}$	
$\frac{40}{50}$	
$\frac{8}{10}$	
$\frac{32}{50}$	
$\frac{9}{20}$	
$\frac{8}{40}$	
$\frac{30}{40}$	

3. Complete the output for each:



Input □	Output △
$\frac{20}{50}$	
$\frac{12}{20}$	
$\frac{23}{25}$	
$\frac{41}{50}$	
$\frac{49}{50}$	
$\frac{12}{40}$	

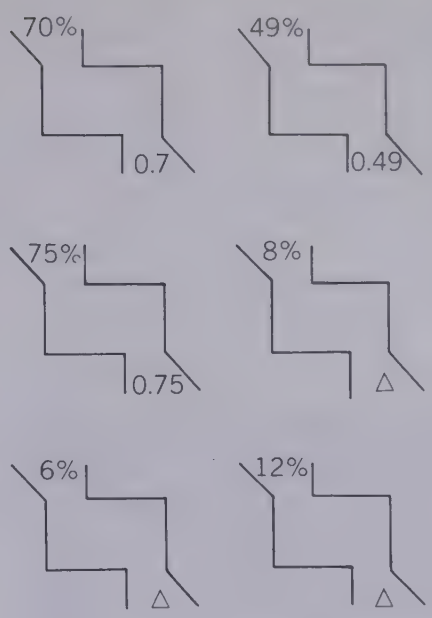
4. Complete the output for each:



Input □	Output △
45%	
33%	
56%	
72%	
12%	

State what this Function Machine does.

5. Complete the output for each:



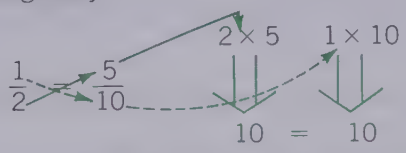
Input □	Output △
63%	
59%	
72%	
9%	
2%	
1%	
	0.83
	0.78
	0.32
	0.9

State in words the operation of this Function Machine.

6. We know that

$$\frac{1}{2} = \frac{5}{10}$$

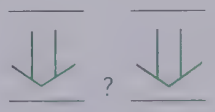
Now observe what we get when we multiply in this strange way:



Does this always happen?

Multiply in these equivalent fractions to see if the above pattern always occurs.

(a) $\frac{3}{4} \times \frac{9}{12}$



(b) $\frac{3}{6} = \frac{1}{2}$

(c) $\frac{5}{8} = \frac{25}{40}$

(d) $\frac{8}{9} = \frac{40}{45}$

(e) $\frac{7}{13} = \frac{56}{104}$

7. We can use what we learned in Question 6 to solve problems like this: $\frac{3}{4} = \frac{n}{20}$.

$$\frac{3}{4} = \frac{n}{20}$$
$$4n = 3 \times 20$$
$$\frac{4n}{4} = \frac{60}{4} \quad \text{(Divide each side by 4.)}$$
$$n = 15$$

Use this method to solve each:

$\frac{5}{6} = \frac{n}{30}$ $\frac{7}{8} = \frac{n}{32}$ $\frac{9}{12} = \frac{n}{75}$ $\frac{4}{16} = \frac{n}{36}$

$\frac{7}{8} = \frac{n}{100}$ $\frac{5}{6} = \frac{n}{100}$ $\frac{41}{60} = \frac{n}{100}$ $\frac{22}{30} = \frac{n}{100}$

Do Questions 1 to 7 in Exercise 3·11.

Review: Three Percent Problems

Type I 75% of 90 = x

Solution

$$\frac{75}{100} = \frac{x}{90}$$
$$100x = 75 \times 90$$
$$\frac{100}{100}x = \frac{6750}{100} \quad \text{(Divide each side by 100.)}$$
$$x = 67.5$$

$\therefore$ 75% of 90 = 67.5

Type II 30 is n% of 70.

Solution

$$\frac{30}{70} = \frac{n}{100}$$
$$70n = 100 \times 30$$
$$\frac{70}{70}n = \frac{3000}{70} \quad \text{(Divide each side by 70.)}$$
$$n = 42.9$$

$\therefore$ 30 is 42.9% of 70.

Type III 45 is 60% of y.

Solution

$$\frac{60}{100} = \frac{45}{y}$$
$$60y = 45 \times 100$$
$$y = \frac{4500}{60}$$
$$= 75$$

$\therefore$ 45 is 60% of 75.

EXERCISE 3·11

1. Simplify each as indicated:

$\frac{6}{8} = \frac{3}{4}$ $\frac{15}{20}$ $\frac{11}{33}$ $\frac{4}{20}$ $\frac{25}{100}$ $\frac{75}{100}$ $\frac{60}{100}$

$\frac{80}{100}$ $\frac{20}{60}$ $\frac{35}{49}$ $\frac{56}{64}$ $\frac{81}{108}$ $\frac{45}{81}$ $\frac{72}{96}$

2. Write each as a fraction with a denominator of 100:

$\frac{3}{4} = \frac{75}{100}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{4}{10}$ $\frac{7}{10}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{7}{20}$

$\frac{13}{50}$ $\frac{19}{20}$ $\frac{23}{50}$ $\frac{4}{5}$ $\frac{17}{20}$ $\frac{19}{25}$ $\frac{13}{20}$ $\frac{24}{25}$

3. Write each as a fraction with denominator of 100:

$70\% = \frac{70}{100}$	55%	35%	41%	62%
10%	5%	1%	3%	100%
110%	300%	500%	2%	38%

4. Write each as a decimal fraction:

74% = 0.74	55%	63%	73%	42%
10%	5%	1%	3%	100%
150%	300%	142%	50%	75%

5. Change the following to fractions with denominators of 100:

$0.73 = \frac{73}{100}$	0.35	0.49	0.85	0.9
0.99	0.10	0.05	0.01	0.1	0.03
1.15	2.35	2.0	3.0	5.1	4.0

6. Change the following to percents:

$\frac{48}{100} = 48\%$ $\frac{35}{100}$ $\frac{98}{100}$ $\frac{39}{100}$ $\frac{18}{100}$ $\frac{40}{100}$

$\frac{10}{100}$ $\frac{5}{100}$ $\frac{1}{100}$ $\frac{114}{100}$ $\frac{125}{100}$ $\frac{155}{100}$

7. Change the following to percents:

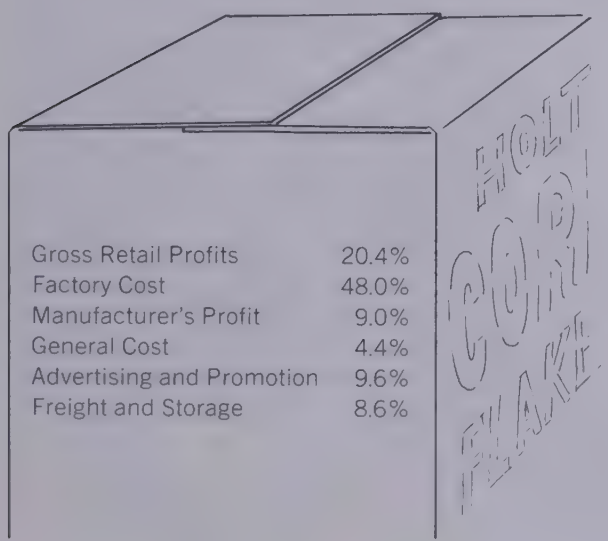
0.38 = 38%	0.41	0.62	0.73	0.84
0.10	0.01	0.03	1.25	1.00
4.00	3.00	3.40	0.40	0.50

8. Solve each of the following:
   (a) 60% of 45 = x (b) 75% of 30 = x
   (c) 62% of 86 = x (d) 33% of 52 = x
   (e) 23% of 48.5 = x (f) 35.5% of 72 = x
   (g) $12\frac{1}{2}$% of 32 = x (h) 37.5% of 80 = x

9. There are 35 students in the grade 9 class.
   40% of the class are absent.
   How many students are absent?
   How many students are present?

10. There are 36 students in Home Room 9B.
    The teacher said, "44.4% of the class are girls."
    How many girls are in 9B?
    How many boys are in 9B?

11. If a box of cornflakes costs 84¢, calculate to the nearest cent each:
    (a) gross retail profit;
    (b) factory cost;
    (c) manufacturer's profit;
    (d) general cost;
    (e) advertising and promotion cost;
    (f) freight and storage cost.



12. Solve each of the following (correct to one decimal place):
    (a) 40 is n% of 50 (b) 32 is n% of 80
    (c) 64 is n% of 80 (d) 38 is n% of 84
    (e) 11.5 is n% of 96 (f) 1.8 is n% of 36
    (g) 28 is n% of 104 (h) 3 is n% of 125

13. There are 32 students altogether in 9F.
    There are 8 students absent today.
    What percent of the class is absent?
    What percent of the class is present?

14. Study this ad which appeared in a newspaper.

    Calculate the percentage each bike has been reduced. Which sale is the best deal?

15. Solve each of the following:
    (a) 36 is 42% of y (b) 54 is 13% of y
    (c) 3 is 8% of y (d) 115 is 90% of y
    (e) 60 is 150% of y (f) 86 is 72% of y
    (g) 88 is 100% of y (h) 88 is 10% of y

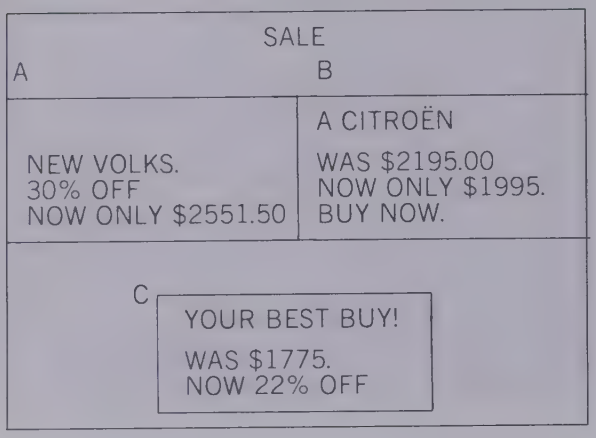
16. Only 24 students in class 9D are present.
    This is 80% of the whole class.
    How many are in the whole class?

17. There are only 496 students in the school today.
    This is 64% of the total enrolment.
    How many are enrolled in the school?

18.

Calculate the regular price of each item.

19.



(a) What was the original price of the Volks?

(b) What percent has the Citroën been reduced?

(c) What is the sale price of the "Best Buy"?

REVIEW EXERCISE 3

1. If point R divides the space between $3\frac{1}{4}$ and $3\frac{3}{8}$ on the Q number line into two equal parts, what number does point R represent?

2. If point A divides the space between $2\frac{1}{3}$ and $2\frac{5}{6}$ on the Q-line into two equal parts, what number does this point represent?

3. Express each in decimal form. Then state whether it is a terminating or repeating decimal.

   (a) $\frac{7}{8}$ (b) $-\frac{5}{16}$ (c) $-7\frac{1}{3}$ (d) $\frac{5}{7}$ (e) $4\frac{23}{40}$

4. Perform the indicated operations.

   (a) $\frac{2}{5}+\frac{3}{4}$ (b) $4\frac{3}{8}-2\frac{9}{16}$

   (c) $\frac{2}{9}+\frac{5}{12}-\frac{5}{6}$ (d) $3\frac{3}{5}-1\frac{5}{8}-\frac{7}{10}$

   (e) $346.2-97.54+13$ (f) $\dfrac{3.69\times 25.6}{1.44\times 82}$

5. Express each in fraction form. Then simplify if necessary.

   (a) 0.008 (b) 6.375 (c) 0.056

   (d) 0.12 (e) 0.0075

6. Simplify:

   (a) $\dfrac{\frac{3}{4}}{\frac{5}{9}}$ (b) $\dfrac{\frac{7}{8}}{\frac{5}{12}}$ (c) $\dfrac{\frac{3}{5}+\frac{7}{10}}{\frac{11}{15}}$

   (d) $\dfrac{\frac{4}{9}-\frac{1}{12}}{\frac{5}{6}-\frac{7}{18}}$ (e) $\dfrac{\frac{3}{4}+\frac{1}{3}}{\frac{3}{4}-\frac{1}{3}}$ (f) $\dfrac{\frac{3}{8}-\frac{1}{12}}{\frac{3}{6}+\frac{1}{8}}$

7. Evaluate and simplify:

   (a) $8a-7b$ for $a=\frac{3}{4}$, $b=\frac{1}{2}$.

   (b) $3x(4y-x)$ for $x=0.75$ and $y=1.2$.

   (c) $\dfrac{2m}{3n}-\dfrac{4n}{5m}$ for $m=\dfrac{3}{4}$ and $n=\dfrac{2}{5}$.

8. Express in standard notation:

   (a) 10 000 000 (b) 7268 (c) 0.084

   (d) 0.000 059 6 (e) 836×10^{-7}

9. Simplify:

   (a) $\frac{3}{4}-\frac{1}{2}\div\frac{3}{4}$ (b) $\frac{2}{3}\div\frac{3}{4}-\frac{4}{5}\times 5$

   (c) $3.2\times 4-6.4\div 2$ (d) $(6-8)\div(4+8)$

   (e) $2\frac{1}{2}\times 4+5.5\div 5$

   (f) $(18-6\div 3)\times(6.4\div 4-2)$

   (g) $(\frac{1}{2}-\frac{3}{4}\div 2)\times(\frac{3}{4}\div 2-1)$

   (h) $(1\frac{1}{2}\div 2-3)\times(2\div 3-\frac{1}{3})$

10. Write each as a fraction with denominator of 100:

    (a) $\frac{4}{5}$ (b) $\frac{13}{20}$ (c) $\frac{121}{25}$

11. Write each as a fraction with denominator of 100:

    (a) 39% (b) 51% (c) 188% (d) 3%

12. Write each as a decimal fraction:
    (a) 65% (b) 25% (c) 133% (d) 8%

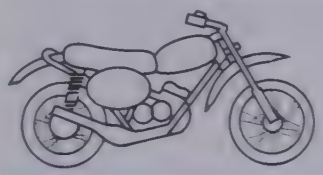
13. Write each as a fraction with denominator of 100:
    (a) 0.39 (b) 0.69 (c) 5.15 (d) 0.04

14. Write each as a percent:
    (a) $\frac{36}{100}$ (b) $\frac{49}{100}$ (c) $\frac{125}{100}$ (d) $\frac{7}{100}$

15. Write each as a percent:
    (a) 0.45 (b) 0.75 (c) 4.05 (d) 0.09

16. Solve each:
    (a) 35% of $55 = x$ (b) 68.5% of $250 = x$
    (c) 80 is n% of 120 (d) 8.4 is n% of 56
    (e) 90 is 45% of y (f) 36 is 28.8% of y

17. What is the percent off on this motorcycle?



SALE
WAS: $560
NOW: $420

Project

Project 4

Working as a class, assume that you all must quit school today. Then survey the class as to what kind of job each classmate would like to obtain. Remember the job would have to be in line with the qualifications of each individual. A student could not say "Doctor", since obviously he is not qualified to be one.

(a) Choose some method of charting your data.

(b) Now survey the class to determine what each classmate would like to be *if* he or she could get the training, qualifications, and jobs they would like best.

(c) Do you notice any differences in the results of the first part and second part of the survey? Discuss.

(d) Do your classmates think it is worth staying in school?

(e) What must you do to obtain the job you would really like to get for a lifetime occupation?

Project 5

Over the next seven days, watch the ads on TV or listen to them on radio. Record the total number. Also record the number of each type listed. (Do you want to add more types?):

	Number	% of Total
Food		
Car		
Soft drinks		
Sports		
Clothes		
Movies		
Personal hygiene		
Others		
Total		100%

Copy and complete the chart

Example

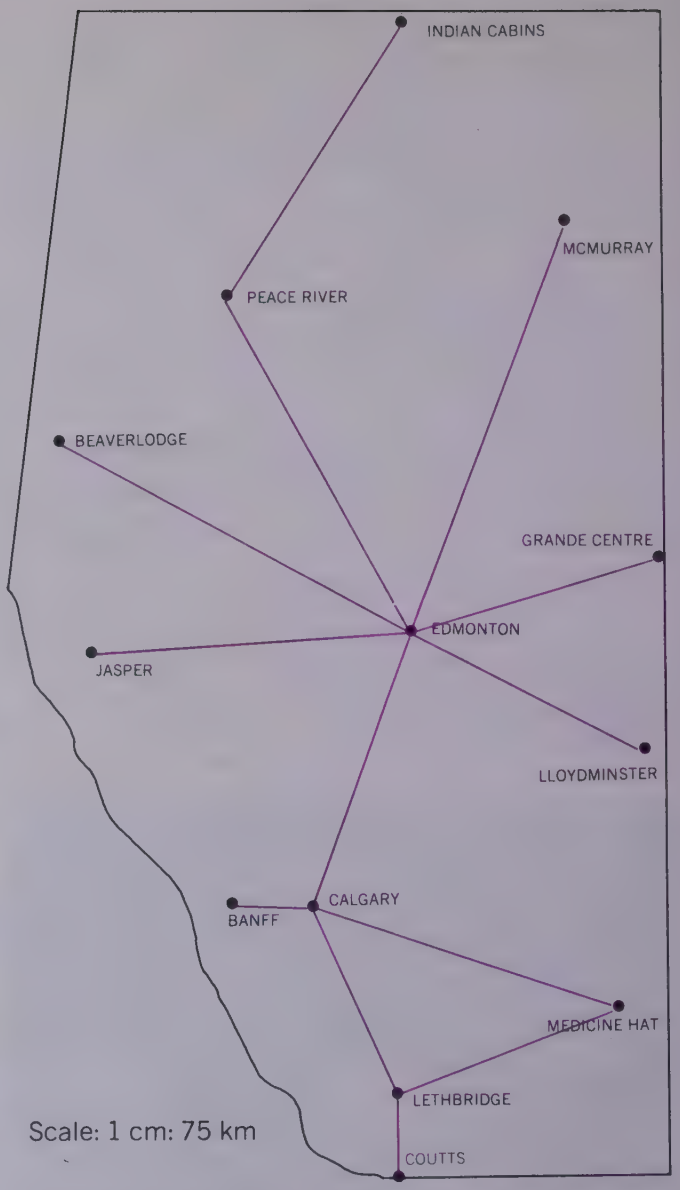
Edmonton to Grande Centre is 3.6 cm.

Solution

1 cm represents 75 km.
3.6 cm represents x km.

$$\frac{1}{3.6} = \frac{75}{x}$$
$$x = 3.6 \times 75$$
$$x = 270$$

The distance from Edmonton to Grande Centre is 270 km.



1. Calculate each of the following:

From	To	Centimetres on Map	Kilometres
Coutts	Lethbridge		
Lethbridge	Calgary		
Calgary	Medicine Hat		
Calgary	Edmonton		
Calgary	Banff		
Edmonton	Peace River		

Refer to the Appendix if you are not familiar with or are having difficulty with metric units.

2. Make up a chart as in Question 1 to find all the distances between the towns given on the map.

3. What is the length of:
   (a) the Alberta-United States border?
   (b) the Alberta-Saskatchewan border?
   (c) the Alberta-Northwest Territories border?
   (d) the Alberta-British Columbia border?

Using the Library

Reading in Mathematics, Book 1, Adler, I., Ginn and Company, (1972).

A Read pages 9-12. Use your hands to multiply these numbers.

(a) 7×9 (b) 6×7 (c) 8×8

(d) 12×13 (e) 12×15 (f) $\square \times 10$

(g) 16×18 (h) 17×18 (i) 19×19

B Read pages 56-64.

(a) What is a [illegible]? What is a [illegible] number?

(b) Write all the [illegible] prime numbers.

(c) How many different ways can [illegible] be written as the product of [illegible]?

(d) What are the factors of 7?

(e) What is a perfect number? Show that 6 and 28 are perfect numbers.

(f) Can you find another perfect number?

(g) Refer to Feedback, Correction, page 63. What is [illegible] about every even number greater than 4 can be written as the sum of two odd prime numbers? Is this statement true or false?

C Read pages 179-181 and then answer (a).

Reading in Mathematics, Book 2, Adler, I., Ginn and Company, (1972). Read page 179. If you like this poem, use the library to get the complete works of *The Pirates of Penzance* by W. S. Gilbert.

D Use the library to prepare a report on the history of the automobile.

E Story of *Mathematics For Young People*, Rogers, J. T., Random House of Canada Ltd., Toronto, (1966). Read page 11. Perform the magic trick described. Can you explain why it works?

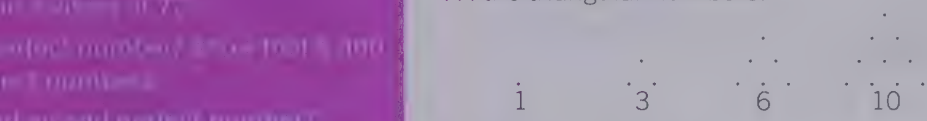
Do You Remember . . .

. . . that in expanded notation, 467.5 is written as:

$$(4 \times 10^2) + (6 \times 10^1) + (7 \times 1) + (5 \times 10^{-1})?$$

How would you write 8 027 359 in expanded notation?

. . . the triangular numbers?



1 3 6 10 etc.

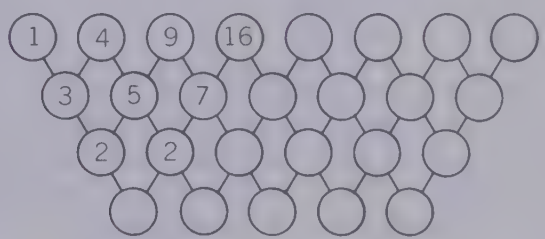
What are the next six triangular numbers? Which of the first ten triangular numbers are multiples of seven? Which are multiples of eleven? What is the quotient when:

(a) the ninth number is divided by the fifth number?

(b) the tenth number is divided by the fourth number?

Something to Think About

Study this pattern. Complete it by filling the empty circles.



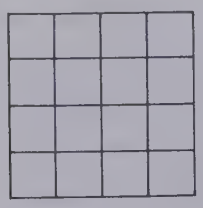
(a) What is the sequence formed by the first row numbers?

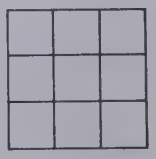
(b) How are the second row numbers related to the first row numbers? Do the second row numbers form a sequence?

(c) If the same operation is continued from Row 2 to Row 3 as from Row 1 to Row 2, what are the numbers of Row 3?

(d) When the pattern is continued, what numbers fill the bottom row?

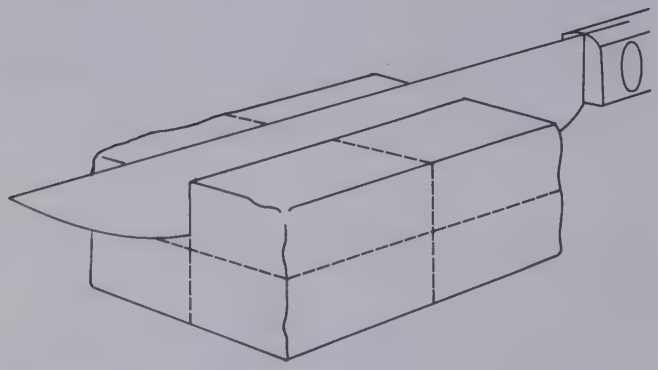
How can the two following squares be cut into four pieces so that the four pieces can be rearranged to form a five by five square? The pattern of the five by five square must also be a checkerboard.





Have You Discovered that . . .

. . . sums are not what they seem?



This is the picture of a family-size block of ice cream. It has been cut into equal pieces by three cuts through its centre as shown. There is one piece each for the mother, the father, and each of their n children, with one piece left over for seconds.

(a) How many children are there?

The piece left over is cut up in the same way as the large piece, giving second helpings all around with one piece left over for third helpings. Suppose that one continues to divide the ice cream once, twice, three times, four times, etc.

(b) What fractions of the block did Father eat for his:

(i) first helping?

(ii) second helping?

(iii) third helping?

(c) What fraction of the ice cream did Father eat altogether?

(d) From (b) we know that Father ate this much ice cream.

$$\tfrac{1}{8} + (\tfrac{1}{8})^2 + (\tfrac{1}{8})^3 + \ldots$$

From (c) we know Father ate $\frac{1}{7}$ of the ice cream. (Since there are 7 in the family, he got $\frac{1}{7}$.)

Therefore,

$$\tfrac{1}{8} + (\tfrac{1}{8})^2 + (\tfrac{1}{8})^3 + (\tfrac{1}{8})^4 + \ldots = \tfrac{1}{7}.$$

Do you believe this?

(e) Now divide a block of ice cream into 4 pieces, serving a piece to each of three people and cutting the piece left over into 4 more equal pieces, and continue to do this four, five, or more times.

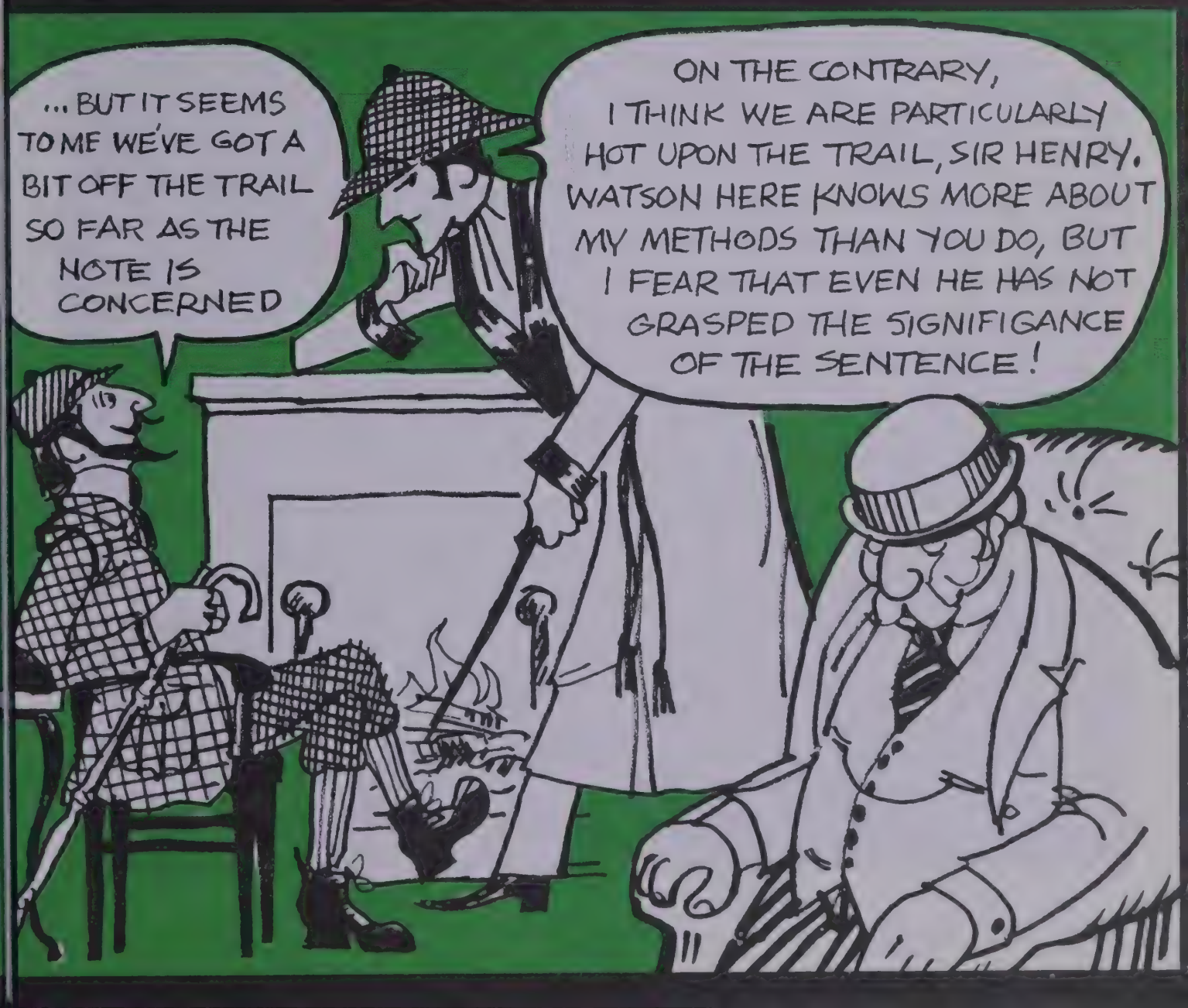
How much does any one person eat by the time all the ice cream is gone?

How much does a person eat on his first serving, second serving, etc?

What is the sum of:

$$\tfrac{1}{4} + (\tfrac{1}{4})^2 + (\tfrac{1}{4})^3 + (\tfrac{1}{4})^4 + \ldots ?$$

Chapter Four
MORE MISSING ELEMENTS
Irrational Numbers


...BUT IT SEEMS TO ME WE'VE GOT A BIT OFF THE TRAIL SO FAR AS THE NOTE IS CONCERNED
ON THE CONTRARY, I THINK WE ARE PARTICULARLY HOT UPON THE TRAIL, SIR HENRY. WATSON HERE KNOWS MORE ABOUT MY METHODS THAN YOU DO, BUT I FEAR THAT EVEN HE HAS NOT GRASPED THE SIGNIFIGANCE OF THE SENTENCE!

Project

Project 1

Purpose

To find relationships in the rectangular numbers, especially those related to prime and composite numbers.

Materials

Geo-paper

Method

Let us represent the natural numbers by dots placed in rows and columns on geo-paper, wherever possible.

The first rectangular number is four. The dots of the number four and the number six form rectangles. A square is a special rectangle!

Why are the numbers 1, 2, and 3 not rectangular numbers?

Rectangular numbers are those natural numbers which can be represented by dots in rectangular form.

Square numbers are those natural numbers which can be represented by dots in a square form.

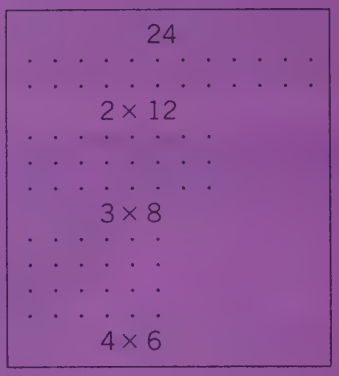
A. Which of the natural numbers between 6 and 50 are rectangular numbers? Which are square numbers? On geo-paper, show the rectangular numbers and the square numbers as rectangular arrangements of dots.

B. Which numbers between 6 and 50 are not square or rectangular numbers? What are these numbers called?

C. Prepare a class bulletin board display of the different arrangements of the rectangular numbers. Each member of the class should take one number. Make all the square numbers the same color.

Notice that:

$2 \times 12 = 24$
$3 \times 8 = 24$
$4 \times 6 = 24$
$1 \times 24 = 24$



We say 2, 12, 3, 8, 4, 6, 1, 24 are *factors* of 24.

What are the factors of 18?

What two factors of any number do not produce a rectangle?

D. "To classify" means to divide into groups according to some rules. How many ways can you classify the natural numbers less than 25? Show each classification. State the rules you used for each classification scheme. Discuss each classification scheme in class.

E. You may have noted that the number 4 has only one rectangular form; that the number 6 has only one rectangular form; and that the number 24 has three rectangular forms. Complete the following chart for the natural numbers less than 40.

Number of Rectangular Forms			
1	2	3	4
$4 = 2 \times 2$	$12 = 2 \times 6$	$24 = 2 \times 12$	$36 = 2 \times 18$
$6 = 2 \times 3$	$12 = 3 \times 4$	$24 = 3 \times 8$	$36 = 3 \times 12$
	$32 = 2 \times 16$	$24 = 4 \times 6$	$36 = 4 \times 9$
	$32 = 4 \times 8$		$36 = 6 \times 6$

Prime numbers are those natural numbers greater than 1 which are not rectangular numbers.

A *composite number* is another name for a rectangular number.

F. List the prime numbers less than 50.

G. List the prime numbers between 50 and 100.

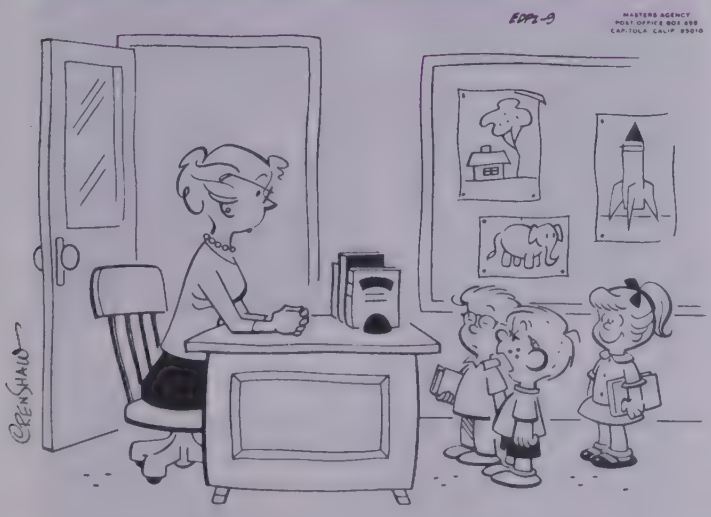
H. List the composite numbers less than 50.

I. List the composite numbers between 100 and 120.

J. List all the factors of each of the following numbers:

(a) 60	(b) 61	(c) 62
(d) 63	(e) 64	(f) 65
(g) 66	(h) 67	(i) 68

K. Which of the numbers in J. have only the number 1 and the number itself as factors? What is the name of these numbers?



" WHEN DO WE LEARN HOW TO MAKE A FAST BUCK ? "

4·1 Areas of Rectangles

Eric was planning a patio. He drew this plan.

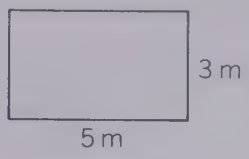


Figure 4-1

He wanted to calculate its area as the first step to calculating the amount of cement needed.

$$A = \ell w$$
$$A = 5 \times 3$$
$$A = 15$$

The area is 15 m². What does the symbol m² mean? Eric decided this patio was too small. He drew three patios as shown.

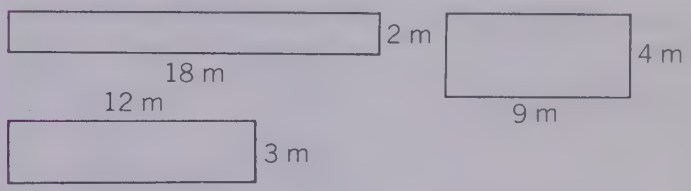


Figure 4-2

Work out the area of these three patios. What do you observe about their areas?

Using only natural numbers for length and width, design two more patios that have areas equal to these three in Figure 4-2. What are the dimensions of your patios?

Did you choose one rectangle 1 m by 36 m?

Another patio could be 6 m by 6 m. This last patio is a square. We label a square as shown here.

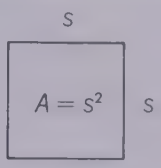


Figure 4-3

Since the length and width are the same, we use the same symbol for both.

Example 1

A square has an area of 121 cm². Find the length of one side of the square.

Solution

$$A = s^2 \text{ or } s^2 = A$$

By substitution: $s^2 = 121 \text{ cm}^2$

$s \times s = 121 \text{ cm}^2$

$\therefore\ s = 11 \text{ cm}\ (11 \times 11 = 121).$

One side of the square is 11 cm.

Example 2

A rectangle is 16 cm long and 4 cm wide. A square has the same area. What are the dimensions of the square?

Solution

Area of rectangle: $A = \ell w$

By substitution: $A = 16 \text{ cm} \times 4 \text{ cm}$

$A = 64 \text{ cm}^2.$

Area of square: $A = s^2$

$s^2 = 64 \text{ cm}^2$

$s \times s = 64 \text{ cm}^2$

$s = 8 \text{ cm}\ (8 \times 8 = 64).$

The square is 8 cm to a side.

EXERCISE 4·1

1. The following are areas of squares. For each of the squares, find the length of one side.

   (a) 49 cm² (b) 169 m²

   (c) 25 m² (d) 81 cm²

   (e) 225 m² (f) 16 m²

   (g) 36 cm² (h) 100 m²

   (i) 144 cm² (j) 121 cm²

   (k) 400 m² (l) 900 cm²

   (m) 1600 m² (n) 3600 m²

   (o) 6400 m² (p) 2500 cm²

   (q) 8100 cm² (r) 4900 m²

2. A rectangle is 12.5 m long and 8 m wide. What are the dimensions of the square that would have an area equal to this rectangle?

3. Jack constructed a rectangle 8 cm long and 2 cm wide. Jane constructed a square of equal area. What was the measure of the side of Jane's square?

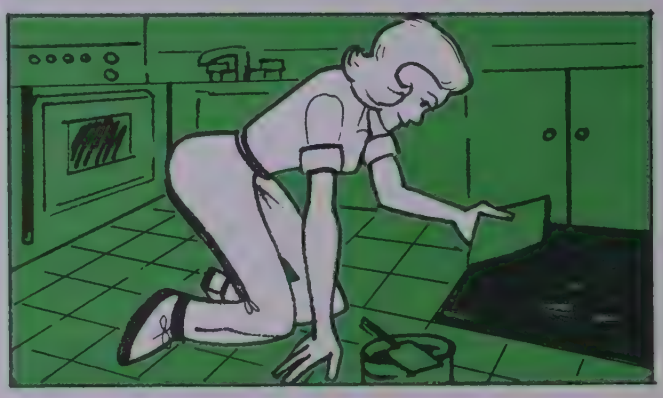
4. A square lot of 10 m on a side is traded for a lot of equal area. The length of the new lot is 25 m. What is the width?

5. A rectangle is 16 cm by 9 cm. What are the dimensions of the square that has an area equal to that of the rectangle?

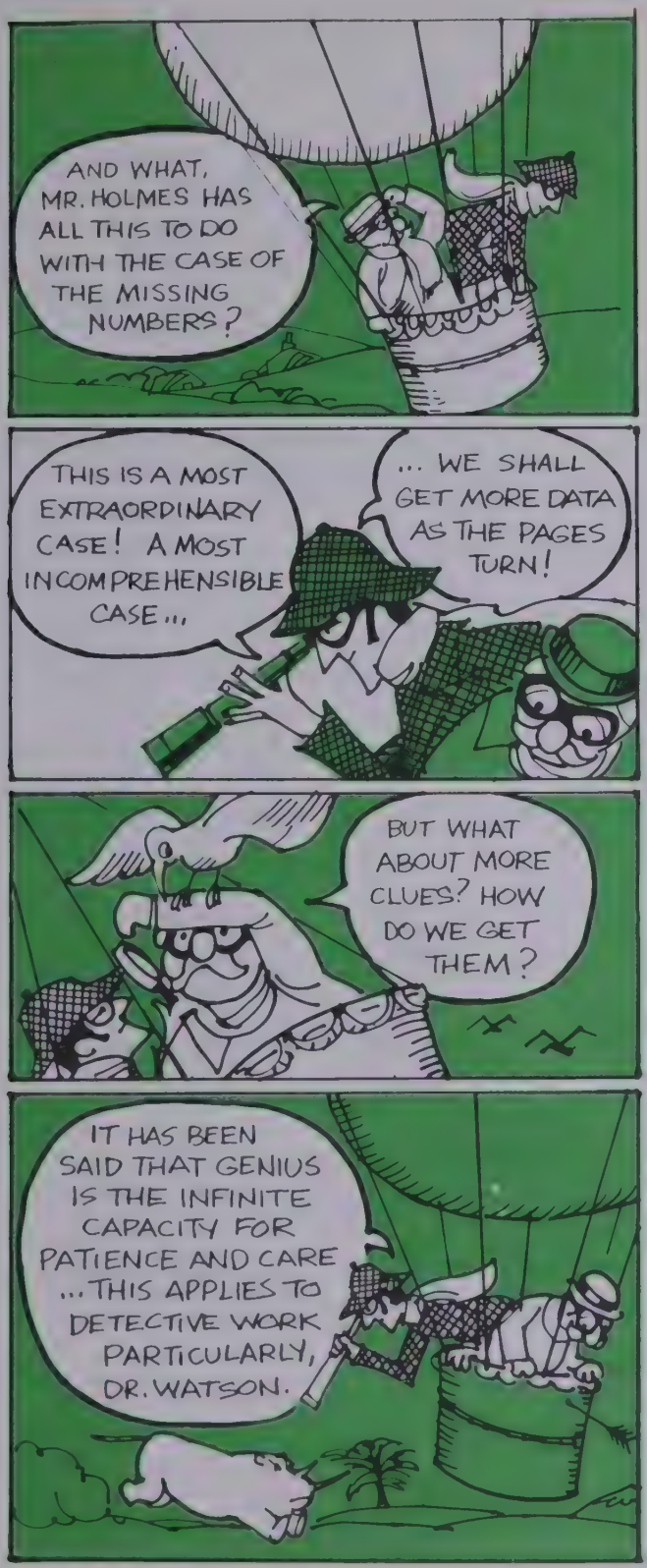
6. What is the area of a square whose side is 6.5 cm?

7. Jim and Bob found that their school hall was 64 m long and 4 m wide. What dimensions would a square room of equal floor area have?

8. Betty bought tile (30 cm × 30 cm) to do the kitchen floor in her home. She bought 10 packages with 24 tiles in each. When she laid the kitchen floor, she had 44 tiles left over. The kitchen floor is square. What are the dimensions of the floor?

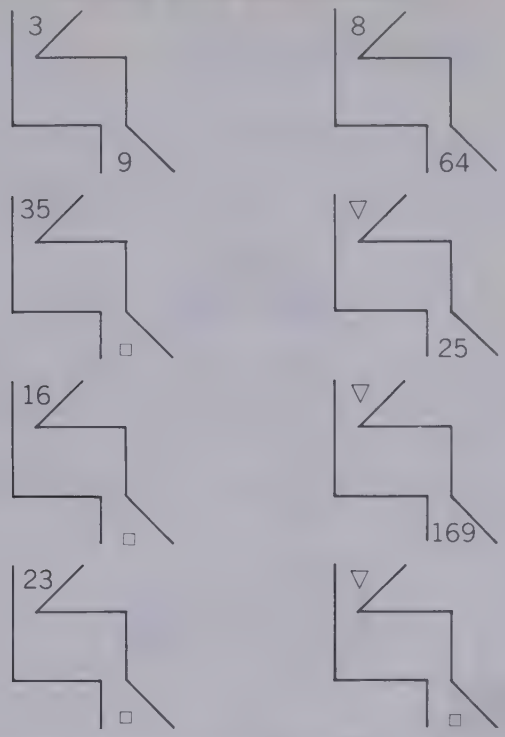


9. In designing the landscape of a public park, it was decided to put two large flower beds into the overall plan. One flower bed was a rectangle 30 m by 9 m. The other flower bed was a square $\frac{8}{15}$ the area of the rectangle. What dimensions did the square flower bed have?


AND WHAT, MR. HOLMES HAS ALL THIS TO DO WITH THE CASE OF THE MISSING NUMBERS?
THIS IS A MOST EXTRAORDINARY CASE! A MOST INCOMPREHENSIBLE CASE...
... WE SHALL GET MORE DATA AS THE PAGES TURN!
BUT WHAT ABOUT MORE CLUES? HOW DO WE GET THEM?
IT HAS BEEN SAID THAT GENIUS IS THE INFINITE CAPACITY FOR PATIENCE AND CARE ...THIS APPLIES TO DETECTIVE WORK PARTICULARLY, DR. WATSON.

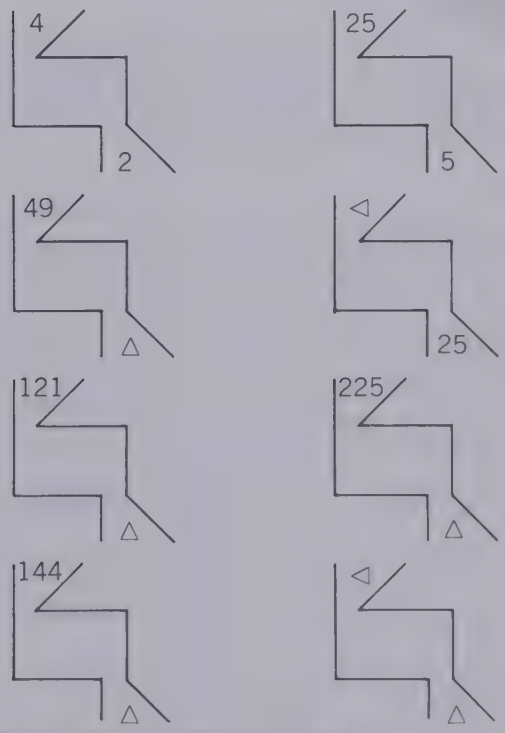
Find the missing inputs and outputs.

1.



What is the rule relating ▽ and □?

2.



What is the rule relating ◁ and △?

4·2 Find Square Roots (by factoring)

When the area of a square is known, calculating s, the length of the side of the square, is the same as calculating the square root of the area measure.

To calculate the square root of a number b, one finds another number a which, when multiplied by itself, gives the original number.

The symbol for square root is $\sqrt{\ }$. For example: if $a \times a = b$, then $a = \sqrt{b}$.

Do Question 1 in Exercise 4·2.

There are various methods of calculating square roots of numbers. Some are more useful than others. The first method is the *factor method.*

Example 1

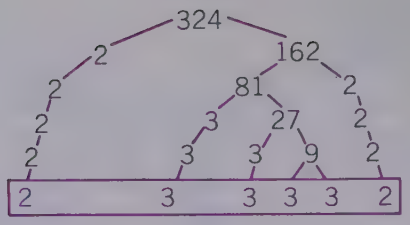
Use the factor method to find the square root of 324.

Solution

The beginning of the list of prime numbers is 2, 3, 5, 7, 11, 13, 17, 19, ... In choosing our factors, we begin with the smallest ones and move up to the larger ones until the factoring is complete. Thus,

1st prime factor	→	2 | 324	(Divide.)
2nd prime factor	→	2 | 162	
3rd prime factor	→	3 | 81	
4th prime factor	→	3 | 27	
5th prime factor	→	3 | 9	
last prime factor	→	3 | 3	
		1	

Also,



$$\begin{aligned}\therefore \sqrt{324} &= \sqrt{2\times2\times3\times3\times3\times3} \\ &= \sqrt{(2\times2)\times(3\times3)\times(3\times3)} \\ &= \sqrt{2\times2}\times\sqrt{3\times3}\times\sqrt{3\times3} \\ &\qquad \text{(Associate in groups of two of a kind.)} \\ &= 2\times3\times3 \quad (\sqrt{2\times2}=2) \\ &= 18\end{aligned}$$

Check: $18 \times 18 = 324$.

Example 2

Use the factor method to calculate the value of $\sqrt{38\,025}$.

Solution

Notice that 2 is not a factor. Go on to the next one. (Try 3.)

3 | 38 025	
3 | 12 675	
5 | 4 225	
5 | 845	
13 | 169	– (Why not use next prime 7?)
13 | 13	
1	

$$\begin{aligned}\therefore \sqrt{38\,025} &= \sqrt{3\times3\times5\times5\times13\times13} \\ &= \sqrt{3\times3}\times\sqrt{5\times5}\times\sqrt{13\times13} \\ &= 3\times5\times13 \\ &= 195\end{aligned}$$

Check: $195 \times 195 = 38\,025$.

Example 3

Calculate $\sqrt{\frac{256}{576}}$.

Solution (Method 1)

Calculate $\sqrt{256}$ and $\sqrt{576}$ independently.

$$\sqrt{\frac{256}{576}} = \frac{\sqrt{256}}{\sqrt{576}}.$$

2 | 256	2 | 576
2 | 128	2 | 288
2 | 64	2 | 144
2 | 32	2 | 72
2 | 16	2 | 36
2 | 8	2 | 18
2 | 4	3 | 9
2 | 2	3 | 3
1	1

$$\begin{aligned}\sqrt{256} &= \sqrt{2\times2\times2\times2\times2\times2\times2\times2} \\ &= \sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2\times2} \\ &= 2\times2\times2\times2 \\ &= 16\end{aligned}$$

$$\begin{aligned}\sqrt{576} &= \sqrt{2\times2\times2\times2\times2\times2\times3\times3} \\ &= \sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{2\times2}\times\sqrt{3\times3} \\ &= 2\times2\times2\times3 \\ &= 24\end{aligned}$$

Hence, $\sqrt{\frac{256}{576}} = \frac{\sqrt{256}}{\sqrt{576}} = \frac{16}{24} = \frac{2}{3}$.

Check: $\frac{16}{24}\times\frac{16}{24} = \frac{256}{576}$.

Alternate method

First simplify: $\frac{256}{576} = \frac{64}{144} = \frac{8}{18} = \frac{4}{9}$.

$$\therefore \sqrt{\frac{256}{576}} = \sqrt{\frac{4}{9}} = \frac{2}{3}.$$

EXERCISE 4·2

1. Calculate:
   (a) $\sqrt{9}$ (b) $\sqrt{36}$ (c) $\sqrt{144}$
   (d) $\sqrt{81}$ (e) $\sqrt{25}$ (f) $\sqrt{121}$
   (g) $\sqrt{400}$ (h) $\sqrt{16}$ (i) $\sqrt{100}$
   (j) $\sqrt{169}$ (k) $\sqrt{225}$ (l) $\sqrt{625}$

2. Calculate the square root of each using the factor method:
   (a) 441 (b) 225 (c) 576
   (d) 729 (e) 4356 (f) 33 124
   (g) 11 025 (h) 5929 (i) 5184

3. Calculate: (Simplify.)
   (a) $\sqrt{\frac{25}{16}}$ (b) $\sqrt{\frac{64}{36}}$ (c) $\sqrt{\frac{121}{100}}$
   (d) $\sqrt{\frac{144}{256}}$ (e) $\sqrt{\frac{1024}{2304}}$ (f) $\sqrt{\frac{2704}{676}}$
   (g) $\sqrt{\frac{5184}{7056}}$ (h) $\sqrt{\frac{5625}{10\,000}}$ (i) $\sqrt{\frac{256}{400}}$

4. Can you calculate:
   (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{8}$
   (d) $\sqrt{12}$ (e) $\sqrt{324}$ (f) $\sqrt{450}$

   Do we have numbers that exactly express the square root of 2?

5. This factor method of finding square roots is of limited use. Can you explain why?

4·3 Square Roots (by the Newton Method)

We recall that the square root of a number, when multiplied by itself, gives the number. Therefore, a number divided by its square root yields a quotient which is also its square root. Thus,

(a) $\because\ 9\times9=81$
$\therefore\ 81\div9=9.$

(b) $\because\ 15\times15=225$
$\therefore\ 225\div15=15.$

Example 1

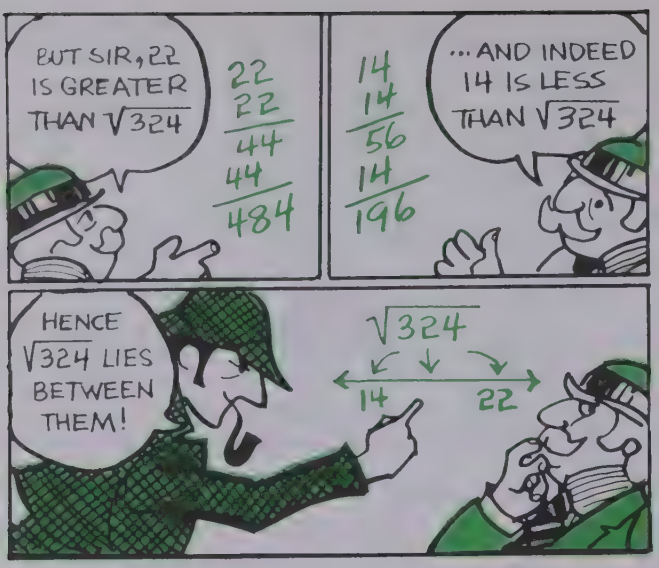
Calculate $\sqrt{324}$.

Solution

Let the guess be 22.

$\frac{324}{22} = 14$, remainder of 16.

What is 22^2? What is 14^2?



$\sqrt{324}$ must lie between 14 and 22. (Why?)

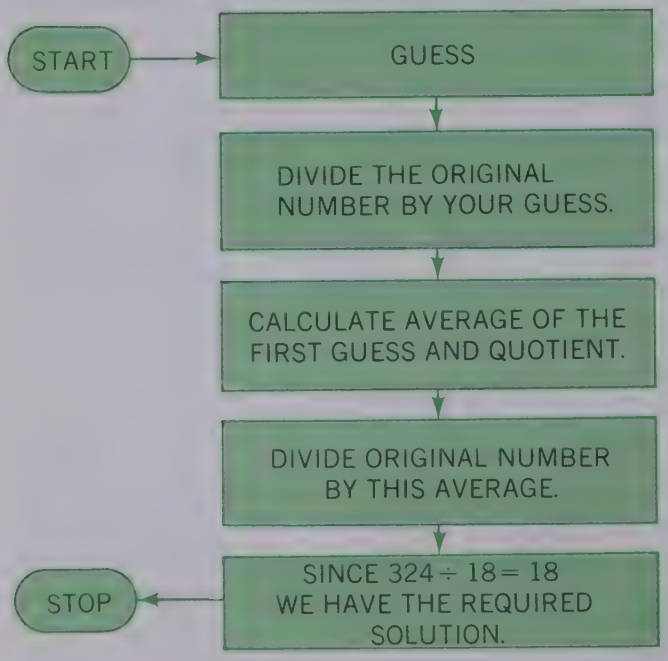
$$\frac{14+22}{2}=\frac{36}{2}=18$$

$$\frac{324}{18}=18$$

$\because \quad 18\times 18=324$

$\therefore \quad \sqrt{324}=18$

This is known as *Newton's Method* of calculating the square root of a number.


START
GUESS
DIVIDE THE ORIGINAL NUMBER BY YOUR GUESS.
CALCULATE AVERAGE OF THE FIRST GUESS AND QUOTIENT.
DIVIDE ORIGINAL NUMBER BY THIS AVERAGE.
SINCE 324 ÷ 18 = 18 WE HAVE THE REQUIRED SOLUTION.
STOP

Example 2

Find $\sqrt{729}$ by Newton's Method.

Solution

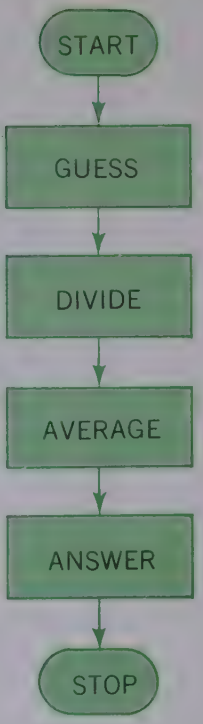
Let us guess that $\sqrt{729}$ is 23.

$$\begin{array}{r} 31 \\ 23\overline{)\,729} \\ \underline{69} \\ 39 \\ \underline{23} \\ 16 \end{array}$$

$\therefore 729 \div 23 = 31$ with remainder 16. We know 23 is not the square root of 729. The average of 23 and 31 is

$$\frac{23+31}{2}=\frac{54}{2}=27$$

Since $729 \div 27 = 27$
we know $\sqrt{729}=27$.



Do Questions 1-3 in Exercise 4·3.

Let's pause for a moment.

Are you good at multiplying decimals?

Try these.

(a) $\begin{array}{r} 2.1 \\ \times 2.1 \end{array}$ (b) $\begin{array}{r} 4.3 \\ \times 4.3 \end{array}$ (c) $\begin{array}{r} 15.6 \\ \times 15.6 \end{array}$

(d) $\begin{array}{r} 0.3 \\ \times 0.3 \end{array}$ (e) $\begin{array}{r} 1.49 \\ \times 1.49 \end{array}$ (f) $\begin{array}{r} 1.79 \\ \times 1.79 \end{array}$

If you got 4.41, 18.49, 243.36, 0.09, 2.2201 and 3.2041, then proceed to Example 3. If you need more practice, try these.

(a) 4.7^2 (b) 6.1^2 (c) 0.8^2

(d) 0.1^2 (e) 1.56^2 (f) 7.5^2

Example 3

By Newton's Method, find $\sqrt{161.29}$.

Solution

We guess the square root is about 13.

$161.29 \div 13 \simeq 12.56$

($\simeq$ means approximately equal to.)
Find the average of 13 and 12.56.

$$\frac{13 + 12.56}{2} = \frac{25.56}{2} = 12.78$$

Again, $161.29 \div 12.78 \simeq 12.62$.

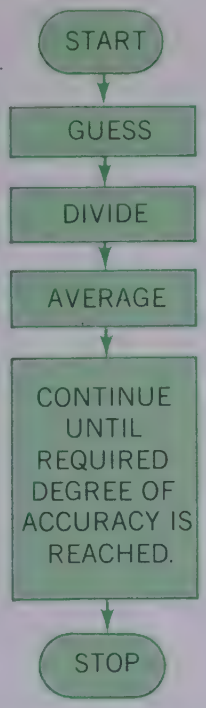
Because the first average did not produce the required square root, find another average (this time of 12.78 and 12.62).

$$\frac{12.78 + 12.62}{2} = \frac{25.40}{2} = 12.70$$

Again, divide 161.29 by the second average.

$$161.29 \div 12.7 = 12.7$$

Hence $\sqrt{161.29} = 12.7$



This method is used when a computer is programmed to calculate square roots.

Project

Project 2

Devise a method to find the square root of a number using your circular slide rule.

EXERCISE 4·3

Calculate the indicated square roots by Newton's Method. Use a calculator (if available).

1. (a) $\sqrt{361}$ (b) $\sqrt{529}$
   (c) $\sqrt{784}$ (d) $\sqrt{961}$
2. (a) $\sqrt{1156}$ (b) $\sqrt{3969}$
   (c) $\sqrt{2401}$ (d) $\sqrt{4356}$
3. (a) $\sqrt{6084}$ (b) $\sqrt{8836}$
   (c) $\sqrt{15\,129}$ (d) $\sqrt{54\,756}$
4. (a) $\sqrt{53.29}$ (b) $\sqrt{21.16}$
   (c) $\sqrt{60.84}$ (d) $\sqrt{88.36}$
5. (a) $\sqrt{0.0484}$ (b) $\sqrt{0.0324}$
   (c) $\sqrt{0.2809}$ (d) $\sqrt{0.7225}$
6. (a) $\sqrt{1.7424}$ (b) $\sqrt{2.4025}$
   (c) $\sqrt{3.4596}$ (d) $\sqrt{11.6964}$
7. Use your circular slide rule to check your answers to Questions 1 and 2 by approximation.
8. Can you find the following?
   (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{6}$ (d) $\sqrt{10}$

BUT MR. HOLMES, WHAT DOES FINDING SQUARE ROOT HAVE TO DO WITH THE CASE OF THE MISSING NUMBER?
THE SOLUTION LIES IN FINDING √2. IF YOU WILL ENDEAVOR TO FIND √2, YOU WILL BEGIN TO SEE THE SOLUTION TO THE CASE!
√2 = 1.41421..., AND WHAT DOES IT MEAN, NOW THAT I'VE FOUND IT?

4·4 Problems Involving Square Roots

Tables are often used to find square roots.

TABLE OF SQUARES AND SQUARE ROOTS

n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$	n	n^2	$\sqrt{n}$	
1	1	1	51	2601	7.14	101			1
2	4	1.41	52	2704	7.32	102			1
3	9	1.73	53			103			1
4	16	2.00	54						
5	25	2.24							
6	36	2.45							
26	676								
			83	6889					
48	2304	6.93	98			148			1
49	2401	7.00	99			149			1
50	2500	7.07	100	10 000	10.00	150			2

This table is for the square root of any number (n) from 1 to 200. To find the square root of $\sqrt{48}$, we go down the column marked n until we reach 48. Then we go across to the column marked $\sqrt{n}$. Here we see 6.93. Therefore, $\sqrt{48} = 6.93$ (correct to two decimals).

To make use of this table for numbers beyond 200, use the n^2 column and refer to the n column for the answer.

For example, to find $\sqrt{676}$, look under n^2 and find 676. The corresponding n reads 26. Therefore, $\sqrt{676} = 26$.

Similarly, for $\sqrt{6889}$, look under n^2 and find 6889. The corresponding n reads 83. Therefore, $\sqrt{6889} = 83$.

If one wishes to find $\sqrt{2376}$, which cannot be found under n^2, then an approximation is possible. Find the number under n^2 which is closest to 2376. The closest number is 2401 and $\sqrt{2376} \simeq 49$.

This table will prove helpful in problem solving.

EXERCISE 4·4

Use the Table of Squares and Square Roots in this book for this exercise.

1. Find the square root of each:

(a) 9	(b) 36	(c) 144
(d) 169	(e) 81	(f) 196
(g) 529	(h) 1024	(i) 2025
(j) 23 104	(k) 25 281	(l) 7921

2. Find the approximate square root of each:

(a) 530	(b) 500	(c) 600
(d) 250	(e) 300	(f) 510

Complete:

____ m = 1 km
500 m = ____ km

____ m = 5 km
3000 m = ____ km

A student could run 1 km in about ? min.

A student could run about ? m in 1 min.

Projects

Project 3

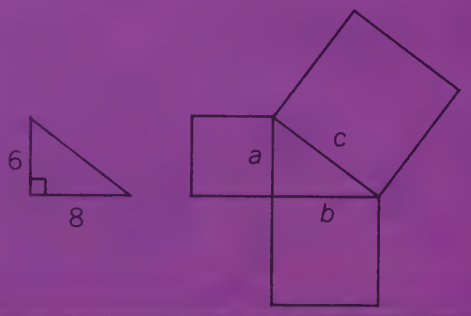
Purpose

To work with the Pythagorean Relation.

Method

A. 1. On a full sheet of 1 cm squared paper, draw a right triangle as indicated:

$a = 6, b = 8, c = ?$



2. Construct squares on each side.
3. Find the area of each square.
4. What are the relations among the areas of the squares?
5. What is the length of side c?

B. Repeat Part A for the right triangle in which $a = 5$ and $b = 12$.

C. Use these symbols for the above right triangles.

Area of square on side a: a^2.
Area of square on side b: b^2.
Area of square on side c: c^2.

Using these symbols, write three different statements for the relations illustrated in Parts A and B.

Project 4

Purpose

To work with the Pythagorean Relation that leads to a proof later.

Method

1. Obtain two sheets of thin cardboard 17 cm × 17 cm. On one sheet, mark the four right triangles as shown. Color each triangle differently.

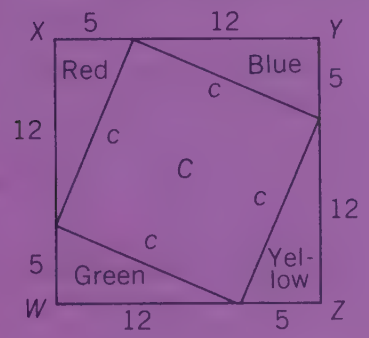


Figure 4-4

(a) What is the shape of region C?
(b) What is the area of the square $XYZW$?
(c) What is the area of the four triangles?
(d) What is the area of the region C?

2. Cut off the four triangles.
3. Arrange the four triangles from Figure 4-4 on the second sheet of cardboard as shown.

(a) What is the sum of the two areas A and B?
(b) How many different ways can you calculate the sum of the areas A and B?
(c) How is the area of region C related to the sum of the areas of A and B?

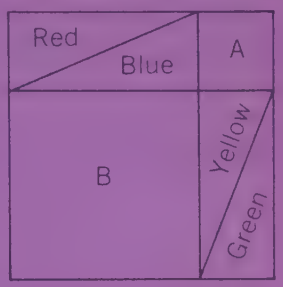


Figure 4-5

4. Refer to Figure 4-6. Copy and complete the following.

(a)	$a = 3$	$b = 4$	$C = ?$	$c = ?$
(b)	$a = 24$	$b = 7$	$C = ?$	$c = ?$
(c)	$a = 1$	$b = 0.75$	$C = ?$	$c = ?$
(d)	$a = ?$	$b = 15$	$C = 289$	$c = ?$
(e)	$a = 25$	$b = 60$	$C = ?$	$c = ?$
(f)	$a = 20$	$b = ?$	$C = ?$	$c = 29$

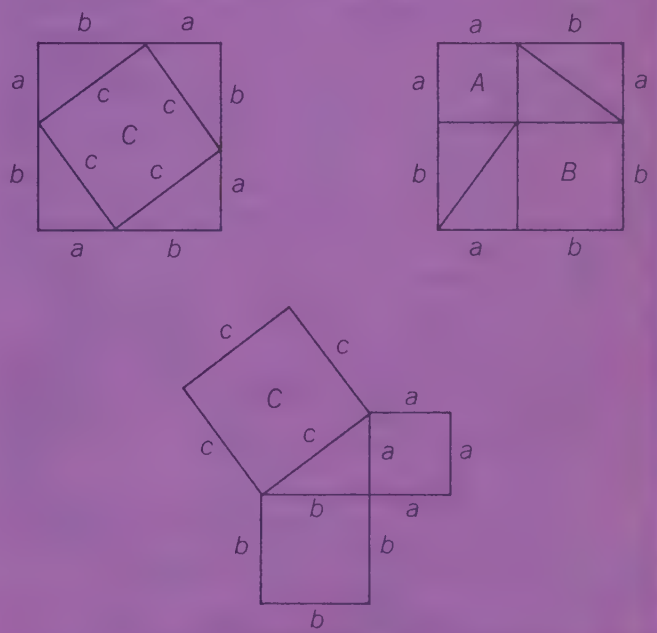
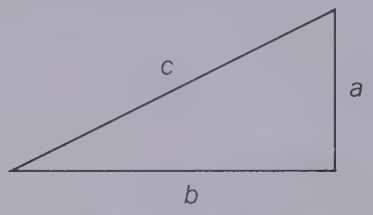


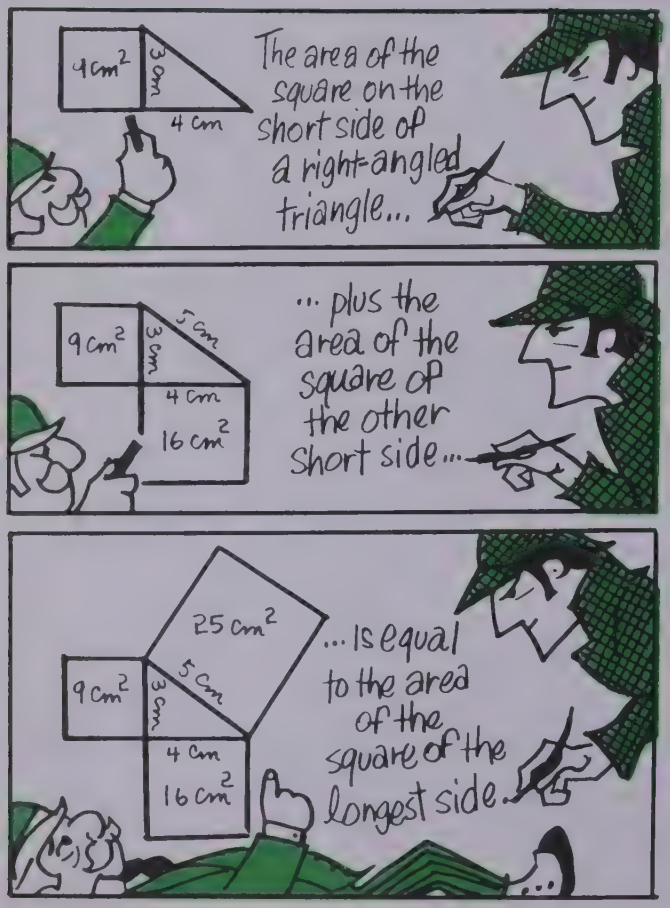
Figure 4-6

4·5 Working With Right Triangles

A right triangle has many applications. It is a subject of much study in trigonometry. One relation of a right triangle was stated by Pythagoras in 500 B.C. He stated that, for any right triangle (as in the diagram), $c^2 = a^2 + b^2$ where a, b, and c are the lengths of the three sides as shown.



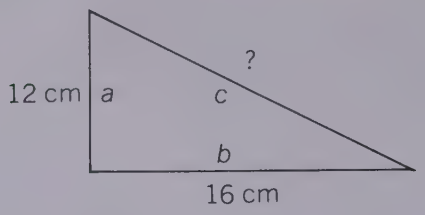
Note! Sides a and b contain the right angle and side c is the hypotenuse.



Example 1

For a right triangle, if $a = 12$ cm and $b = 16$ cm, find the length of c.

Solution



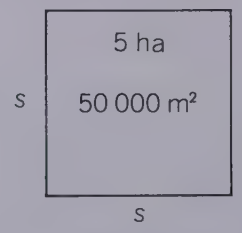
$c^2 = a^2 + b^2$
$c^2 = 12^2 + 16^2$ (Make replacements.)
$c^2 = 144 + 256$ (Multiply and add.)
$c^2 = 400$
$c = \sqrt{400}$ (Find the square root.)
$c = 20$

The length of side c is 20 cm.

Example 2

A farmer wishes to enclose a five-hectare square piece of land. How many metres of fencing does he need? (1 ha is equivalent to 10 000 m².)

Solution

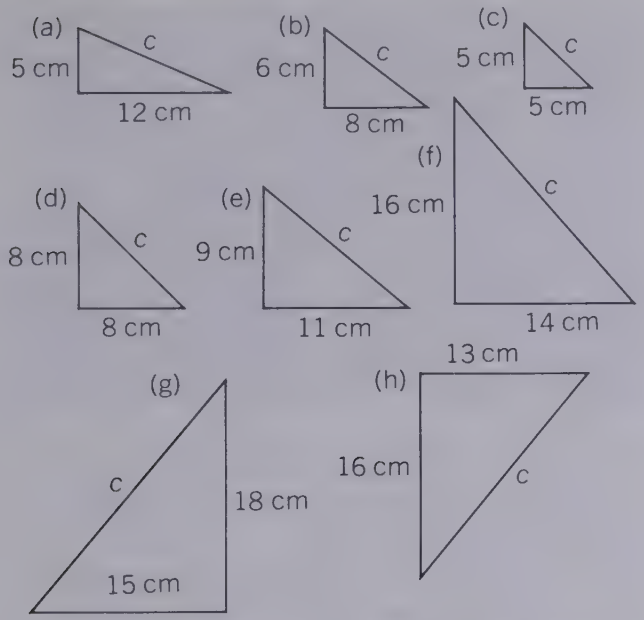


Area of square lot $= 5$ ha
$= 5 \times 10\,000$ m²
$= 50\,000$ m²
$s^2 = 50\,000$
$s = \sqrt{50\,000}$
$= \sqrt{5 \times 10\,000}$
$= \sqrt{5} \times \sqrt{10\,000}$
$= \sqrt{5} \times 100$
$s \simeq 2.24 \times 100$ (Use tables.)
$s \simeq 224$

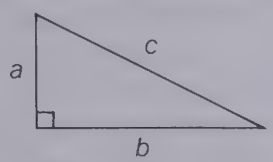
The length of one side is 224 m, correct to the nearest metre. The amount of fencing required is 4×224 or 896 m.

EXERCISE 4·5

1. Solve for side c in each right triangle (to two decimals where applicable).

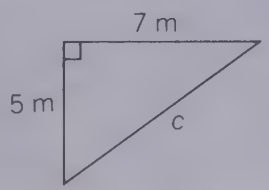


2. Calculate the length of side c (to two decimal places).

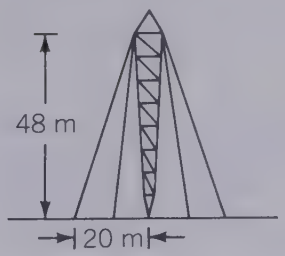


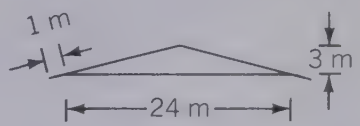
(a)	$a = 9$ cm	$b = 18$ cm	$c =$
(b)	$a = 17$ cm	$b = 17$ cm	$c =$
(c)	$a = 20$ cm	$b = 24$ cm	$c =$
(d)	$a = 41$ cm	$b = 22$ cm	$c =$
(e)	$a = 22$ cm	$b = 16$ cm	$c =$
(f)	$a = 10$ cm	$b = 20$ cm	$c =$
(g)	$a = 21$ cm	$b = 19$ cm	$c =$
(h)	$a = 18$ cm	$b = 13$ cm	$c =$

3. A surveyor laid out a right triangle as illustrated. What was the length of the hypotenuse (the longest side)?



4. A taxi company set out its broadcasting and receiving antenna on top of a building as illustrated. Four guy-wires were secured 48 m up the tower for stability. Each guy-wire was anchored 20 m away from the base of the tower. How many metres of guy-wire were used?

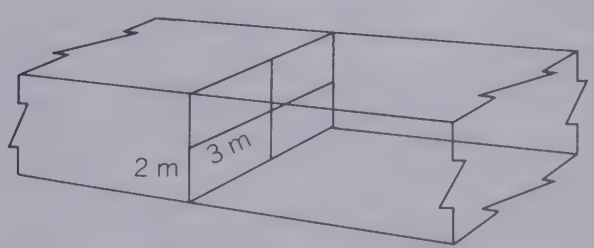




5. A housebuilder must buy lumber for rafters to build a gable roof on his alpine cottage. The cottage is 24 m wide and the rise of the roof peak is 3 m. The plans call for an overhang of 1 m (see illustration). What is the length of each rafter?

6. The study of physics reveals that a very heavy falling body drops from rest a distance of s metres in t seconds, as related by the formula $s = 5t^2$ (approximately). How many seconds will it take to fall 125 m? 500 m? 200 m? 1 km?

7. Four rectangular ducts in a heating system, each 3 m by 2 m, are fed from a single square duct. For proper airflow, the cross-section area of the square duct must be equal to the combined cross-section areas of the four ducts leading away from it. What is the approximate area of the square duct?

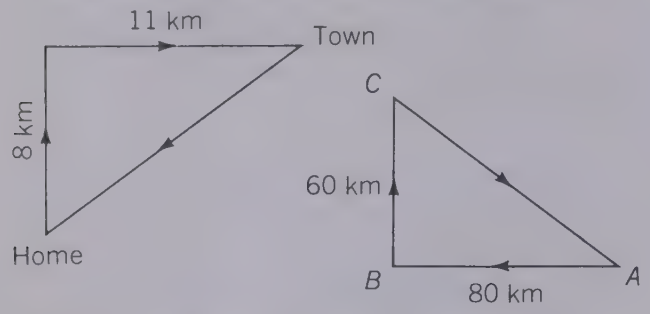


8. In an electrical circuit, the impedance (z) in ohms is found by the formula $z = \sqrt{R^2 + x^2}$, where R is resistance in ohms and x is the reactance in ohms. If R is 14.8 ohms and $x = 11.1$ ohms, find z.

9. If the area of a circle is known, then the approximate radius of that circle can be found by using the formula $r = \sqrt{\frac{7A}{22}}$. For a circle with area $A = 5544\ \text{cm}^2$, find r.

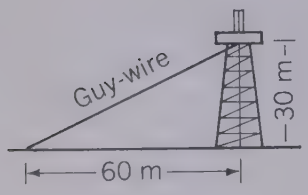
Hint! First divide A by 22, then multiply by 7.

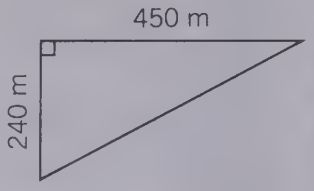
10. Lou travels 8 km north and 11 km east when he goes from home to town. How far is his home from town "as the crow flies" (cross country)?



11. Pilot Jane flies from town A, to town B, to town C, and back to A. If B is 80 km due west from A, and if C is 60 km due north from B, how many kilometres is the round trip?

12. As a safety measure, a cable is strung from a platform 30 m up an oil-drilling derrick to an anchor 60 m away from the middle of the base of the derrick. In case of an emergency, a worker on the platform rides the cable down to safety. How long a cable is used for this oil rig?

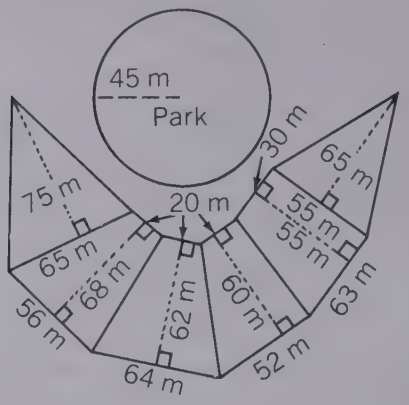




13. Mr. Rich bought the city lot shown in the diagram. How many hectares are in his lot?

14. Mr. and Mrs. Pinto purchased a lot on the outskirts of their city. The description of the lot was given as "a rectangular lot 0.3 km by 0.4 km". What is the number of hectares in the lot?

15. Calculate the number of hectares in each lot shown in the subdivision.



Which has the most hectares, the park or the largest lot?

16. The city of London, England, has about 22.8 m^2 of park per person. London has a population of approximately 9 000 000.
    (a) Approximately, how many square metres of park are there in London?
    (b) Approximately, how many hectares of park are there in the city?
    (c) How many square kilometres of park are there in the city?
    (d) How many square kilometres would there have to be in your community (or city) to have the same ratio as London? Does your community (or city) have this much? If not, what should, or can you do about it?

17. In 1970, Japan had 17.5 million motor vehicles and about 100 000 km of paved roads; and Britain had 13 million vehicles on 300 000 km of paved roads.

(a) Which country had the most cars per kilometre of paved roads?

(b) In 1973, Japan had about 21 000 000 vehicles while Britain's total rose to about 14 000 000. Which country had the most cars per kilometre of roads in 1973 if the number of kilometres of roads remained the same in both cases?

(c) Use the *Canada Handbook* from your library to find the number of cars per kilometre of paved highway (or of all highways) in Canada.

Early man needed numbers to count: 1, 2, 3, 4, ... These are the counting or **natural numbers**.

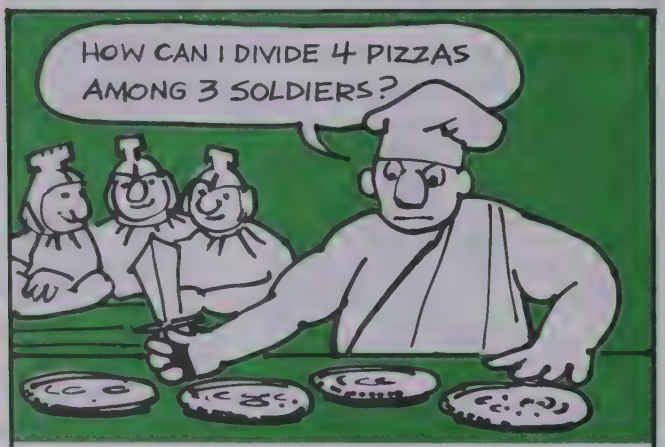
Then the number 0 was invented. Now we have the whole numbers: 0, 1, 2, 3, 4, . . .

4·6 From One Number System On To Another . . .

Let us review the number system we have developed to this point in time. We will do this by examining what we believe is the historical development of the number system through the ages.

We believe that our present number system developed in stages as the need arose.

Then came a puzzle for early mathematicians— the only solution was to invent the **negative numbers.**



Fractions or rational numbers were also needed.

Archimedes was killed by a warrior. He refused to leave his math problem drawn in the sand.

Of course, need was not the only reason why new number systems were developed. As people progressed from being cave dwellers and hunters to being farmers and city dwellers, more spare time was gained for art, philosophy, sports, etc. In times of meditation and deep-thinking, people might have thought like this: "If a rational number multiplied by itself produced another rational number, then every rational number should have two equal factors."

For example:

$$4 \times 4 = 16 \rightarrow 16 = 4 \times 4 \rightarrow \sqrt{16} = 4.$$

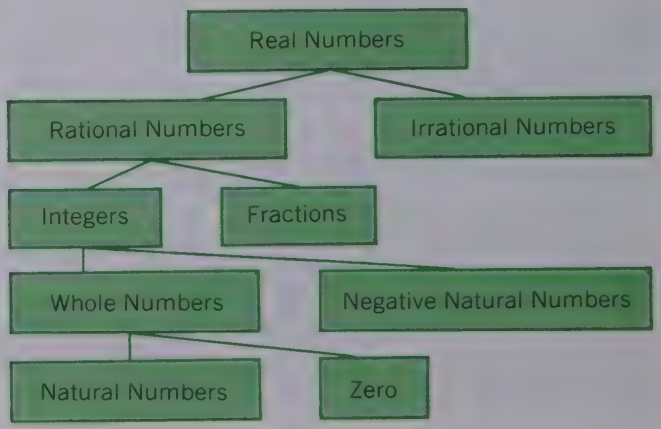
But:

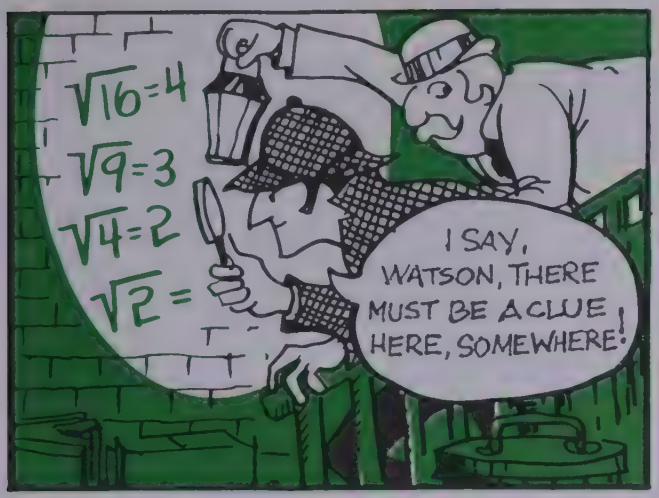
$$\underline{?} \times \underline{?} = 2 \rightarrow 2 = \underline{?} \times \underline{?} \rightarrow \sqrt{2} = \underline{?}.$$

Can you fill in the blanks? Is there a number missing from our system?

The philosopher-mathematician was quite amazed when he discovered that he was posing many questions for which he had no answers. He could not find solutions to $\sqrt{2}$, $\sqrt{3}$, and others in the set of rational numbers. He went on to study new numbers like $\sqrt{5}$, $\sqrt{11}$, $\sqrt{17}$, etc. For each one of them he discovered corresponding points on the Q-line which were vacant points. That is, he discovered that the Q-line is not a continuous line of adjacent points but has empty spots in it.

This new set of numbers, namely $\sqrt{2}$, $\sqrt{3}$, and so on, make up what we call the *Irrational Numbers*. When the irrational numbers were combined with the rational numbers, a new system of numbers called the *Real Numbers* was created. This new system of numbers requires extensive study and will not be dealt with in this book.





EXERCISE 4·6

1. Starting with $\sqrt{1}$, then $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, and so on, how many of the first 50 natural numbers have exact square roots? What is the name for the set containing the other numbers?

2. Copy the table below and fill each space with "yes" or "no" to indicate whether the number set is true for each property under the operation.

Properties	Under Operations	Natural Numbers	Whole Numbers	Integers	Rationals
Closure	+				
	−	No			
	×				
	÷				
Commutative	+				
	−				
	×				
	÷			No	
Associative	+				
	−				
	×				
	÷				
Distributive	Multiplication over Addition				
Identity Element					

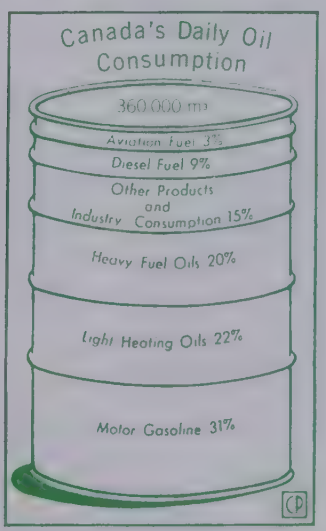
Energy

1. Calculate the number of cubic metres of each type of oil consumed in Canada daily.

2. Canada's oil production is about 360 000 m³ a day. Canada uses about 170 000 m³ and exports about 180 000 m³. The balance is in the pipelines and in inventory. How much is in inventory and the pipelines?

3. Of Canadian production, 85% comes from Alberta, 11% from Saskatchewan, 3% from British Columbia, and 1% from Manitoba. Calculate the number of cubic metres produced daily by each of the four provinces.

4. Canada imports about 180 000 m³ daily: 44% from Venezuela, 30% from Arab countries, 16% from Iran, 5% from Angola, 4% from Nigeria, and 1% from Colombia. How many cubic metres are imported from each country daily?



5. Of the oil produced and used in Canada, 45% is used in the Western Provinces, 50% is used in Ontario, and 5% in Quebec. How many cubic metres are used in each of the three regions?

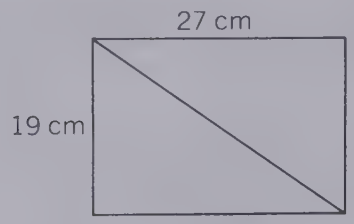
6. What is the biggest user of oil (and gasoline) in Canada? Name four ways that Canadians can conserve on oil and gasoline without appreciably affecting their standard of living. Discuss these in class. Should Canada cut off oil exports to the United States? Discuss.

REVIEW EXERCISE 4

1. A carpet layer used 12 m of carpet which was 6.5 m wide to cover the floor of the family room. What were the dimensions of the room if it was a square? Round your answer to the nearest larger square metre.

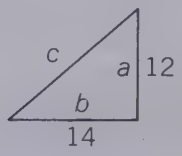
2. (a) By the factor method, find $\sqrt{3969}$.
   (b) By the Newton Method, find $\sqrt{7056}$.

3. A carpenter took a rectangular piece of plywood, 27 cm long and 19 cm wide, and cut it into two congruent triangles as indicated in the illustration. How long was the saw-cut?



4. Calculate r given: $r = \sqrt{\frac{121}{625}}$.

5. Calculate the length of side c to the nearest whole number.



6. Which of these numbers are prime?
   (a) 31 (b) 35 (c) 36
   (d) 37 (e) 39 (f) 41

7. A piece of land, 4 km × 49 km, is traded for a square piece of land of equal area. What is the length of a side of the square?

8. A square bar of side 10 cm is milled (cut) from a round rod. Rods with diameters of 14 cm, 14.5 cm, 15 cm, and 15.5 cm are available. From which rod can the bar be cut so that there is a minimum of waste?

Using the Library

1. *Reading In Mathematics*, Book 1. Adler I. Ginn and Company (1972)
   (a) Read page 76. Do you think this is funny? Why?
   (b) Read pages 34-36. Build a triangle of dots to represent T(5), T(7), T(10), T(12). Is there a short way to calculate the value of T(9)? T(13)?
   (c) Read pages 37-39. What is the value of T(5) + T(6)? T(7) + T(8)? What is the short way to calculate T(9) + T(10)? T(11) + T(12)?

2. *Reading In Mathematics*, Book 2. Adler I., Ginn and Company (1972). Read pages 32-35. Try folding, gluing, and cutting paper strips as indicated on page 32. What do you get?

Who is M. C. Escher? Is there a book of his work in your library? On page 35 is a box with a question about the picture. Can you answer the question? *Do not look at the answer!*

3. Canada has many famous inventors and scientists. Use the library to find the names of several. Identify what each is famous for.

Do You Remember that . . .

. . . there are definite relationships among certain numbers? Study the following example of one such relationship.

1	$= 1$	$= 1^2$
$1 + 3$	$= 4$	$= 2^2$
$1 + 3 + 5$	$= \underline{?}$	$= \underline{?^2}$
$1 + 3 + 5 + 7$	$= \underline{?}$	$= \underline{?^2}$

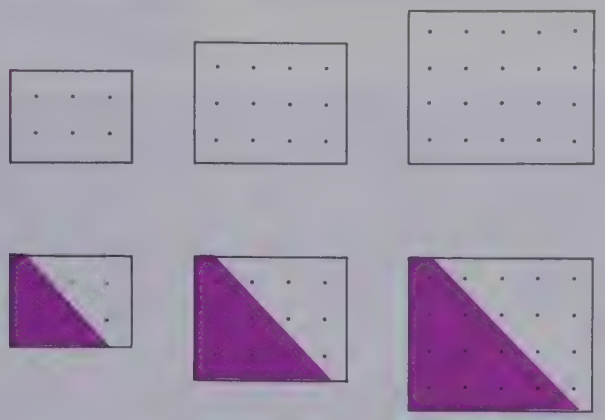
Copy, complete, and expand this series to 10 lines. Express the relations in as many ways as you can. What is the sum of the first 15 odd numbers? 20 odd numbers? 25 odd numbers? 30 odd numbers? 100 odd numbers?

. . . the triangular numbers form a pattern? The first three triangular numbers are shown. In this exercise, 1 is not a triangular number.

1. Continue the pattern by drawing the next four triangular number patterns.
2. How many dots are there in each of the first seven patterns?
3. What relations are there between each of the successive triangular numbers? Discuss. State their relations in as many ways as you can.
4. Can you find an easy way (besides counting) to calculate how many dots are in each pattern?

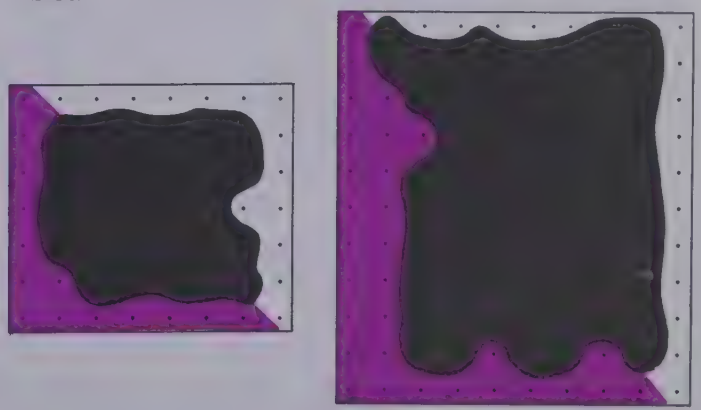
   Discuss each method. Test each method to see if the method works.
5. How many dots are there in the 10th triangular pattern? the 15th triangular pattern? the 29th triangular pattern?
6. If you had difficulty with Questions 3, 4, and 5 above, these suggestions may help. If you did not have difficulty, go on to Question 7.

Study these diagrams:



Find an easy way to calculate quickly the number of dots in each colored region.

An ink blot blotted out some dots. How many dots were in the colored portion before the ink blot?



7. Find the sum of each two consecutive triangular numbers up to the tenth.

first and second:	$3 + 6 = 9 = 3^2$
second and third:	$6 + 10 = 16 = ?$
third and fourth:	$10 + 15 = ? = ?$

   What patterns do you see in the sums? How are the sums related to the numbers of the triangular numbers, i.e., first and second, third and fourth, etc.? What are the sums of the following pairs of consecutive triangular numbers?

   (a) 13th and 14th (b) 18th and 19th

   (c) 23rd and 24th (d) 38th and 39th

   (e) 98th and 99th

Something to Think About

Project

Project 5

Purpose

To determine speed ratio, gear ratio, and other numeral ratios of a bicycle.

Materials

Bicycle, metric tape measure.

Method

1. Determine the distance travelled in one complete revolution of a pedal. The best way is to measure the distance for five or ten complete turns then calculate the average.

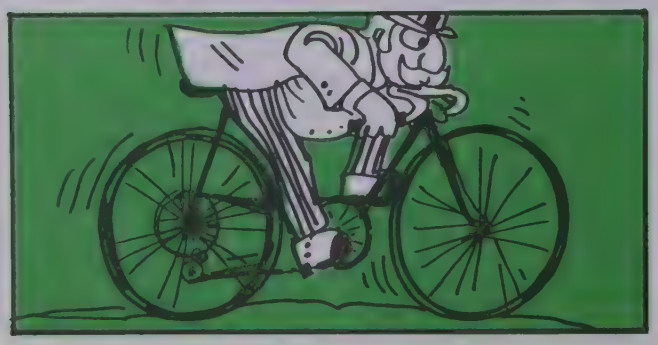
2. (a) Determine the gear ratio. If your bike is a three-, five-, or ten-speed, calculate all the gear ratios.

   The gear ratio is equal to the number of teeth on the drive-pedal sprocket wheel divided by the number of teeth on the rear sprocket wheel.

   (b) Which bike is the hardest to pedal up an incline? With which speed ratio is it the hardest to pedal up an incline?

   (c) At which speed ratio can one go the fastest?

   (d) Why do you think five- and ten-speed bikes are popular and in wide use?



3. Determine the stopping distance for bikes at various speeds.

10 km/h	30 km/h
20 km/h	40 km/h

   (a) As you ride the bike, get it going at the desired speed (a speedometer on the bike is necessary), and as the front wheel crosses a line marked on the pavement, apply the brakes. Measure the distance from the line to the point where the front wheel comes to a stop. Do not "spin out".

   (b) Draw a graph to illustrate the data.

   (c) Repeat (a) and (b) for a

      (i) grass surface

      (ii) gravel surface

      (iii) asphalt surface.

4. Discuss the Highway Code and traffic regulations. Are there traffic regulations that apply specifically to bicycles?

5. Is there a reason for using your own bike and not others when riding on the street or road?

6. Compare the initial cost, cost of upkeep, and cost of repairs of a standard bike and a multiple-speed bike.

7. Discuss the question of stopping distances of bicycles, motorbikes, and cars with your teacher or the local police or the local Automobile Association representative.

Have You Discovered that . . .

. . . according to Pythagoras there are certain natural numbers x, y, and z, such that $x^2 + y^2 = z^2$? Such numbers are called Pythagorean triples. The triples 3, 4, 5 and 5, 12, 13 satisfy the Pythagorean relation. Are there other such triples? If 3, 4, 5 is one such triple, what about 6, 8, and 10? What about 9, 12, and 15?

Chapter Five
THE SOLUTION IS NOT OBVIOUS

Equations, Formulas, and Problems

I THINK THAT I SHALL BE IN A POSITION TO MAKE THE SITUATION RATHER MORE CLEAR TO YOU BEFORE LONG...
THE PIG AND WHISTLE
IT HAS BEEN AN EXCEEDINGLY DIFFICULT AND MOST COMPLICATED BUSINESS. THERE ARE SEVERAL POINTS UPON WHICH WE STILL WANT LIGHT - BUT IT IS COMING, ALL THE SAME!

5·1 Equations

An equation implies a statement of equality. To have equality means that at least one thing (whatever it might be) is equal to at least one other thing. In mathematics, a statement in symbol form which has an equal sign is called an equation.

In algebra, much of our work involves equations with one or more symbols which represent some unknown quantity or quantities. Statements of this type are the equations with which we are concerned:

$$2x + 4 = 18$$
$$3x - 7y = 15$$
$$2x - 3 = 4y$$

Note in this equation:

$3x - 7y$ (left side) $=$ (is equal to) 15 (right side)

In the above equation, both x and y represent some unknown numbers.

Symbols, such as x and y, which represent unknown numbers are called *variables*.

The equation

$$x^2 + 3x - 5 = 0$$

has only one variable, x; and the equation

$$3ab - 4bc = 12$$

has three variables, a, b, and c.

We will now solve some simple equations.

Example 1

Solve: (a) $x + 5 = 9$ (b) $7y = -42$

Solution

(a) $x + 5 = 9$

We must think of a number which, when added to 5, yields 9.

$$(?) + 5 = 9$$

The number is 4.

$$(4) + 5 = 9$$

Thus, the solution is

$$x = 4.$$

(b) $7y = -42$

What number multiplied by 7 yields -42?

$$7(?) = -42$$

The number is -6.

$$7(-6) = -42$$

Thus, the solution is

$$y = -6.$$

EXERCISE 5·1

1. How many variables are there in each of the following?

   (a) $2ab = 5$ (b) $x - y + z = 8$

   (c) $\frac{2km}{n} = 5m^2$ (d) $8x^2 + 7x - 2 = 0$

   (e) $t^2 - \frac{1}{t} = 5$ (f) $x^2 - 2xy + y^2 = 1$

   (g) $a^2 + b^2 + c^2 - 2ab - 2ac + 2bc = 0$

   (h) $3x^2 - 2x + 5 = y^2 + y - 3$

2. Solve:

   (a) $x - 7 = 5$ (b) $13 - y = 4$

   (c) $14 - 2(3) = x$ (d) $6z = 42$

   (e) $2x + 1 = 17$ (f) $5x + 4 = 14$

   (g) $12a - 24 = 0$ (h) $9y - 5 = -5$

   (i) $7m + 1 = 22$ (j) $10x = 120$

   (k) $284y = 284$ (l) $3z + 5 = 8$

   (m) $15 - 2x = 11$ (n) $8 - 3y = 2$

   (o) $11 - 3z = 2$ (p) $12x + 44 = 188$

5·2 Building and Undoing Equations

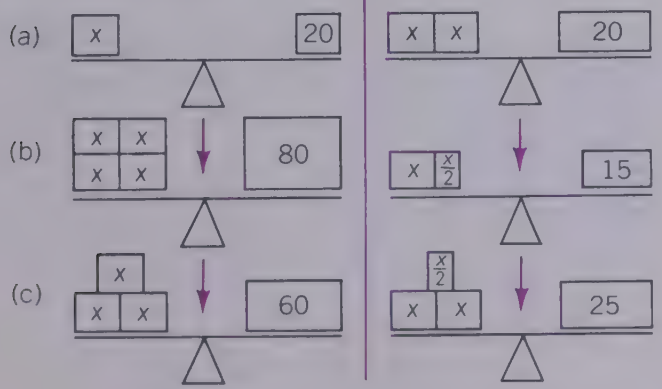
The equations in Question 2 of Exercise 5·1 were relatively simple to solve. Most of them were quite obvious and required very little thinking. Some, on the other hand, were not so obvious and did require some logical thinking to solve them.

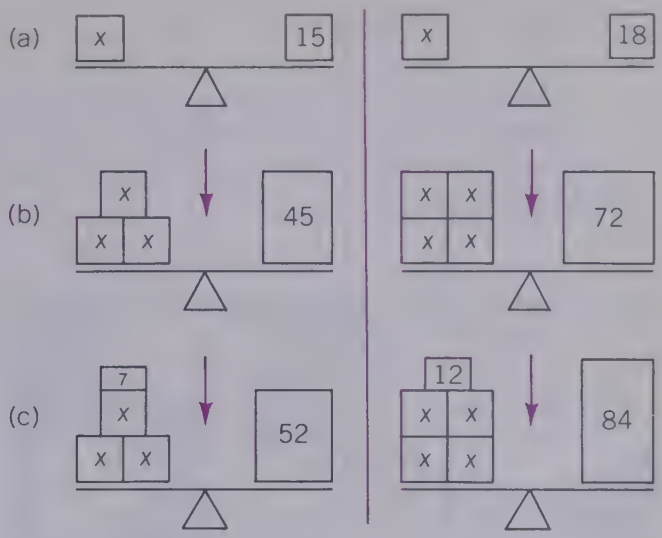
Because an equation is a statement of equality, it is very much like an equal arm balance in equilibrium. Whatever is in the pan on one side of the balance in equilibrium has exactly the same mass as the object(s) in the pan on the other side. If the mass on one side is changed (added to, subtracted from, multiplied, or divided), then the equilibrium is upset. To re-establish equilibrium, whatever was done to one side must be done to the other.

Addition	Addition
Subtraction	Subtraction
Multiplication	Multiplication
or Division	or Division

An equation is kept in balance if the same operation is performed on each side.

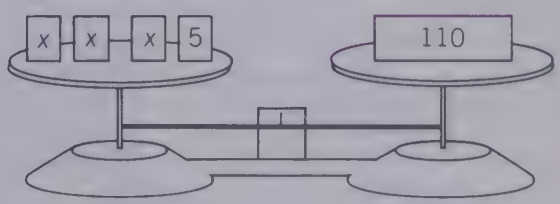
Study the following illustrations of maintaining equilibrium using symbols. Explain what is done as you go from step to step.





We conclude that, whatever is done to one side of a balance at equilibrium must also be done to the other side, in order that equilibrium be maintained. It is exactly the same for solving a mathematical equation.

Whatever operation is performed on one side of an equation, the same operation must be performed on the other side for the equation to maintain equality.



The mass on the right is 110 g. What are the masses of the three unknowns on the left? Explain how you reached your answer.

EXERCISE 5·2

1. Study each step in the following series of equivalent equations. Tell what was done to each side of the previous equation to get the subsequent equivalent equation.

(a)
$x = 2$
$x + 6 = 8$ — (i) Add 6 to each side.
$x + 6 = 8$
$x = 2$ — (ii)

(b)
$x = 3$
$2x = 6$ — (i)
$2x = 6$
$x = 3$ — (ii)

(c)
$x = 6$
$5x = 30$ — (i)
$5x = 30$
$x = 6$ — (ii)

(d)
$x = 8$
$x - 6 = 2$ — (i)
$x - 6 = 2$
$x = 8$ — (ii)

(e)
$x = 8$
$3x = 24$ — (i)
$3x + 5 = 29$ — (ii)
$3x + 5 = 29$
$3x = 24$ — (iii)
$x = 8$ — (iv)

2. Copy and complete the following. Indicate what has been done in each step.

(a)
$x = 11$
$-3x = -33$ — (i)
$4 - 3x = -29$ — (ii)
$4 - 3x = -29$
$\_\_\_\_ = \_\_\_\_$ — (iii) Subtract 4 from each side.
$x = \_\_\_\_$ — (iv) Divide each side by -3.

Remember to *undo* the operations in the reverse order to which you *did* them.

(b)
$x = 8$
$4x = 32$ — (i)
$4x + 6 = 38$ — (ii)
$4x + 6 = 38$
$\_\_\_\_ = \_\_\_\_$ — (iii)
$x = \_\_\_\_$ — (iv)

(c)
$x = -7$
$-4x = +28$ — (i)
$3 - 4x = 31$ — (ii)
$3 - 4x = 31$
$\_\_\_\_ = \_\_\_\_$ — (iii)
$x = \_\_\_\_$ — (iv)

(d)
$x = 9$
$8x = 72$ — (i)
$8x + 14 = 86$ — (ii)
$8x + 14 = 86$
$\_\_\_\_ = \_\_\_\_$ — (iii)
$x = \_\_\_\_$ — (iv)

5·3 Solving Equations

The simplest equations are of this form:

$x = 3$ $\quad y = -4$ $\quad a = 1\frac{1}{2}$ $\quad m = 0.3$

Study these two equations. What is the solution for each?

(a) $x = 4$ $\qquad$ (b) $x + 6 = 10$

The solution to (a) is obvious. Since it is in the simplest form we know, the replacement for x is 4.

$x = 4$
$4 = 4$

In (b), the solution can be found readily. The clues are all there! The solution for x is also 4.

$x + 6 = 10$
$4 + 6 = 10$

Since the two equations, (a) and (b), have identical solutions, we say the two equations are equivalent. Given an equation, there are a number of ways of obtaining equivalent equations.

Given the equation $x = 7$, we can write equivalent equations by:

(1) multiplying each side by the same number,

$x = 7 \leftrightarrow 3x = 21$

($\leftrightarrow$ means "is equivalent to")

(2) dividing each side by the same number,

$$x = 7 \leftrightarrow \frac{x}{3} = \frac{7}{3}$$

(3) adding the same number to each side,

$$x = 7 \leftrightarrow x + 6 = 7 + 6$$

(4) subtracting the same number from each side.

$$x = 7 \leftrightarrow x - 3 = 7 - 3$$

Do Questions 1 through 5 in Exercise 5·3.

Example 1

Solve for x in $2x - 7 = 11$. Verify.

Solution

$$\begin{array}{r} 2x - 7 = 11 \\ +\ 7 = +7 \\ \hline 2x - 7 + 7 = 11 + 7 \end{array}$$ (Add 7 to both sides.)

$$\therefore \quad 2x = 18$$

$$\frac{2x}{2} = \frac{18}{2}$$ (Divide both sides by 2.)

$$x = 9$$

Verify this in the original equation:

Left Side $= 2x - 7$ Right Side $= 11$

$= 2(9) - 7$

$= 18 - 7$

$= 11$

L.S. = R.S.

$\therefore \quad x = 9$ is correct.

Example 2

Solve for a in $31 - 2a = 15$. Verify.

Solution

$$\begin{array}{r} 31 - 2a = 15 \\ -31 \quad\ = -31 \\ \hline 31 - 31 - 2a = 15 - 31 \end{array}$$ (Subtract +31 from both sides.)

$$-2a = -16$$

$$\frac{-2a}{-2} = \frac{-16}{-2}$$ (Divide both sides by −2.)

$$a = 8$$

Verify this in the original equation:

L.S. $= 31 - 2a$ R.S. $= 15$

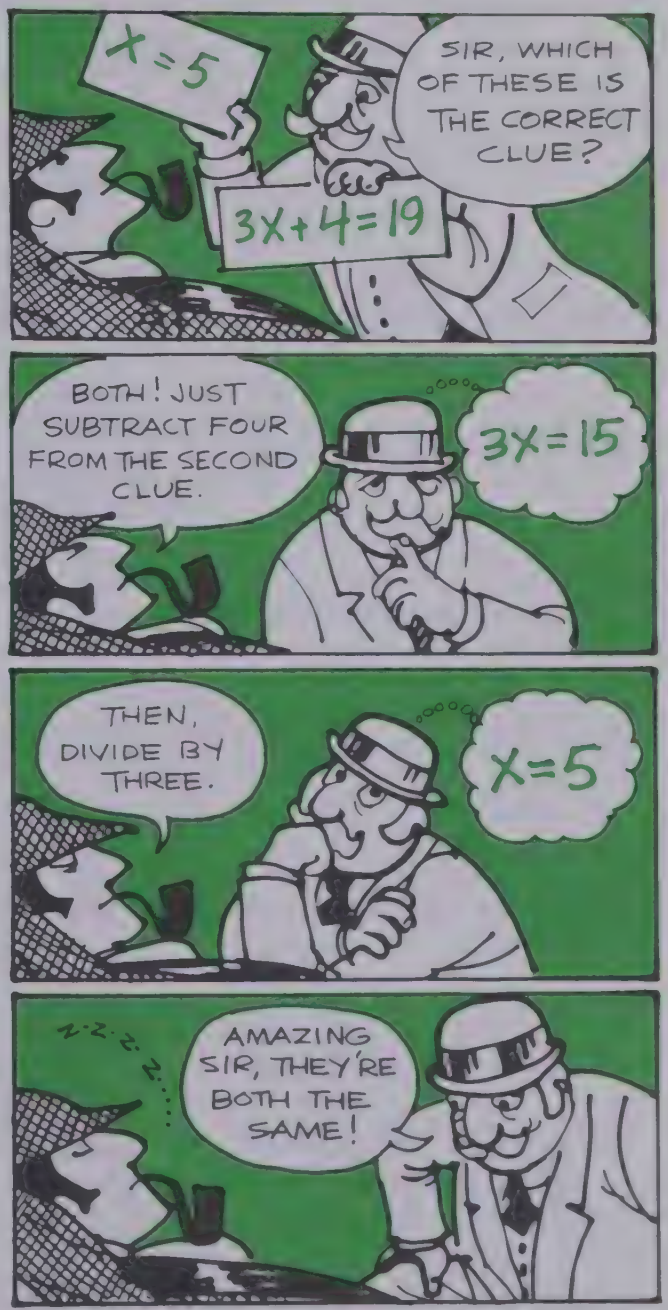
$= 31 - 2(8)$

$= 31 - 16$

$= 15$

L.S. = R.S.

$\therefore \quad a = 8$ is correct.



EXERCISE 5·3

Show your work in Questions 1-5.

1. Solve these by adding the same positive number to each side.

   (a) $t-2=6$ (b) $m-4=2$

   (c) $x-28=16$ (d) $r-36=-44$

   (e) $m-97=-32.75$ (f) $t-198=-221.5$

2. Solve these by subtracting the same positive number from each side.

   (a) $s+4=12$ (b) $t+47=-32$

   (c) $m+51=-8\frac{1}{2}$ (d) $r+191=198$

   (e) $8+m=-191$ (f) $15+s=-79.8$

3. Solve these by adding or subtracting the same number to (from) each side.

   (a) $5+r=91$ (b) $m-19=-22$

   (c) $m-7=19$ (d) $32=m+5$

   (e) $26=s-4\frac{1}{3}$ (f) $y+4=-28$

   (g) $x-91=-101$ (h) $y+47=-6\frac{3}{4}$

   (i) $m-6.7=-41.2$ (j) $t+3.5=-6.2$

4. Solve these by dividing each side by the same number.

   (a) $4x=8$ (b) $9y=108$

   (c) $7m=-35$ (d) $-6y=42$

   (e) $-11x=121$ (f) $13y=108$

   (g) $26y=130$ (h) $32x=-96$

   (i) $48m=-41$ (j) $-48y=128$

5. Solve these by multiplying each side by the same number.

   (a) $\frac{x}{2}=8$ (b) $\frac{y}{3}=9$ (c) $\frac{m}{4}=8$

   (d) $\frac{y}{3}=-3$ (e) $\frac{m}{4}=-5$ (f) $\frac{n}{-3}=4$

   (g) $\frac{r}{-2}=-1.7$ (h) $\frac{s}{-4}=-3.2$ (i) $\frac{t}{-5}=-6\frac{2}{5}$

   (j) $\frac{x}{12}=10$ (k) $\frac{x}{11}=-5$ (l) $\frac{y}{43}=22$

   (m) $\frac{n}{44}=13$ (n) $\frac{m}{101}=100$ (o) $\frac{r}{281}=28$

6. Copy and solve these by writing equivalent equations to get the simplest equations. Indicate what has been done in each step.

   (a) $3x-2=13$ (i)
   $3x=$ ___ (ii)
   $x=$ ___

   (b) $2x-3=-13$ (i)
   $2x=$ ___ (ii)
   $x=$ ___

   (c) $4-2x=14$ (i)
   $-2x=$ ___ (ii)
   $x=$ ___

   (d) $5x+4=34$ (i)
   $5x=$ ___ (ii)
   $x=$ ___

   (e) $6-3x=15$ (i)
   $-3x=$ ___ (ii)
   $x=$ ___

   (f) $2x-12=-12$ (i)
   $2x=$ ___ (ii)
   $x=$ ___

7. *Note!* This exercise is not only set out for the purpose of obtaining an answer for each question. It is also the intention in this exercise that the method of solution be learned. By showing the steps of the solution (as in the examples above), we indicate our thinking procedures.

   Solve each equation for the variable, show each step, and verify.

   (a) $3x-5=4$ (b) $7y+3=17$

   (c) $2z+13=25$ (d) $5a-4=21$

   (e) $4b-\frac{7}{10}=\frac{9}{10}$ (f) $9x-0.6=20.1$

   (g) $6x+\frac{2}{8}=\frac{3}{8}$ (h) $12m+7.1=3.1$

   (i) $21t-3=60$ (j) $35b+7=112$

   (k) $13a-31=60$ (l) $26x+11=219$

   (m) $14-3x=8$ (n) $27-4y=7$

   (o) $56-6m=20$ (p) $82-8a=42$

   (q) $143=15b-83$ (r) $49-12x=25$

   (s) $216-13z=47$ (t) $181-9a=73$

   (u) $306-15x=81$ (v) $114+23m=183$

   (w) $425-25a=300$ (x) $253+18x=379$

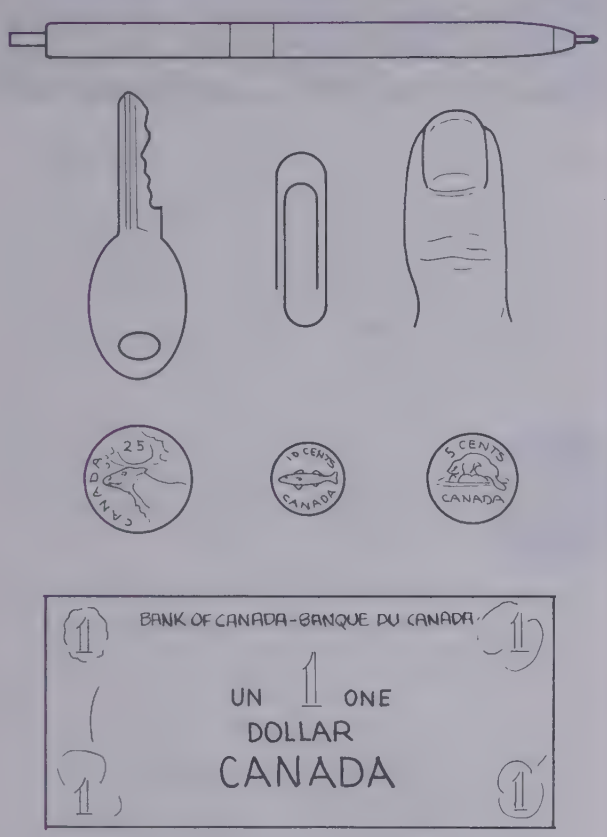
   (y) $2m-4=26$ (z) $5z+4=99$

(aa) $8 - 6n = -52$ (bb) $13s - 26 = 104$

(cc) $25y + 25 = 650$ (dd) $24x - 16 = 464$

(ee) $23 - 9k = 131$ (ff) $2.5q + 2.7 = 7.7$

(gg) $\frac{3}{4}m - \frac{1}{2} = 1$ (hh) $\frac{n}{3} + \frac{1}{4} = \frac{3}{4}$



Estimate, in metric units, the measures of the items shown.

5·4 More on Equations

The study of equations is an important part in the study of algebra. The foundation laid here is of great importance, for no matter how advanced a study of algebra is made, none of this is discarded.

In this section, the equations that are covered are not readily solved by mere observation or inspection. There is a greater need here for a formal method of solution than in the previous section.

Writing equivalent equations is an important part of solving equations. Follow this illustration:

$$\begin{array}{r} x = 9 \\ +x = +x \\ \hline 2x = 9 + x \end{array}$$ (Add x to each side.)

$$\begin{array}{r} 2x = 9 + x \\ -x = \quad -x \\ \hline x = 9 \end{array}$$ (Subtract x from each side *OR* add $-x$ to each side.)

Study one more example:

$$\begin{array}{r} y = -6 \\ -3y = \quad -3y \\ \hline -2y = -6 - 3y \end{array}$$ (Add $-3y$ to each side *OR* subtract $3y$ from each side.)

$$\begin{array}{r} -2y = -6 - 3y \\ +3y = \quad +3y \\ \hline y = -6 \end{array}$$ (Add $3y$ to each side.)

Do Questions 1 to 4 in Exercise 5·4.

Now let us put several of these types of operations together:

$$\begin{array}{r} x = 9 \\ 8x = 72 \\ 8x + 14 = 86 \\ 7x + 14 = 86 - x \end{array}$$

(i) Multiply each side by 8.
(ii) Add 14 to each side.
(iii) Subtract x from each side.

$$\begin{array}{r} 7x + 14 = 86 - x \\ 8x + 14 = 86 \\ 8x = 72 \\ x = 9 \end{array}$$

(iv) Add x to each side.
(v) Subtract 14 from each side.
(vi) Divide each side by 8.

Do Question 5 in Exercise 5·4.

Here we show a method of keeping track of what to do. You should use this method until you are good at doing the operations mentally.

Example 1

Solve for x: $4x + 5 = x + 14$.

Solution

$$\begin{aligned} 4x + 5 &= x + 14 \\ -5 &= \quad -5 \quad \text{(Subtract 5 from both sides.)} \\ \hline 4x &= x + 9 \\ -x &= -x \quad \text{(Subtract } x \text{ from both sides.)} \\ \hline 3x &= 9 \\ \frac{3x}{3} &= \frac{9}{3} \quad \text{(Divide each side by 3.)} \\ x &= 3 \end{aligned}$$

Verify. Replace x by 3 in each part of the equation.

$$\begin{aligned} \text{L.S.} &= 4x + 5 \\ &= 4(3) + 5 \\ &= 12 + 5 \\ &= 17 \\ \text{L.S.} &= \text{R.S.} \end{aligned} \qquad \begin{aligned} \text{R.S.} &= x + 14 \\ &= 3 + 14 \\ &= 17 \end{aligned}$$

$\therefore$ $x = 3$ is correct.

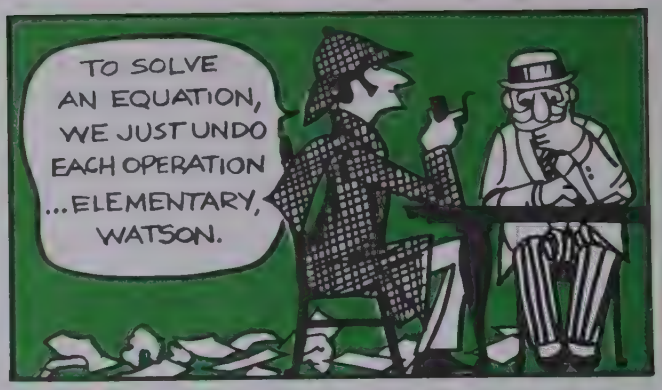
Do Questions 6(a) to 6(h) in Exercise 5·4 before continuing.

Example 2

Solve for y: $7 - 3y = y - 13$

Solution

$$\begin{aligned} 7 - 3y &= y - 13 \\ -y &= \quad -y \quad \text{(Subtract } y \text{ from both sides. Why?)} \\ \hline 7 - 4y &= -13 \\ -7 &= \quad -7 \quad \text{(Subtract 7.)} \\ \hline -4y &= -20 \\ \frac{-4y}{-4} &= \frac{-20}{-4} \quad \text{(Divide by } -4.) \\ y &= 5 \end{aligned}$$



Verify. Replace y by 5 in each part of the equation.

$$\begin{aligned} \text{L.S.} &= 7 - 3y \\ &= 7 - 3(5) \\ &= 7 - 15 \\ &= -8 \\ \text{L.S.} &= \text{R.S.} \end{aligned} \qquad \begin{aligned} \text{R.S.} &= y - 13 \\ &= 5 - 13 \\ &= -8 \end{aligned}$$

$\therefore$ $y = 5$ is correct.

Do Questions 6(i) to 6(n) in Exercise 5·4 before continuing.

Example 3

Solve for z: $7 - 3z = 12 - 2z$.

Solution

$$\begin{aligned} 7 - 3z &= 12 - 2z \\ 2z &= \quad 2z \quad \text{(Add } 2z.) \\ \hline 7 - z &= 12 \\ -7 &= \quad -7 \quad \text{(Subtract 7.)} \\ \hline -1z &= 5 \\ \frac{-1z}{-1} &= \frac{5}{-1} \quad \text{(Divide by } -1.) \\ z &= -5 \quad \text{(Remember } -z = -1z.) \end{aligned}$$

Verify. By making the appropriate replacements we get:

$$\begin{aligned} \text{L.S.} &= 7 - 3z \\ &= 7 - 3(-5) \\ &= 7 + 15 \\ &= 22 \\ \text{L.S.} &= \text{R.S.} \end{aligned} \qquad \begin{aligned} \text{R.S.} &= 12 - 2z \\ &= 12 - 2(-5) \\ &= 12 + 10 \\ &= 22 \end{aligned}$$

$\therefore$ $z = -5$ is correct.

EXERCISE 5·4

1. Solve by adding the same variable to each side.

   (a) $-2x = 6 - 3x$ (b) $-x = 8 - 2x$

   (c) $-m = 14 - 2m$ (d) $-5x = 12 - 6x$

   (e) $8 - 7x = -6x$ (f) $49 - 12x = -11x$

   (g) $-8x = -7x - 7$ (h) $-5m = -6m - 4$

   (i) $-51 - 10m = -9m$ (j) $\frac{m}{2} = \frac{5m}{2} - 10$

2. Solve each by adding the same variable to each side (or by subtracting the same variable from each side).

   (a) $2x = 6 + x$ (b) $4x = 1 + 3x$

   (c) $6x = 3 + 5x$ (d) $8 + 4x = 5x$

   (e) $12 + 7x = 8x$ (f) $14x = 15x - 2$

   (g) $5x - 9 = 6x$ (h) $7x - 14 = 8x$

   (i) $6y = 28 + 5y$ (j) $8.6x - 17.1 = 14.3x$

3. Solve each:

   (a) $-2y = 5 - 3y$ (b) $9s = 5 + 8s$

   (c) $3p = 84 + 2p$ (d) $4 - 8y = -7y$

   (e) $15 - 7y = -6y$ (f) $14 + 8r = 9r$

   (g) $33p = 18 + 32p$ (h) $15t = 8 + 14t$

   (i) $-8m = 6 - 9m$ (j) $41t = 21 + 40t$

   (k) $62s = 61s - 49$ (l) $-58t = 5 - 59t$

4. Solve these tricky ones:

   (a) $4y + y = 25$ (b) $6 - 3y - 4 = 14$

   (c) $3y - 4 - y = 10$ (d) $21 = 8y - 5y$

   (e) $16y - 9y = 35 - 14$ (f) $7y + 8y = 19 + 3$

   (g) $13 - 1 = 17m - 11m$

   (h) $6 - 13 = 89m - 88m$

   (i) $29y - 30y = 14 - 15$

   (j) $1.1 = 2.3x - 3.4x$

   (k) $\frac{2x}{3} - \frac{x}{3} = \frac{5}{6}$

   (l) $\frac{y}{4} + \frac{3}{4} = \frac{y}{2} - \frac{1}{2}$

   (m) $13.4 + x = 18.2 - 0.2x$

   (n) $\frac{2x}{3} - 1 + \frac{4x}{6} = \frac{5}{3}$

5. Copy and complete each. Indicate what has been done in each step.

   (a)
   $x = 15$ (i)
   $9x = 135$ (ii)
   $9x + 10 = 145$ (iii)
   $7x + 10 = 145 - 2x$
   $7x + 10 = 145 - 2x$
   $9x + 10 =$ _____ (iv)
   $9x =$ _____ (v)
   $x =$ _____ (vi)

   (b)
   $x = 4$ (i)
   $-5x = -20$ (ii)
   $9 - 5x = -11$ (iii)
   $9 - 3x = 2x - 11$
   $9 - 3x = 2x - 11$
   $9 - 5x =$ _____ (iv)
   $-5x =$ _____ (v)
   $x =$ _____ (vi)

   (c)
   $m = -0.5$ (i)
   $-8m = 4$ (ii)
   $14 - 8m = 18$ (iii)
   $14 - 12m = 18 - 4m$
   $14 - 12m = 18 - 4m$
   $14 - 8m =$ _____ (iv)
   $-8m =$ _____ (v)
   $m =$ _____ (vi)

   (d)
   $x = 5$ (i)
   $-6x = -30$ (ii)
   $13 - 6x = -17$ (iii)
   $13 + x = 7x - 17$
   $13 + x = 7x - 17$
   $13 - 6x =$ _____ (iv)
   $-6x =$ _____ (v)
   $x =$ _____ (vi)

   (e)
   $x = 5$ (i)
   $-6x = -30$ (ii)
   $x = 7x - 30$ (iii)
   $13 + x = 7x - 17$
   $13 + x = 7x - 17$
   $x =$ _____ (iv)
   $-6x =$ _____ (v)
   $x =$ _____ (vi)

(f)

$x = -3$ (i)

$6x = -18$ (ii)

$18 + 6x = 0$ (iii)

$18 + 4x = -2x$

$18 + 4x = -2x$ (iv)

$\_\_\_\_ = \_\_\_$ (v)

$\_\_\_\_ = \_\_\_$ (vi)

$x = \_\_\_$

(g)

$x = -3$ (i)

$6x = -18$ (ii)

$4x = -18 - 2x$ (iii)

$18 + 4x = -2x$

$18 + 4x = -2x$ (iv)

$\_\_\_\_ = \_\_\_$ (v)

$\_\_\_\_ = \_\_\_$ (vi)

$x = \_\_\_$

Compare (d) and (e). What do you notice?
Compare (f) and (g). What do you notice about the order of the operations in writing equivalent equations?

6. Solve each equation for its variable, show each step, then verify.

(a) $2x - 3 = x + 4$ (b) $3y + 5 = y + 9$

(c) $4a + 1 = 10 + a$ (d) $5b - 7 = 5 + b$

(e) $z + 6 = 2z - 1$ (f) $y + 12 = 3y + 2$

(g) $a + 21 = 9a - 3$ (h) $19 + z = 2z + 8$

(i) $14 - 2x = 8 - x$ (j) $23 - 3y = 17 - y$

(k) $4 + 7z = 9 - z$ (l) $17 - 2a = a - 23$

(m) $\frac{4}{10} - 3b = \frac{9}{10} - b$ (n) $\frac{8}{10} + 5x = \frac{2}{10} - x$

(o) $\frac{25}{100} + 7y = \frac{5}{100} + 3y$ (p) $\frac{9z}{10} + 11 = \frac{5z}{10} + 3$

(q) $8b + 15 = 3b = 10$ (r) $5x - 7 = 12x + 7$

(s) $4y + 7 = 3y - 4$ (t) $9z - 28 = 20 + 5z$

(u) $7 + 13m = 18m - 13$ (v) $64 - 3x = 8 + x$

(w) $13 + 7b = 10b - 62$ (x) $87 - 6y = 15y - 57$

7. Solve:

(a) $2y - 4 = y - 13$

(b) $6y + 7 = 2y - 9$

(c) $2y - 6 = 3y - 17$

(d) $17m - 4 = 6m + 117$

(e) $23 - 5y = -19 - 11y$

(f) $4w - 6 = -6 + 5w$

(g) $10 - 17y = -38 - 14y$

(h) $30 - 17y = 70 - 7y$

(i) $14s - 32 = 19s + 38$

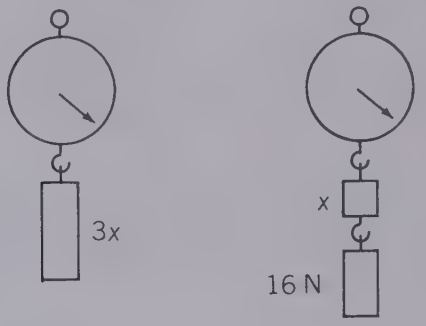
(j) $8t + 3t = 14 + 8$

(k) $12p + 8p = 13 + 27$

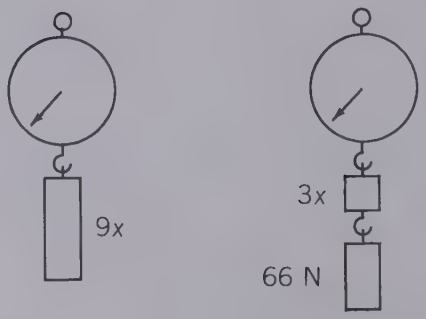
(l) $19p - 8p - 3 = 1p - 17 - 16$

8. The readings on each pair of scales are the same. Calculate the value of x in each case in newtons.

(a)



(b)



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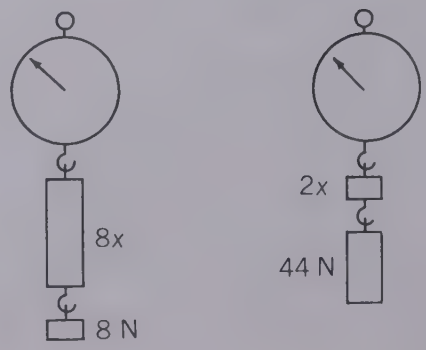


5·5 Inequations

Most inequations can be solved much like equations. In this book, we will only deal with those inequations that can be solved as equations are solved.

Remember! $13 > 5$ is read, "13 is greater than 5."
$5 < 7$ is read, "5 is less than 7."

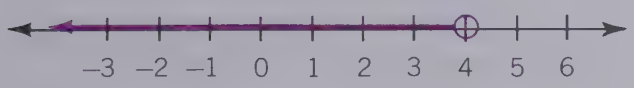
Example 1

Solve and show the solution on a number line.

(a) $3x < 12$ (b) $4x - 8 > 15$

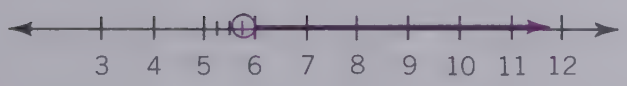
Solution

(a)
$$3x < 12$$
$$\frac{3x}{3} < \frac{12}{3} \quad \text{(Divide by 3.)}$$
$$x < 4$$



How do we show that 4 is not included?

(b)
$$4x - 8 > 15$$
$$4x - 8 > 15 \quad \text{(Add 8.)}$$
$$+8 \quad +8$$
$$4x > 23$$
$$\frac{4x}{4} > \frac{23}{4} \quad \text{(Divide by 4.)}$$
$$x > 5\tfrac{3}{4}$$



EXERCISE 5·5

Solve and show your solution on a number line.

1. $8x < 48$
2. $6x > 40$
3. $\frac{1}{3}x < 22$
4. $\frac{x}{3} > 12$
5. $\frac{x}{6} > 13$
6. $\frac{2x}{3} > 8$
7. $x + 5 > 9$
8. $x - 6 < -15$
9. $3x + 3 > 17$
10. $9x - 6 > 27$
11. $8x - 7 < -23$
12. $7x + 4 < -37$
13. $18 + 6x > -54$
14. $18 + 3x > 42$
15. $19 + 24x < -41$
16. $23 + 12x < -1$
17. $16.46 + 2.8x > 7.5$
18. $6.6 + 16x > 78.6$

5·6 Using Reciprocals

Can you think of two numbers whose product is 1? How many such pairs of numbers can you think of?

There are many pairs of numbers whose product is 1. Some of these pairs are shown here:

3 and $\frac{1}{3}$ $\frac{2}{9}$ and $\frac{9}{2}$ $\frac{6}{5}$ and $\frac{5}{6}$

Let's check each:

$3 \times \frac{1}{3} = 1$ $\frac{2}{9} \times \frac{9}{2} = 1$ $\frac{6}{5} \times \frac{5}{6} = 1$

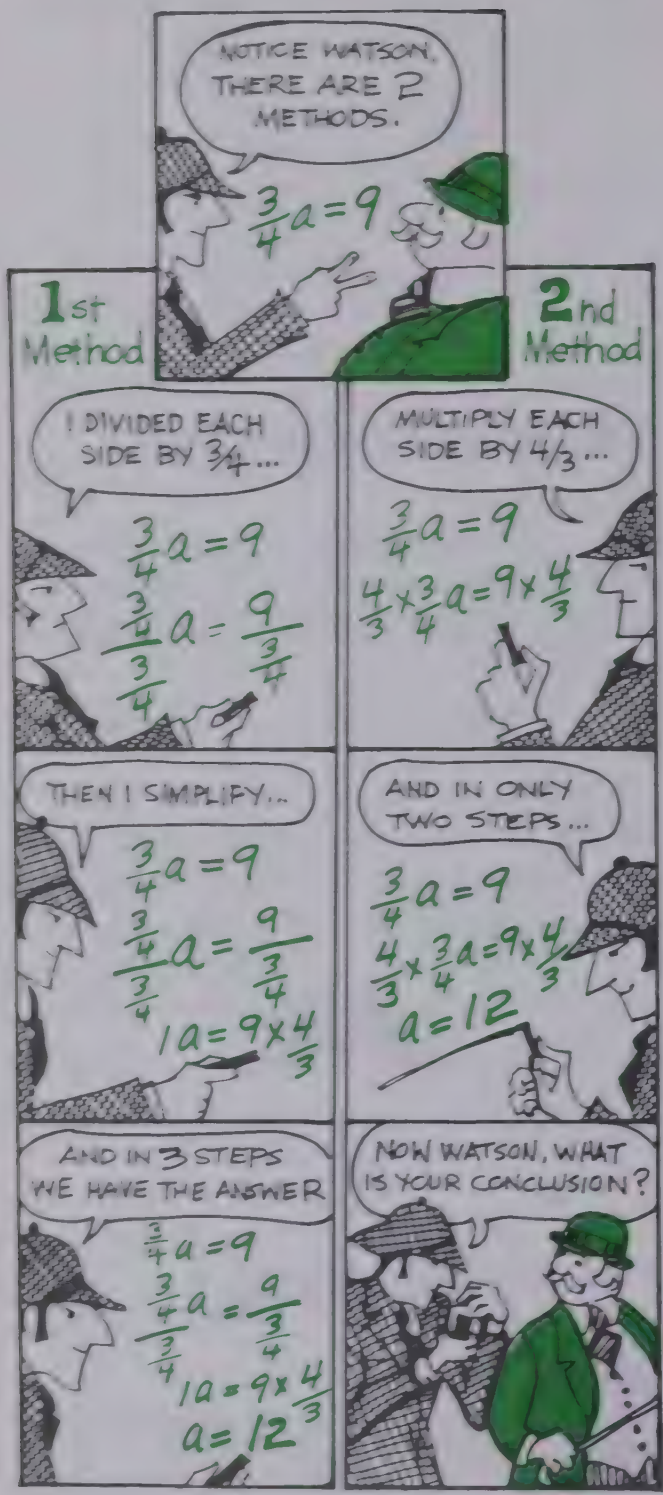
The reciprocal of $\frac{6}{5}$ is $\frac{5}{6}$. Also, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}$. What is the reciprocal of $\frac{2}{9}$? of $\frac{9}{2}$? of 3? of $\frac{1}{3}$? We say $\frac{6}{5}$ and $\frac{5}{6}$ are reciprocals. Name five pairs of reciprocals.

The product of a number and its reciprocal is 1.

$$\frac{a}{b} \times \frac{b}{a} = 1$$

Do Questions 1 to 5 in Exercise 5·6.

We can solve equations in two ways as Sherlock Holmes illustrates.



Example 1

Solve for y: $-\frac{3}{4}y=27$

1st Solution: $-\frac{3}{4}y=27$

Step 1: $(-\frac{4}{3})(-\frac{3}{4}y)=-\frac{4}{3}\cdot 27$ (Multiply each side by $-\frac{4}{3}$.)

Step 2: $y=-36$

2nd Solution: $-\frac{3}{4}y=27$

Step 1: $\frac{-\frac{3}{4}y}{-\frac{3}{4}}=\frac{27}{-\frac{3}{4}}$ (Divide each side by $-\frac{3}{4}$.)

Step 2: $(-\frac{4}{3})\cdot(-\frac{3}{4}y)=27\cdot(-\frac{4}{3})$

Step 3: $y=-36$

Did you notice that Step 2 in the second solution is equivalent to Step 1 in the first solution?

Verify in original equation:

$$\text{L.S.}=-\tfrac{3}{4}y \qquad \text{R.S.}=27$$
$$=(-\tfrac{3}{4})\cdot(-36)$$
$$=27$$
$$\text{L.S.}=\text{R.S.}$$

$\therefore y=-36$ is correct.

Do Questions 6 to 22 in Exercise 5·6.

Example 2

Solve for z: $23-\frac{2}{3}z=15$.

Solution:

$$23-\tfrac{2}{3}z=15$$
$$-23 \quad =-23 \qquad \text{(Subtract 23.)}$$
$$-\tfrac{2}{3}z=-8$$
$$(-\tfrac{3}{2})(-\tfrac{2}{3}z)=(-\tfrac{3}{2})(-8) \qquad \text{(Multiply by } -\tfrac{3}{2}\text{.)}$$
$$z=12$$

Verify:

$$\text{L.S.}=23-\tfrac{2}{3}(12) \qquad \text{R.S.}=15$$
$$=23-8$$
$$=15$$
$$\text{L.S.}=\text{R.S.}$$

$\therefore z=12$ is correct.

Do Questions 23 to 29 in Exercise 5·6.

Mathematicians have special descriptions or names for some of the things we have been doing and saying in this section. Since you may want to read and use other books, you should be familiar with these descriptions.

In examples like

$$x + 3 = 9 \text{ and } y - 6 = 12,$$

we solved each by "adding the same number to each side".

We shall call this the "**Addition Property of Equality**" and abbreviate the name by APE.

$$\begin{array}{rl} x + 3 = 9 & \textit{Addition Property of Equality} \\ \underline{-3 = -3} & \text{We added } -3 \text{ to each side of the} \\ x = 6 & \text{equality.} \end{array}$$

Similarly, for examples like

$$\tfrac{4}{5}x = 12 \text{ and } 4y = 60,$$

we have solved each by "multiplying each side by the same number".

We will call this the "**Multiplication Property of Equality**" and abbreviate the name by MPE.

$$\begin{array}{l} \tfrac{4}{5}x = 12 \\ \tfrac{5}{4} \times \tfrac{4}{5}x = \tfrac{5}{4} \times 12 \qquad \text{(MPE.)} \\ x = 15 \end{array}$$

Example 3

Solve for x: $16 - \dfrac{3x}{8} = -13 + \dfrac{5x}{6}$.

Solution

$$16 - \frac{3x}{8} = -13 + \frac{5x}{6}$$

$$\underline{-16 \qquad = -16} \qquad \text{(APE.)}$$

$$-\frac{3x}{8} = -29 + \frac{5x}{6}$$

$$\underline{-\frac{5x}{6} = \qquad -\frac{5x}{6}} \qquad \text{(APE – since we want to collect all the terms containing } x \text{ on the left side.)}$$

$$-\frac{29x}{24} = -29$$

$$\left(-\frac{24}{29}\right)\left(-\frac{29x}{24}\right) = \left(-\frac{24}{29}\right)(-29) \qquad \text{(MPE.)}$$

$$x = 24$$

Verify:

$$\begin{array}{ll} \text{L.S.} = 16 - \tfrac{3}{8}(24) & \text{R.S.} = -13 + \tfrac{5}{6}(24) \\ \quad = 16 - 9 & \quad = -13 + 20 \\ \quad = 7 \longleftrightarrow & \quad = 7 \end{array}$$

$$\text{L.S.} = \text{R.S.}$$

$\therefore x = 24$ is correct.

EXERCISE 5·6

1. Copy and calculate the answer to each:

(a) $2 \times \frac{1}{2}$	(b) $\frac{3}{4} \times \frac{4}{3}$	(c) $\frac{7}{8} \times \frac{8}{7}$
(d) $\frac{1}{9} \times 9$	(e) $\frac{2}{3} \times \frac{3}{2}$	(f) $\frac{7}{9} \times \frac{9}{7}$
(g) $(-\frac{1}{4}) \times (-4)$	(h) $(-\frac{5}{7}) \times (-\frac{7}{5})$	(i) $(-\frac{8}{3}) \times (-\frac{3}{8})$

2. Calculate each quotient:

(a) $\frac{2}{2}$	(b) $\frac{\frac{3}{4}}{\frac{3}{4}}$	(c) $\frac{\frac{7}{8}}{\frac{7}{8}}$
(d) $\frac{\frac{1}{9}}{\frac{1}{9}}$	(e) $\frac{\frac{2}{3}}{\frac{2}{3}}$	(f) $\frac{\frac{7}{9}}{\frac{7}{9}}$
(g) $\frac{-\frac{1}{4}}{-\frac{1}{4}}$	(h) $\frac{-\frac{7}{5}}{-\frac{7}{5}}$	(i) $\frac{-\frac{8}{3}}{-\frac{8}{3}}$

3. What similarities do you see between each part in Question 1 and its matching part in Question 2?

4. Write the reciprocal of each:

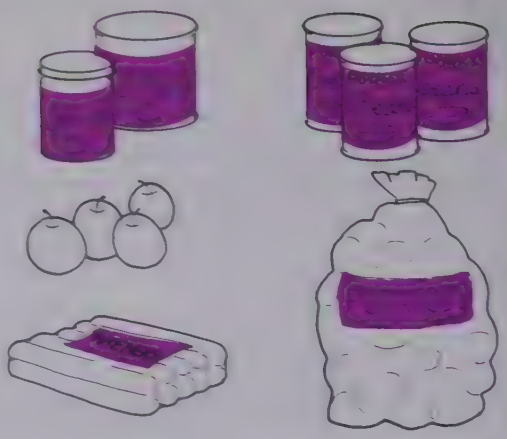
(a) $\frac{1}{2}$	(b) 5	(c) $\frac{6}{7}$
(d) $\frac{8}{9}$	(e) 12	(f) $\frac{1}{9}$
(g) $\frac{3}{1}$	(h) 3	(i) 7
(j) 17	(k) $\frac{1}{15}$	(l) -5
(m) $-\frac{10}{3}$	(n) $-\frac{1}{4}$	(o) -7

5. Write what *reciprocal* means to you. Discuss your answer with a friend and listen to his answer. Does *reciprocal* mean the same to both of you? If not, who is correct? Why?

Using reciprocals, solve each equation and show your work. Verify.

6. $\frac{1}{5}m = 13$
7. $\frac{2}{5}x = 32$
8. $\frac{1}{4}a = 11$
9. $\frac{3}{5}x = 12$
10. $\frac{6}{5}x = 55$
11. $\frac{4}{5}c = 28$
12. $\frac{1}{13}x = 36$
13. $3x = 15$
14. $\frac{1}{3}n = 6$
15. $\frac{3}{5}y = 15$
16. $\frac{5}{6}y = 30$
17. $-7a = 14$
18. $-\frac{1}{3}x = 16$
19. $-\frac{1}{8}a = 56$
20. $-\frac{4}{5}a = -20$
21. $-\frac{1}{3}x = 30$
22. $-\frac{1}{4}n = -70$
23. $\frac{4}{7}x - 8 = 4$
24. $\frac{3x}{10} - 18 = 3$
25. $\frac{7x}{12} - 35 = 0$
26. $14 - \frac{x}{3} = 2$
27. $7 - \frac{3}{7}y = 16$
28. $30 - \frac{4}{5}x = 22$
29. $18 - \frac{3}{5}a = 45$
30. $34 - \frac{2x}{11} = 40$
31. $22 - \frac{4x}{13} = 2$
32. $3 - \frac{1}{2}m = 11 - \frac{1}{4}m$
33. $\frac{1}{3}y + 7 = 5y - 1$
34. $\frac{3}{10}b + 9 = \frac{5}{4}b - 10$
35. $8 - \frac{3}{7}x = \frac{7}{4}x + 69$
36. $\frac{5}{6}z + 17 = \frac{1}{2}z - 6$
37. $3n - 21 = \frac{8}{5}n - 7$
38. $14 - \frac{5}{12}y = \frac{1}{15}y - 15$
39. $\frac{5}{16}x - 13 = 50 + \frac{10}{8}x$

Unit Pricing



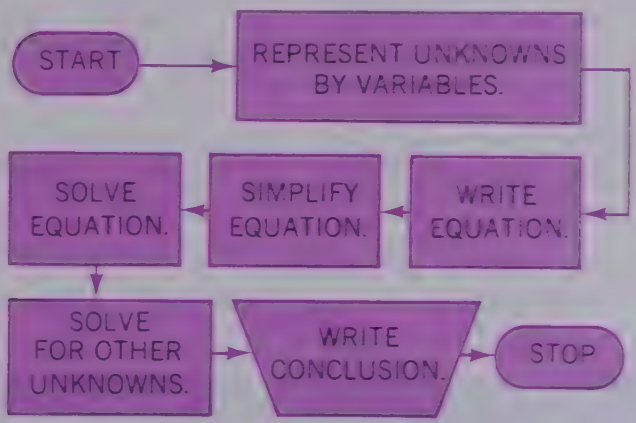
The use of metric units makes unit pricing very easy. Determine which is the better buy.

1. 400 g of jam at 80¢
   or
   1.5 kg at $3.15?
2. Three 0.5 ℓ cans of pork and beans for $1.11
   or
   Four 0.75 ℓ cans at $2.25?
3. 7 kg of potatoes for 98¢
   or
   500 g at 8¢?
4. Apples at 2 kg for 98¢
   or
   Apples at 750 g for 40¢?
5. Orange juice: 350 ml at 49¢
   or
   1 ℓ at $1.36?
6. Wieners: 2 kg at $3.60
   or
   700 g at $1.26?
7. Ribbon: 2 m for $3.30
   or
   300 cm for 50¢?
8. Styrofoam sheets: 900 mm × 700 mm for $2.24
   or
   0.75 m × 2.5 m for $6.65?

5·7 Solving Verbal Problems

Now that we have developed further skills in solving algebraic equations, we shall extend these skills to word problems. In Section 2·8, we dealt with problems of a simpler nature. In this section, we will probe deeper into the same type of problems.

First, let us outline the steps to follow in solving word problems.



Example 1

The width of a rectangular lot of land is two thirds of its length. If the perimeter of the lot is 1200 m, find its dimensions.

Solution

We want to find the length and width of the lot. Therefore, let x represent the length of the lot. Then, $\frac{2}{3}x$ will represent the width.

We remember the perimeter of a rectangle is $P = 2 \times \text{length} - 2 \times \text{width}$.

$$P = 2l + 2w$$

The perimeter is 1200 m.

$$1200 = 2x + 2(\tfrac{2}{3}x)$$
$$1200 = 2x + \tfrac{4}{3}x$$
$$1200 = \tfrac{10}{3}x \qquad (2 + \tfrac{4}{3} = \tfrac{10}{3}.)$$
$$\tfrac{10}{3}x = 1200$$
$$\tfrac{3}{10} \cdot \tfrac{10}{3}x = \tfrac{3}{10} \cdot 1200 \text{ (MPE.)}$$

Length: $x = 360$ m

Width: $\frac{2}{3}x = \frac{2}{3} \cdot 360$
$= 240$ m

Verify:

$$P = 2l - 2w$$
$$\text{L.S.} = 1200$$
$$\text{R.S.} = 2(360) - 2(240)$$
$$\text{R.S.} = 720 + 480$$
$$= 1200$$

$\therefore$ length = 360 m, width = 240 m.

Do Questions 1 to 4 in Exercise 5·7.

Example 2

A board is 5 m long. It is to be cut into two parts. The shorter part is two thirds the length of the longer part. Find the lengths of the two parts.

Solution

Step

1 Let x be the length in metres of the longer piece. Then, $\frac{2}{3}x$ is the length in metres of the shorter piece. We know the two pieces together are 5 m long, so

2 $$x + \tfrac{2}{3}x = 5$$

3 $$\tfrac{3}{3}x + \tfrac{2}{3}x = 5$$
$$\tfrac{5}{3}x = 5$$

4 $$\tfrac{3}{5} \times \tfrac{5}{3}x = \tfrac{3}{5} \times 5 \qquad \text{(MPE.)}$$
$$x = 3$$

5 $$\tfrac{2}{3}x = \tfrac{2}{3} \times 3$$
$$x = 2$$

Verify:

The two lengths are 5 m:

6 $2 + 3 = 5$. This checks.

The shorter is $\frac{2}{3}$ of the longer:

$2 = \frac{2}{3} \times 3$. This checks.

Do Questions 5 to 9 in Exercise 5·7.

Example 3

Bill is 1.2 times as old as his brother Louis. Four years ago their ages totalled 25 years. What are their ages now?

Solution

We want to find the age of Louis and of Bill.

Let y represent Louis' age now.
Then, $1.2y$ represents Bill's age now.
Louis' age 4 years ago was $y - 4$.
Bill's age 4 years ago was $1.2y - 4$.
Louis' age 4 years ago + Bill's age 4 years ago = 25.

$$\therefore \quad (y-4)+(1.2y-4)=25$$
$$y-4+1.2y-4=25$$
$$2.2y-8=25$$
$$2.2y-8+8=25+8 \qquad \text{(APE.)}$$
$$2.2y=33$$
$$\frac{1}{2.2}\times\frac{2.2y}{1}=\frac{33}{1}\times\frac{1}{2.2} \qquad \text{(MPE.)}$$
$$y=15$$
$$1.2y=18$$

Verify:

$$(15-4)+(18-4)=11+14=25.$$

$\therefore$ Louis' age is 15 years and Bill's is 18 years.

EXERCISE 5·7

Solve the following word problems.

1. The length of a rectangle is three times the width. The perimeter is 72 cm. Calculate the length and width.

2. The length of a rectangle is four times the width. The perimeter is 72 cm. Calculate the length and width.

3. The width of a rectangle is one half its length. The perimeter is 42 cm. Calculate the length and width.

4. The width of a rectangle is three fourths of its length. The perimeter is 56 cm. Calculate the length and width.

5. Harry is one third as old as Mary. The sum of their ages is 24. How old are Harry and Mary?

6. A board is 150 cm long. It is cut into two parts. The shorter is one half the length of the longer part. Calculate the lengths of the two parts.

7. A ribbon is 40 cm long. It is cut into two parts. The shorter part is three fifths the length of the longer part. Calculate the lengths of the two parts.

8. Mary and Jean together walked 6 km. Mary walked half as far as Jean. Calculate how far each walked.

9. Harry and Jim went cross-country skiing. Harry skied four fifths the distance Jim skied. Together they went 9 km. How far did each ski?

Wooden skis need waxing.
The type of wax needed depends on
– the temperature
– the snow conditions.
One wax company prepared this chart:

Wax	Temperature	Snow
special green	*−8°C and below*	*extreme cold*
green	*−3°C to −12°C*	*new, fine-grained*
blue	*0°C to −6°C*	*old, fine-grained*
violet	*around 0°C*	*new, or fine-grained settled*
red	*above 0°C*	*moist, fine-grained settled*
yellow (Klister)	*above 0°C*	*moist, new snow glazed tracks*

On several consecutive winter days, determine the wax you would use for cross country skiing.

10. Ivor compared his age with his father's. His father is 2.5 times as old as he is. The sum of their ages 4 years ago was 55. How old are Ivor and his father now?

11. Refer to Example 3. Let Bill's age equal y years. How would you express Louis' age? Complete the question using this approach.

12. Alice is twice as old as her sister Joan. In ten years, the sum of their ages will be 41 years. What are their ages now?

13. Two numbers differ by 84 and their sum is 348. What are the numbers?

14. Adele is 0.75 times as old as Marie. Eight years from now, their ages will total 44 years. How old are they now?

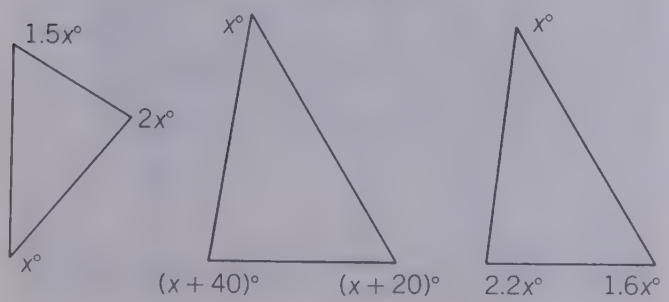
15. A farmer sells 7 cows and 17 pigs for \$2252. Each cow sells for \$140 more than each pig. What is the selling price of each cow?

16. Two girls operate a hotdog stand at the fair. Since Jane built the stand, it was agreed that she receive seven fourths as much of the profits as her partner Kathy. How would \$435.50 in profits be divided?

17. Sam saved 2.5 times as much as Bill. After Sam spent \$3, he still had \$9 more than Bill. How much did Bill save?

18. The perimeter of a school gymnasium is 112 m. If the length is four thirds the width, what are the dimensions of the gym?

19. The sum of the measures of the angles of a triangle is 180°. Calculate the measures of the angles in these triangles.



20. A board is 200 cm long. It is cut into two parts such that one part is three sevenths as long as the other. Find the lengths of the two parts.

21. Two basketball teams share the gate receipts of \$68.00. The losing team gets three fifths of what the winning team receives. How much will each team get?

22. A house and lot cost \$33 660. The house cost 5.6 times as much as the lot. How much does each cost?

23. The total length of three pieces of board is 270 cm. One piece is $1\frac{1}{2}$ times as long as the shortest piece. The other is twice as long as the shortest piece. How long are each of the pieces?

24. Jane is 4 years older than Lewis. Adele is 7 years younger than Jane. If the total of their ages is 40, how old is each of them?

Metric Canada

Refer to the Appendix if you are not familiar with or are having difficulty with metric units.

1. Measure and calculate the distances:

From	To	cm	km
Victoria	St. John's		
Edmonton	Yellowknife		
Halifax	Toronto		
Ottawa	Winnipeg		
Quebec City	Churchill		
Fredericton	Whitehorse		
Dawson	Vancouver		
Resolute	Windsor		

5·8 Percentage Problems

There are many problems in which percentages are used. We do not intend to cover all such problems in this section. However, the variety that will be dealt with should provide practical examples of how to solve them. Because of the variety, it will be necessary to become acquainted with certain formulas. These will be included in each question.

Example 1

Mr. Farmer borrowed a sum of money at an 8% rate of interest. The interest on the loan for two years is $672. What sum of money did he borrow?
(Use $i = prt$.)

i – interest
p – principal amount borrowed
r – rate of interest
t – time in years of loan

Solution

The interest formula $i = prt$ has four variables, three of which are known from the question.

$i = \$672 \qquad r = 8\% = \frac{8}{100} = 0.08 \qquad t = 2$

The unknown variable is p.

Making replacements in the formula $i = prt$:

$$i = p \times r \times t$$
$$672 = p \times 0.08 \times 2$$
$$672 = 0.16p \qquad \text{(Multiply.)}$$
$$\frac{1}{0.16} \times 672 = \frac{1}{0.16} \times 0.16p \qquad \text{(MPE.)}$$
$$p = 4200$$

The sum of money borrowed was $4200.

STOP Do Questions 1 through 5 in Exercise 5·8.

Example 2

A merchant sold a portable humidifier for $59.26, and this price included a gain of 15% on the cost. What did the humidifier cost? (Use $S = C + Cr$ where S = selling price, C = cost price, and r = rate of gain.)

Solution

For the formula $S = C + Cr$:

$S = \$59.26 \quad r = 15\% = \frac{15}{100} = 0.15 \quad C = ?$

Make replacements in the formula:

$$S = C + Cr$$
$$59.26 = C + C\left(\tfrac{15}{100}\right)$$
$$59.26 = \tfrac{100}{100}C + \tfrac{15}{100}C \quad \text{(Do you follow this step?)}$$
$$59.26 = \tfrac{115}{100}C$$
$$\tfrac{100}{115}(59.26) = \tfrac{100}{115}\left(\tfrac{115}{100}C\right) \qquad (\text{MPE} \times \tfrac{100}{115}.)$$
$$52.4 = C$$

$\therefore$ the cost is $52.40. Verify.

EXERCISE 5·8

1. Lois borrowed \$200 to buy a motorbike. The bank charged $11\frac{1}{2}\%$ interest per year. How much interest would she pay in two years? ($i = prt$)

2. Mike had to buy new clothes for his job as a sales clerk. He borrowed \$150 from a bank for $1\frac{1}{2}$ years. The interest rate was 9%. What was the interest he paid on the loan? How much did he repay the bank?

3. A loan of \$4800 at 5% per annum results in \$720 interest after a number of years. Find the time in years. (Use $i = prt$)

4. \$7000 is invested, one quarter at 8% and the balance at 5%. What is the annual interest on this total investment? (Use $i = pr + PR$.)

5. A woman invested \$5000 at 6% and \$2500 at $8\frac{1}{2}\%$. How much interest did she gain in one year?

6. A shop marked up coats by 35% above cost. If a coat cost \$64, find the selling price. (Use $S = C + Cr$.)

7. A house was sold for \$23 400 at a loss of $2\frac{1}{2}\%$ of cost. What was the cost of the house? (Use $S = C - Cr$.)

8. A sales representative works on a salary plus commission basis. The salary is \$320 per month plus 8% of the total sales for the month. Find the earnings for the month of June when sales totalled \$13 850. (Use commission = rate × sales. Then add salary and commission.)

9. A commission sales representative collected \$1400 in commission from sales in one month. If the rate of commission was $12\frac{1}{2}\%$, what were the total sales for the month? (Use $c = r \times s$.)

5·9 Problems Involving Ratios

When we compare two numbers by saying "*five to eight*", then 5:8 and $\frac{5}{8}$ are two ways of expressing that relationship in symbol form. For our needs in problem solving, ratios will be expressed as fractions.

Also, you will recall that one fraction can be written in many equivalent fraction forms.

$$\frac{2}{3} = \frac{4}{6} = \frac{8}{12} = \frac{14}{21} = \frac{50}{75} = \frac{222}{333}$$

When we write two ratios equal to each other, such as $\frac{3}{5} = \frac{9}{15}$, we have a proportion. When a proportion such as $\frac{3}{5} = \frac{9}{15}$ is stated, then we discover that

$$3 \times 15 = 5 \times 9.$$

Similarly, for the proportion

$$\frac{2}{7} = \frac{4}{14}$$

$$2 \times 14 = 7 \times 4.$$

We generalize:

if $\frac{a}{b} = \frac{c}{d}$,

then $a \times d = b \times c$.

Do Questions 1 to 3 in Exercise 5·9.

We shall use proportion in solving the problems that follow.

Example 1

Two numbers are in a ratio of 2:3. If one number is larger than the other by 4. what are the numbers?

Solution

Let the smaller number be x and the larger number be $x + 4$.

Then $\frac{x}{x+4} = \frac{2}{3}$

$3(x) = 2(x+4)$ (Why can we perform this operation?)

$3x = 2x + 8$

$3x - 2x = 2x - 2x + 8$ (Why?)

$x = 8$

$x + 4 = 12$

$\therefore$ the two numbers are 8 and 12.

Example 2

Divide \$91 into two parts which are in a ratio of 2 to 5.

Solution

Let one part be x and the other part be $91 - x$.

$\therefore \quad \frac{x}{91-x} = \frac{2}{5}$

$5(x) = 2(91 - x)$

$5x = 182 - 2x$

$5x + 2x = 182 - 2x + 2x$ (Why?)

$7x = 182$

$\frac{7x}{7} = \frac{182}{7}$ (Why?)

$x = 26$

$91 - x = 91 - 26 = 65$

$\therefore$ the two parts are \$26 and \$65.

Example 3

In a supermarket, apples are selling at 2 kg for 29¢. How much will 7 kg cost?

Solution

Let 7 kg cost y cents.

If 2 kg cost 29¢

then 7 kg cost y¢.

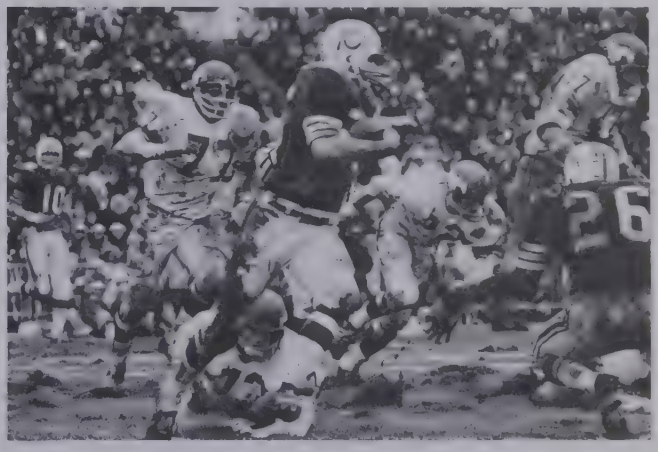
$\frac{2}{7} = \frac{29}{y}$

$2y = 203$

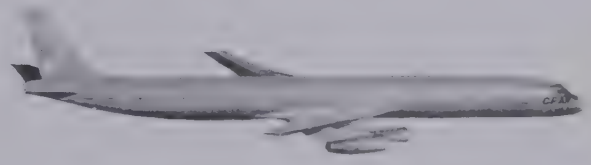
$y = 101\frac{1}{2}$

$\therefore$ 7 kg of apples cost \$1.02.

(Why do you think a business would charge \$1.02 instead of \$1.01$\frac{1}{2}$ or \$1.01?)



At the Grey Cup Game in Toronto in 1971, 36 000 people were in the grandstand. In Canada, this many people die in road mishaps in 5 years. How many die per year?



If 160 DC-8 jets were to crash in 5 years, there would be the same number of deaths. How many passengers and crew are there in a DC-8?

Is it safer to drive or fly?

EXERCISE 5·9

1. Test each of the following to see if $a \times d = b \times c$ when you are given $\frac{a}{b} = \frac{c}{d}$.

   (a) $\frac{1}{2} = \frac{3}{6}$ (b) $\frac{1}{3} = \frac{4}{12}$ (c) $\frac{2}{3} = \frac{8}{12}$

   (d) $\frac{3}{4} = \frac{9}{12}$ (e) $\frac{12}{15} = \frac{4}{5}$ (f) $\frac{12}{16} = \frac{3}{4}$

2. If $a \times d \neq b \times c$, then $\frac{a}{b} \neq \frac{c}{d}$.

   Which of the following are correct statements?

   (a) $\frac{3}{4} = \frac{40}{60}$ (b) $\frac{5}{6} = \frac{25}{30}$ (c) $\frac{8}{11} = \frac{88}{132}$

   (d) $\frac{15}{20} = \frac{3}{4}$ (e) $\frac{111}{1332} = \frac{1}{12}$ (f) $\frac{45}{99} = \frac{2015}{4455}$

3. Find the values of x in the proportions:

   (a) $\frac{3}{7} = \frac{9}{x}$ (b) $\frac{x}{6} = \frac{5}{4}$ (c) $\frac{3}{x} = \frac{-21}{28}$

   (d) $\frac{2}{5} = \frac{x}{3}$ (e) $\frac{x}{5} = \frac{10}{-2}$ (f) $\frac{4}{x} = \frac{x}{16}$

   (g) $\frac{8-x}{x} = \frac{4}{7}$ (h) $\frac{12}{2x} = \frac{x}{6}$ (i) $\frac{3x}{16} = \frac{x-5}{5}$

4. Two numbers are in the ratio of 7:6. One number is 8 larger than the other. What are the two numbers?

5. One number is 36 less than another. The two numbers are in the ratio of 9:13. What are the two numbers?

6. The heights of two girls are in a ratio of 8:7. If the taller girl is 164 cm, how tall is the shorter girl?

7. A park ranger restocks a children's fishing pond with perch and sunfish. A ratio of two sunfish to five perch is kept in the restocking. If 165 perch are put into the pond, how many sunfish should also be put in?

8. $65.00 is divided into two parts which are in the ratio of 3:7. What are the two parts?

9. In a cross-country relay race of 3600 m, the distance run by the first runner, to that of the second, is 3:5. How far did each runner run?

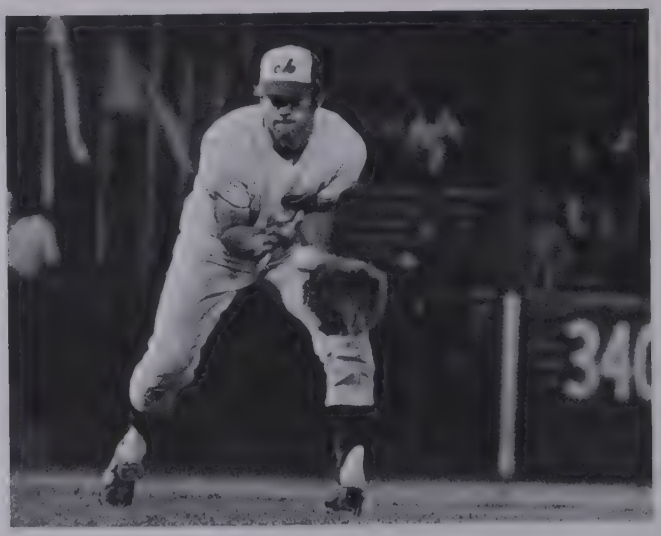
10. Mary saved $1.45 in the past three weeks. At this rate, how much will she save in 12 weeks?

11. Susan picked 14 baskets of berries in 20 minutes. At that rate, how long will it take her to pick 35 baskets?

12. A painter needs to mix 5 ℓ of paint for a job. The mixing guide says that three parts of yellow paint should be used with seven parts of brown paint. How many litres of each yellow and brown paint should be used?

13. Sam discovered that his car made 143 km on 25.4 ℓ of gasoline. At that rate, how much gasoline will the car use for the whole trip of 1573 km?

14. Sam and Bill bought new baseball gloves. Sam's glove cost him $12.75. If the ratio of the cost of Sam's glove to Bill's glove is 17:14, find the cost of Bill's glove.



15. A bricklayer knows that 486 bricks are used to build 6 m^2 of wall. How many bricks must be ordered to build a house which has 1080 m^2 of wall space?

16. By prior agreement, the profits from a sales venture were to be divided in a ratio of four to seven, four parts to Jim and seven parts to Helen. How much does each get from a total profit of $49.50?

17. Assuming the most pleasing shape of a picture frame is 16:11, what should the length of the picture frame be if it is 25.3 cm wide?

Oil and Gas Pipelines

1. Approximately five billion dollars has been spent from 1950 to 1974 to build oil and natural gas pipelines in Canada – lines that would go round the earth about $3\frac{1}{2}$ times. (The circumference of the earth is approximately 140 000 km.)
   (a) How many dollars per year have been spent on pipelines from 1950 to 1974?
   (b) How many kilometres of pipelines have been built in Canada from 1950 to 1974?
   (c) What is the average cost per kilometre for the pipeline?

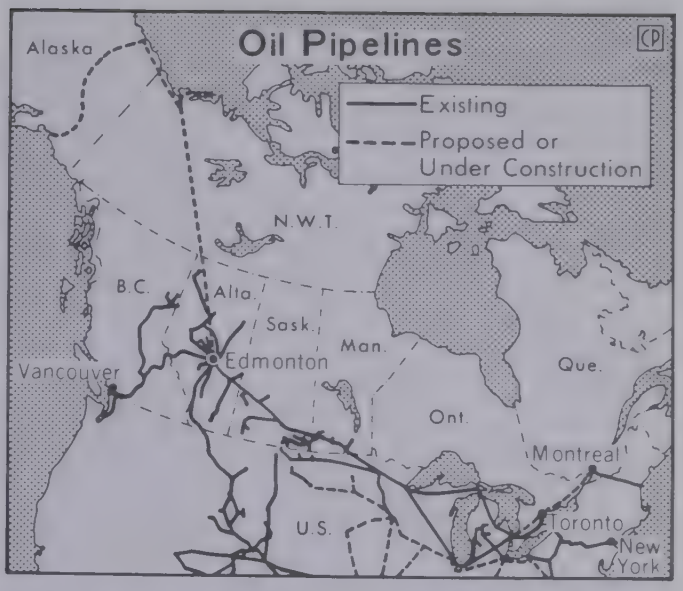
2. Natural gas lines account for a major portion of the total distance. Approximately 111 000 km of gas lines make up city grids, connections, and cross-Canada trunk lines. What fraction of the total are the natural gas lines? State your answer as a decimal fraction.

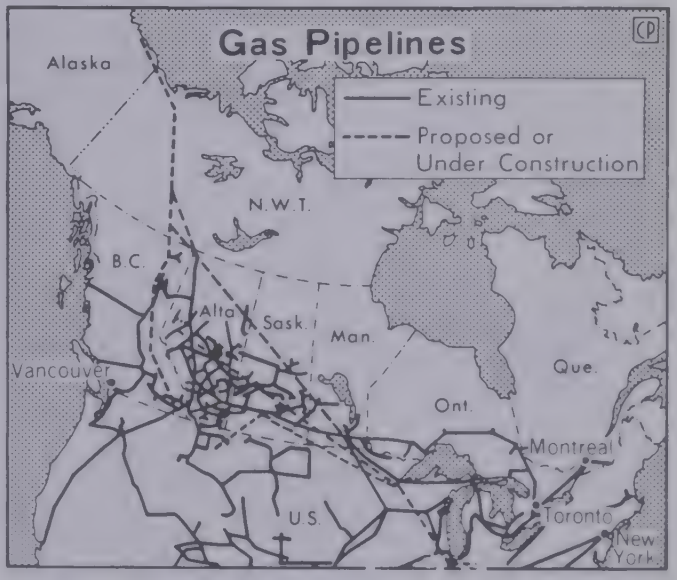
3. Oil moves slowly in the lines; the average speed is 7.2 km/h. It takes approximately 35 days for a litre of oil from Alberta to reach a refinery in Ontario. At that rate, how many kilometres is it from the oil well to the refinery?

4. The average speed for natural gas is 28.8 km/h. How long would it take natural gas to travel from Alberta to the Ontario refinery?

5. The Trans-Canada Pipeline (natural gas line) is about 6440 km long. The Interprovincial Pipeline (oil line) system is about 8400 km long – including portions that pass through the United States south of Lake Superior and Lake Huron. How long would it take a cubic metre of natural gas to travel the length of the Trans-Canada Pipeline? How long would it take a litre of oil to travel the length of the Interprovincial Pipeline?

6. Discuss the advantages and disadvantages of constructing extensive oil and gas supply lines from the Arctic oil and gas fields to Southern Canada and the United States.





5·10 The Language of Algebra

It is often convenient to speak in general terms about certain things. If we want to talk about the amount of money that would be collected if everyone contributed three dollars, we can say, "If x students contributed, we will collect $3x$ dollars." If we want to talk about how far we will ski if we ski 12 km/h, we can say, "If we ski y hours, we will ski $12y$ km."

Often we can give instructions more clearly in the language of algebra than we can in words.

$$\frac{(2x+10)+(4y-10)+\left(\frac{z}{2}+50\right)}{3}$$

To many people, this statement is easier to understand than "one third the sum of two times the first number plus ten, four times the second number less ten, and one half the third number plus fifty."

Exercise 5·10 is designed to give you practice in writing the language of algebra. Maybe you'll use this type of language in your everyday conversations. Try it!

EXERCISE 5·10

Write an expression in the language of algebra for each of the following. Then write a sentence answering the question using the language of algebra.

1. A club consists of x boys. Two new members join. How many boys are now in the club?
2. A man went shopping with $\$y$. He spent \$8. How much has he left?
3. How many centimetres are there in 3 m? in x m?
4. How many grams are there in 1.27 kg? in x kg?
5. How many cents are there in k quarters? in y dimes?
6. One boy is 6 yr older than another. If the age of the older boy is y yr, what is the age of the younger?
7. The sum of two numbers is 32 and one of the numbers is x. What is the other number?
8. A man had a mass of w kg and then lost 6 kg. What is his mass now?
9. A man who had a mass of w kg lost z kg a week for 5 weeks. What is his mass now?
10. If the perimeter of a square is k cm, what is the length of a side?
11. What is the next integer above 18? the next below 18? If n is an integer, state the next integer above n. State the next below n.
12. The smallest of three consecutive integers is k. What are the others?
13. The largest of three consecutive integers is m. What are the others?
14. If r is an even integer, what is the next even integer above r?
15. What is the area of a square whose side is x units?
16. The length of a rectangle is 3 times its width. If the width is y m, what is the length?
17. John is x yr old. How old will he be in 5 yr? How old was he 3 yr ago? How old will he be in y yr? How old was he m yr ago?
18. A boy earns $\$d$ per week and his salary is increased by 20%. What is his present salary?
19. In a hockey season, a player on defence scored $2x$ goals. A forward on the same team scored 5 more than twice as many. How many goals did the forward score?
20. On a trip of $25y$ km, how far has a man gone when he has completed $\frac{3}{5}$ of his journey? How far has he still to go?

21. One rectangle is x cm long and y cm wide. A second rectangle is p cm long and q cm wide. What is the difference in their areas:
    (a) when the first rectangle is the larger?
    (b) when the second rectangle is the larger?

22. Find the total mass of three trucks whose masses are $(3m + 200)$ kg, $(5m + 500)$ kg, and $(4m + 700)$ kg respectively.

23. A television antenna, $5h$ m in height, is erected on the roof of a building which is $7h + 52$ m high. What is the total height of the antenna above the street?

24. If you are now m yr old, what must be your father's present age if he is 5 yr more than three times your age?

25. How many days are there in $3x$ weeks? in $2m - 5$ weeks?

26. A rugby team has n games in its schedule. It has played $\frac{3}{4}$ of its season. How many games remain to be played?

27. Write an algebraic expression for the number of nickels in k quarters and in d dollars.

28. An article cost $\$b$ and is sold at a gain of 10%. What is the selling price?

Using the Library

1. *Reading In Mathematics*, Book 1, Adler I., Ginn and Company (1972)
    (a) Read pages 42 and 43. What are the first six cubic numbers? What is the sum of (i) the first three cubic numbers? (ii) the first four cubic numbers?
    (b) Read pages 103 and 104. Are the instructions clear and complete enough to help you knit? Perhaps you will need some help to get started.

2. *Reading In Mathematics*, Book 2, Adler I., Ginn and Company (1972)
    (a) Read page 53. At the rate given in the box, how many babies are born in an hour? At the rate given, how many people die per hour? How was the figure 190 000 calculated?
    (b) Read page 54. At the rate given in the top box, how many people die of starvation each day? hour? minute? second? What can your school do about the number who starve?
    (c) On page 54, read the box at the bottom of the page. What is the total mass of waste per year as listed in the U.S.A.? How many tonnes pollute the environment per day? per hour? per minute? per second? What can your school do about pollution in your community?

1. Use the library to prepare a report on the history of sea transportation.

1. *Math Menagerie*, Kadeach, R. R., Harper and Row (1970)
    (a) What is a googol? a googolplex? a Mega? a Megiston?
    (b) What is a hexagon and a hexahedra? Make one of each. What can you do with them?

REVIEW EXERCISE 5

1. Solve and verify:
   (a) $3x - 9 = 2x - 2$ (b) $11 - 2n = 3n + 21$
   (c) $2t - 15 = 21 - t$ (d) $42 + 6x > 15$

2. Sue has five times as much money as her sister Jean. After Jean had earned another $2.00, they both had the same amount. How much did each have originally?

3. Solve and verify:
   (a) $\frac{5}{6}x - 7 = 8$ (b) $7 - \frac{3}{4}y = 4$
   (c) $\frac{1}{2}z + 2 = 7 - \frac{1}{3}z$ (d) $\frac{2}{5}a + 9 = \frac{3}{2}a - 2$

4. Two neighbours, the Bloors and the Trapps, together bought 18 t of coal for heating their houses. The Bloors took $\frac{5}{7}$ of the coal the Trapps took. How many tonnes of coal did each family buy?

5. A house was sold for $27 295 at a profit of 3% of the selling price. What was the profit on the house?

6. On a biology project, Mildred found 50 more than 3 times the number of insects in a square-metre plot than Harry did. Harry found 5 times as many as George did. Together, Harry and George found 96 insects. How many insects did Mildred, Harry, and George find?

7. Find the value of each unknown.
   (a) $\frac{25}{32} = \frac{x}{40}$ (b) $\frac{n}{18} = \frac{17}{-3}$ (c) $\frac{14}{3z} = \frac{z}{42}$

8. Divide 72 into three parts so that the second part is three times the first, and the third is twice the second less 8.

9. The formula for the area of a trapezoid is
   $$A = \tfrac{1}{2}(a + b)h.$$
   (a) Calculate h when $A = 228\ m^2$, $a = 16$ m, and $b = 22$ m.
   (b) Calculate h when $A = 134.125\ cm^2$, $a = 17$ cm, and $b = 12$ cm.

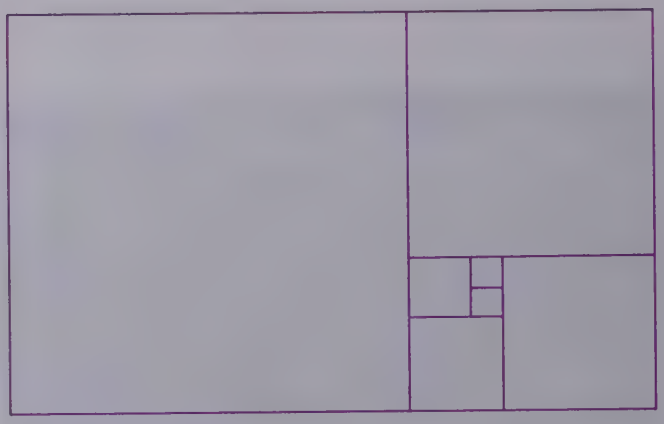
Do You Remember that...

. . . a sequence of numbers is a pattern determined by a certain law or rule? Study the following sequence known as the *Fibonacci sequence*.

1, 1, 2, 3, 5, 8, 13, 21, 34, . . .

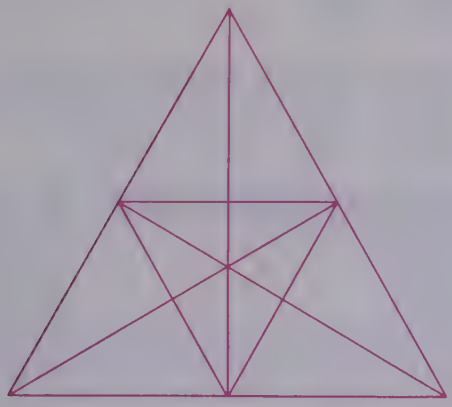
Have you discovered the law for determining the numbers of the sequence?

(a) What are the next three terms of the above Fibonacci sequence?



(b) On a clear sheet of paper, represent the first eight numbers of the Fibonacci sequence by a diagram like this one. Use half a centimetre for the side of the smallest square(s).

Something to Think About



As we observed above, patterns can be represented by diagrams. One can produce this diagram by (i) joining the midpoints of the sides of an equilateral triangle, and (ii) by joining the midpoints with the three vertex points. How many triangles can you count in the diagram? Would you be surprised to learn that there are more than 40? Compare your answers with your friends to see how many they get. (*Hint!* The answer is a prime number.)

A large passenger ship such as the Empress of Britain may use as much as 1 200 000 ℓ of oil every 3 weeks. If the ship operates 42 weeks in a year, how many litres of oil are required annually? Use scientific notation to express the answer.

It is very important that the people of the world do everything they can to conserve energy. In 1975, the Canadian Government initiated a plan to conserve energy. The following questions indicate ways the government urged the people of Canada to save energy.

1. Large cars use more gas per kilometre than compact cars.

   Large Car – 3.5 km/ℓ

   Compact Car – 8.5 km/ℓ

   The average car is driven 16 000 km in a year. There are 6.3×10^6 cars in Canada. If all cars were compacts rather than larger cars, how many litres of gas would be saved per year?

2. It is estimated that there is a savings of $\frac{1}{4}$ if cars are driven at 85 km/h rather than 110 km/h. If the average driver spends \$230/a on gas for long distance driving, how much can he save by slowing down?

3. Now the lights of the Energy, Mines and Resources Tower in Ottawa are turned off from 22 00 to 06 00 and on weekends. There is a savings of \$900 a week. If this is a 20% savings, what does it cost to operate the lights per week now?

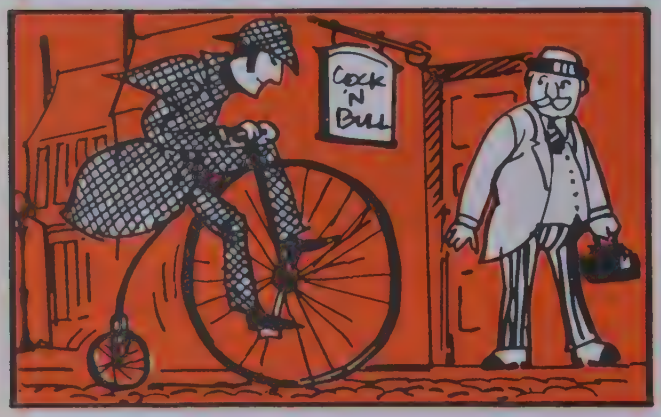
4. By lowering the maximum heating temperature during the day from 23°C to 21°C and from 20°C to 18°C at night, and by reducing the cooling during the summer; it is expected that an 18% savings will be achieved. If the average home achieved the same savings, how much would the average home save per year if its heating bill is $15 a month? There are about 5.4×10^6 households in Canada. How much would be saved if temperatures in all households were reduced?

5. By using a "Blue Flame" on an oil or gas furnace, about 10% can be saved on the fuel bill. If the average fuel bill per household is $14 per month, how much can be saved in a year in Canada? (Only 56% of all households are heated with oil or gas.)

6. By using proper insulation in a house, about 12% can be saved on a fuel and air-conditioning bill. If this bill is $18 per month and if 65% of the homes are improperly insulated, how much could be saved in Canada in a year?

7. A 12-storey building was put through the government's Energy Systems Analysis. The computer showed that proper energy-conservation measures could save $100 000 a year in that building. This amounts to $1.8¢/m^2$ for the building's floor space. At this rate, how many square metres of floor space are there in the building?

8. Name ways that you can conserve energy in your daily life. Where possible, estimate the dollar savings in a year.



Have You Discovered that...

. . . there is a definite relation between the distance a projector is from the screen and the size of the picture projected?

Project

Project 1

Purpose

To find the relations that exist between the distance a projector is from a wall and the size of the picture.

Method

1. Use a slide projector.
2. Set up the projector 3 m from a plain wall. Record the size of the picture.
3. Move the projector to 4 m from the wall. Record the size of the image.
4. Repeat step 3 for several other distances.
5. What relationships do you find between the distance the projector is from the wall and the size of the picture?
6. How many ways can you define *size*?
7. Repeat step 5 for each definition of size that you have in step 6.

Chapter Six
FACTORS TO BE CONSIDERED
Polynomial Expressions

REALLY WATSON, YOU EXCEL YOURSELF. I AM BOUND TO SAY THAT IN ALL THE ACCOUNTS OF MY OWN ACHIEVEMENTS YOU HAVE HABITUALLY UNDERRATED YOUR OWN ABILITIES...
SOME PEOPLE, WITHOUT POSSESSING GENIUS, HAVE A REMARKABLE POWER OF STIMULATING IT!
HE HAS NEVER SAID SO MUCH BEFORE.
I'M PROUD TO THINK I HAVE MASTERED HIS SYSTEM, SO AS TO APPLY IT IN A WAY WHICH GAINS HIS APPROVAL!

6·1 Thinking Mathematically

YOU SAY THIS COMPANY OFFERED TO PAY $3 FOR EACH MAGAZINE SUBSCRIPTION AND $4 FOR EACH NEWSPAPER SUBSCRIPTION YOU SOLD.
THE CASE IS AS YOU STATED IT, SIR.
IF YOU SOLD Y MAGAZINES AND X NEWSPAPER SUB-SCRIPTIONS YOUR COMMISSION WOULD BE 3X+4Y DOLLARS.
MR. HOLMES, YOU HAVE STATED THE COMMISSION SUCCINCTLY AND PRECISELY... AS THE SUM OF TWO MONOMIALS.

EXERCISE 6·1

Determine the commission earned, at the rate quoted by Sherlock Holmes, if the following number of subscriptions is sold.

	Magazines	Newspapers
1.	5	8
2.	15	19
3.	36	54
4.	75	50
5.	50	75
6.	250	325

What method did you use? Describe your method.

6·2 Monomials

The following are examples of *algebraic expressions*.

$$2a \qquad -5x \qquad y-7 \qquad z^2 \qquad x^2-3x+8$$

In the above examples,

$$a \qquad x \qquad y \qquad z$$

are called variables.

A variable represents some unknown number or numbers.

In the above examples of algebraic expressions, the following are called *monomials*.

$$2a \qquad -5x \qquad z^2$$

An algebraic expression containing one term is called a monomial.

Each term or monomial is composed of coefficients. There are two types of coefficients:

Numerical – refers to the number.
Literal – refers to the letter.

$3x$ — numerical coefficient; literal coefficient

The number of coefficients may vary.

$3x$	– monomial in one variable.
$\frac{1}{5}xy$	– monomial in two variables.
$37xyz$	– monomial in three variables.
7	– monomial with no variable.
x	– monomial with one variable.

Two important points about monomials are:

1. We can think of x as $1x$.
2. The exponents of the variables must be from the set of whole numbers: $\{0, 1, 2, 3, \ldots\}$.

We state the *degree* of each monomial as shown:

Monomial	Degree	
7	0	(zero)
$5x$	1	
$3x^4$	4	
$4m^2n^3$	5	
$175xy^3z^4$	8	

The *degree* of a monomial is the *sum* of the exponents of the variables.

The product of two or more monomials can be determined by using two properties:
– associative property;
– commutative property.

The product of $3x^2y$ and $5y^2$ is:

$(3x^2y)(5y^2)$ (A common way to write a product.)

$= (3 \cdot x \cdot x \cdot y)(5 \cdot y \cdot y)$ (Rewritten and expanded.)

$= (3 \cdot 5)(x \cdot x \cdot y \cdot y \cdot y)$ (Commutative and associative properties.)

$= 15x^2y^3$

It is important to remember this rearranging procedure in multiplying.

$$(12x^2y^3)(-8x^4y^2z) = (12)(x^2)(y^3)(-8)(x^4)(y^2)(z)$$
$$= (12)(-8)(x^2)(x^4)(y^3)(y^2)(z)$$
$$= -96x^6y^5z$$

Do you also remember that

10^3 means $10 \times 10 \times 10$?

Similarly, x^2 means $(x) \cdot (x)$

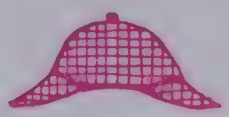
and a^5 means $(a) \cdot (a) \cdot (a) \cdot (a) \cdot (a)$.

Do you recall that

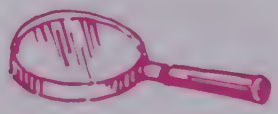
$$a^2 \times a^3 = (a \cdot a) \times (a \cdot a \cdot a)$$
$$= a \cdot a \cdot a \cdot a \cdot a$$
$$= a^5$$

and that $a^2 \times a^3 = a^{2+3}$?

We can summarize this procedure of multiplication with exponents by



$$a^x \cdot a^y = a^{x+y}$$



Example 1

Find the following products:

(a) $(ab)\cdot(ac)$ (b) $(-3m^2n)\cdot(2mn^3)$

(c) $(2x^2y)\cdot(-yz^2)\cdot(-3xz^3)$

Solution

(a) $(ab)\cdot(ac) = a^2bc$

(b) $(-3m^2n)\cdot(2mn^3) = -6m^3n^4$

(c) $(2x^2y)\cdot(-yz^2)\cdot(-3xz^3)$
$= (2)(-1)(-3)\cdot(x^2)(x)\cdot(y)(y)\cdot(z^2)(z^3)$
$= 6x^3y^2z^5$

Do Questions 1 to 9 in Exercise 6·2.

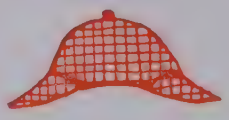
Let us review division involving exponents.

$$x^5 \div x^2 = \frac{x^5}{x^2}$$
$$= \frac{x\cdot x\cdot x\cdot x\cdot x}{x\cdot x}$$
$$= x^3$$

Also $x^5 \div x^2 = x^{5-2}$

and $y^8 \div y^5 = \frac{y^8}{y^5}$
$= y^{8-5}$
$= y^3$

This procedure of division with exponents can be summarized by



$$a^x \div a^y = \frac{a^x}{a^y} = a^{x-y}$$

Monomials can be divided, but the quotient may or may not be a monomial.

Example 2

Find the quotient of:

(a) $30x^3 \div 6x^2$ (b) $64m^4n \div 16m^3n^2$

Solution

(a) $30x^3 \div 6x^2 = \frac{30x^3}{6x^2}$
$= \frac{30x\cdot x\cdot x}{6x\cdot x}$
$= \frac{30x}{6} \times \frac{x\cdot x}{x\cdot x}$ $\left(\frac{x\cdot x}{x\cdot x} = 1\right)$
$= 5x$

This quotient is a monomial.

(b) $64m^4n \div 16m^3n^2 = \frac{64m^4n}{16m^3n^2}$
$= \frac{64\cdot m\cdot m\cdot m\cdot m\cdot n}{16\cdot m\cdot m\cdot m\cdot n\cdot n}$
$= \frac{64}{16} \times \frac{m\cdot m\cdot m\cdot m}{m\cdot m\cdot m} \times \frac{n}{n\cdot n}$
$= 4 \times m \times \frac{1}{n}$
$= \frac{4m}{n}$

The quotient $\frac{4m}{n}$ is not a monomial. Why?

It is not a monomial because $\frac{1}{n}$ is equivalent to n^{-1} and the variables of monomials must have exponents from the set of whole numbers.

Example 3

Find the quotients of:

(a) $-18a^3b^2 \div 6ab$ (b) $\frac{12x^5y^4 - 9x^3y^7}{3x^2y^2}$

Solution

(a) $-18a^3b^2 \div 6ab = \frac{-18a^3b^2}{6ab} = -3a^2b$

(b) $\frac{12x^5y^4 - 9x^3y^7}{3x^2y^2} = \frac{12x^5y^4}{3x^2y^2} - \frac{9x^3y^7}{3x^2y^2} = 4x^3y^2 - 3xy^5$

EXERCISE 6·2

1. "A monomial consists of the product of a variable and a number."

   Explain how $\frac{x}{5}$ can be considered a monomial under this rule.

2. List both the literal coefficient and the numerical coefficient in each of the following:

   (a) $73x$ (b) $56xy$ (c) $3xyz$

   (d) $(2 \times 6)x$ (e) $76m^3p$ (f) $8x^2yz^2$

3. Which of the following are not monomials? Tell why each you pick is not a monomial.

   (a) $7xy$ (b) $\frac{56x}{y}$ (c) $7x - 3y$

   (d) 91 (e) ab (f) $\frac{rst}{15}$

   (g) $x - 7$ (h) $x + y$ (i) $7 + 3m$

4. Use a dictionary to look up the meaning of *coefficient*. Which meaning best fits our use?

5. Give the degree of each monomial.

   (a) $37a^2$ (b) 51 (c) x^2y

   (d) $7x^2y^3$ (e) $2x^3yz^2$ (f) 32

   (g) a^2b (h) mn^3r (i) $8p^{36}s^{17}t^{51}$

6. Multiply the following:

   (a) $(x^2)(x^3)$ (b) $(r)(r^4)$ (c) $m^3 \cdot m^7$

   (d) $y^4 \cdot y^8$ (e) $a^7 \times a^{12}$ (f) $z^9 \times z^6$

7. In each of the following, expand the monomial to find the product.

   (a) $(5x)(15x^3)$ (b) $(4m^3)(7m^2)$

   (c) $(3x^2)(5y^1x^2)$ (d) $(17m^3)(3m^2n)(2n^3m)$

   (e) $(4r^3)^2(6r^2s)$ (f) $(5v^3)(2v^2w^3)^3$

8. Find the following products:

   (a) $(3x^2) \cdot (4x^3)$ (b) $(a^2b)(ab^2)$

   (c) $(5mn^4)(3m^7n)$ (d) $(-ac^2)(3a^4)$

   (e) $(x^2yz^3) \times (xy^2)$ (f) $(-8b^2)(-bc^3)$

   (g) $(3a^2b^3)(4bc^2)(-abc^3)$ (h) $(m^5n^8)(mnp^2)(m^3p)$

9. Calculate the product of each:

   (a) $(2m^3n)(3mn)$ (b) $(4m^2)(2m^3)$

   (c) $(56r^2t)(\frac{1}{8}t^2r)$ (d) $(\frac{1}{4}v^2w)(144w^3v)$

   (e) $(3x^2yz)(5xy^2)$ (f) $(81zw^2v)(\frac{1}{3}z^4w^5v^2q)$

10. Divide:

    (a) $b^4 \div b$ (b) $h^7 \div h^5$ (c) $n^6 \div n^2$

    (d) $p^{12} \div p^4$ (e) $r^{20} \div r^5$ (f) $x^{24} \div x^8$

11. Expand to calculate each quotient:

    (a) $\frac{4x^5}{2x^4}$ (b) $\frac{x^6y^4z}{x^3y^3}$

    (c) $\frac{32m^2n}{16mn}$ (d) $\frac{164m^3s^5}{82r^2s^5}$

12. Calculate each quotient. Identify which quotients are *not* monomials.

    (a) $m^3 \div m^2$ (b) $4m^2n \div 16mn$

    (c) $152r^2st \div 8rs^2t^2$ (d) $83t^2r^3 \div 17tr^2$

    (e) $100v^2w^3 \div 1000w^2z$ (f) $72x^2y^3z^5 \div 25x^2y^2z^4$

Musical notes are sound waves produced by vibrations. These sound waves, and hence music itself, can be expressed very precisely in mathematics. This student at the Ontario Science Centre, Toronto, is producing musical notes by striking the plates with hammers.

13. Find the quotients:

(a) $4at^3 \div 2t$ (b) $-6m^5n^3 \div 2m^2n$

(c) $16x^7y^5 \div 4x^5$ (d) $28y^8z^8 \div (-4y^2z^5)$

(e) $\dfrac{32a^2b^2 - 12a^3b^3}{4ab}$ (f) $24x^5y^4z^3 \div (-8xy^2z)$

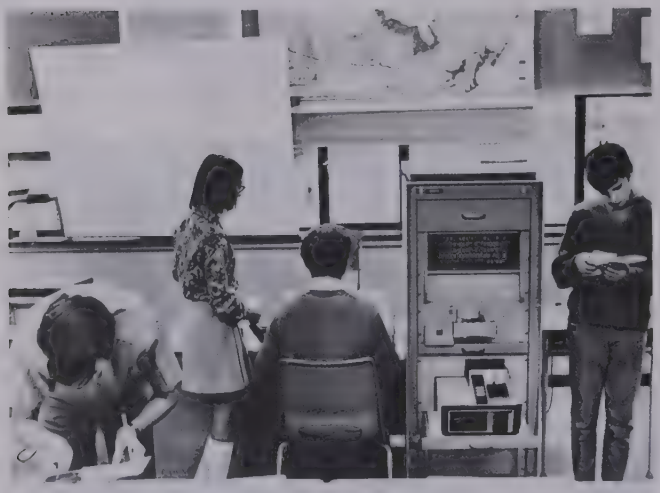
(g) $\dfrac{16m^5n^2 - 20m^3n^6}{4m^3n^2}$ (h) $\dfrac{36x^5y^6 - 42x^6y^4}{6x^4y^4}$

(i) $\dfrac{39r^4t^7 + 52r^3t^4}{13r^3t^4}$ (j) $\dfrac{64y^2x^8 + 80x^4y^4}{16x^4y^2}$

14. Evaluate each when $a = 2$, $b = -3$, $c = 4$, and $d = -5$.

(a) a^2b (b) a^2b^2c (c) $\dfrac{a^2b^3c^2}{ab^2c^2}$

(d) $\dfrac{30a^4b^2cd}{15a^4b^2c}$ (e) $\dfrac{32a^8b^5}{8a^6b^4}$ (f) $\dfrac{45a^3b^3c^2d^2}{15d^2c^2b^4a^5}$



These students are writing their own programs for a computer. The student who is sitting is typing his program into a teletype terminal which is connected to the computer. The girl beside him is checking his work. The other two students have put their programs on punch cards. The flowcharts used in this book illustrate how you must program a computer.

6·3 Sums of Monomials

We can also add monomials. Again, the sums may or may not be monomials. If we add $3m$ and $5m$, we get:

$$3m + 5m = 8m.$$

This appears simple, but because we will be doing harder problems later, let us make certain we understand what we did mathematically.

$$3m + 5m = (3 + 5)m \quad \text{(Distributive property.)}$$
$$= 8m$$

Before proceeding, do Questions 1 through 3 in Exercise 6·3.

Note! In order to add by using the distributive property, two requirements must be met:

1. The same variable or variables must occur in each term.
2. The degrees of the variables must be the same in each term.

If we wish to add $3m$ and $4x$, we have difficulty since the distributive property will not work:

$$3m + 4x = ?$$

Here, the literal coefficients of the two terms are *not identical!*

Monomials with identical literal coefficients are called **like terms**.

Monomials with literal coefficients which are not identical are called **unlike terms**.

Sometimes in algebra, it is difficult to tell when the literal coefficients are identical. As already stated above, each variable must be of the same degree.

1. x^2 and x are not identical.
2. $17m^3$ and $17m^2$ do not have identical literal coefficients since the exponents of the variables are not the same.

When adding (or subtracting) monomials, you may find it more convenient to use the commutative property before using the distributive property.

$$\begin{aligned} 3a - 4b + 2a &= 3a + 2a - 4b \text{ (Commutative property.)} \\ &= (3 + 2)a - 4b \text{ (Distributive property.)} \\ &= 5a - 4b \end{aligned}$$

You will have noticed that the sum of two or more monomials is not always a monomial. The example immediately above illustrates this. There we had the sum of three monomials yielding the sum of two monomials. This means that there were three terms, $3a$, $-4b$, and $2a$, in the problem and two terms in the sum, $5a$ and $-4b$.

We shall refer to all sums or differences as sums since subtraction can be represented as addition:

$$\begin{aligned} 3a - 4a &= 3a + (-4a) \\ &= [3 + (-4)]a \\ &= -1a \end{aligned}$$

For example:

$3a + 4a = 7a$ ← (Monomial.)

$(3a) + (-4b) + (2a) = 5a - 4b$ ← (Binomial.)

$$\begin{aligned} (6m) + (8m^2) + (-12m) + (3m^3) &= (6m) + (-12m) + (8m^2) + (3m^3) \\ &= [6 + (-12)]m + 8m^2 + 3m^3 \\ &= -6m + 8m^2 + 3m^3 \end{aligned}$$

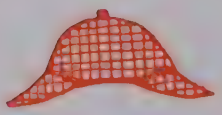
(Trinomial.)

$4s + 7t - 7x + 4y = 4s + 7t - 7x + 4y$

(More than three terms.)

For ease of communication, mathematicians often use the word *polynomials* to refer to monomials, binomials, trinomials, *and* polynomials.

DR. WATSON, WITNESS THIS EXPLANATION.
THE PREFIX 'MONO-' MEANS ONE AND 'BI-' MEANS TWO.
ALSO 'TRI-' MEANS THREE AND 'POLY-' MEANS SEVERAL OR AN INDEFINITE NUMBER.
HENCE, MONOMIAL MEANS ONE TERM, BINOMIAL MEANS TWO TERMS, ...
... TRINOMIAL MEANS THREE TERMS AND POLYNOMIAL MEANS SEVERAL TERMS OR AN INDEFINITE NUMBER OF TERMS.
MY DEAR WATSON, CONGRATULATE ME! I HAVE MADE THE WHOLE THING AS CLEAR AS DAY!



A polynomial is an algebraic expression composed of a monomial or the sum of monomials.

We can add polynomials in the same way we add monomials.

Add: $3m^2+4m+5x$ and $6m^2-7m-8y$.

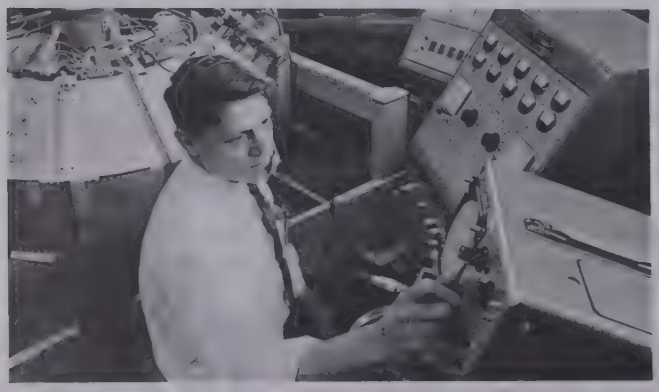
Method 1

$$
\begin{aligned}
&(3m^2+4m+5x)+(6m^2-7m-8y)\\
&=3m^2+4m+5x+6m^2-7m-8y\\
&=(3m^2+6m^2)+(4m-7m)+5x-8y \qquad \text{(Commutative prop.)}\\
&=(3+6)m^2+(4-7)m+5x-8y \qquad \text{(Distributive prop.)}\\
&=9m^2+(-3)m+5x-8y\\
&=9m^2-3m+5x-8y
\end{aligned}
$$

Method 2

$$
\begin{array}{l}
3m^2+4m+5x\\
\underline{6m^2-7m\qquad -8y}\\
9m^2-3m+5x-8y
\end{array}
$$

Place one polynomial under the other so that only like terms are in vertical columns. Then add.



A technician checks out the Alouette B satellite. With the advent of computers, the value of mathematics has skyrocketed beyond all imagination. Satellites make it possible to forecast weather more accurately and also are used to improve television and radio communications.

EXERCISE 6·3

1. If $3xy$ and $5xy$ are like terms, are $7x^2y$ and $8xy^2$ also like terms? Discuss.

2. Use the distributive property to expand each of the following. The first two are done for you.

   (a) $3(a+b)=3a+3b$ (b) $3(x+y)=3x+3y$

   (c) $4(x+y)$ (d) $15(x-y)$

   (e) $(2m-3n)7$ (f) $3(2r+4s)$

   (g) $63(\frac{1}{7}x-\frac{1}{9}y)$ (h) $(3a-4b)4$

   (i) $\frac{2}{3}(6r-9t)$ (j) $(\frac{1}{2}x-\frac{1}{3}z)\frac{3}{4}$

   (k) $1.7(1.5x-2.3y)$ (l) $(p^2-s^2)5$

   (m) $(w-v^2)7$ (n) $(125x^2-75)\frac{1}{5}$

   (o) $\frac{5}{6}(\frac{2}{3}a-\frac{3}{4}b)$ (p) $(2.5x-1.3y)2$

3. Use the distributive property to calculate the sum or difference. Show your work.

   (a) $3m-2m$ (b) $16r^2-5r^2$

   (c) $32rs-16rs$ (d) $4x^2+3x^2$

   (e) $15z^3-8z^3$ (f) $\frac{4}{3}x-\frac{2}{3}x$

   (g) $\frac{3}{4}t-\frac{9}{16}t$ (h) $\frac{13}{15}x-\frac{2}{3}x$

   (i) $\frac{4}{5}y^2-\frac{9}{10}y^2$ (j) $84t-79t$

   (k) $16t^2-4t^2+5t^2$ (l) $7tp-23tp-46tp$

   (m) $22x^2-18x^2-17x^2$ (n) $(42m-72m+33m)$

   (o) $3\pi-7\pi-8\pi-10\pi$ (p) $12y+16y-20y$

4. Add these sets of like terms.

   (a) $4m$, $2m$, $3m$

   (b) $6t^2, -t^2, -3t^2$

   (c) $2xy$, $7xy$, $-5xy$

   (d) $-3a^2b$, $-8a^2b$, $-4a^2b$

   (e) $\frac{3}{4}mnq$, $\frac{2}{5}mnq$

   (f) $-xy^2z$, $5xy^2z$, $-2xy^2z$

   (g) $\frac{2}{3}a^2b^3c$, $\frac{1}{4}a^2b^3c$, $-\frac{1}{6}a^2b^3c$

   (h) $\frac{5}{8}t^4$, $-\frac{5}{6}t^4$, $-\frac{2}{3}t^4$

   (i) $2.5a$, $3.2a$, $4.8a$

   (j) $1.71b^3$, $4.06b^3$, $2.88b^3$

   (k) $6.5r^2t$, $3.26r^2t$, $1.07r^2t$

   (l) $0.03x^5y^3$, $0.71x^5y^3$, $0.084x^5y^3$

5. How many terms are in each of the following expressions? Give a name to each expression.

(a) $6 - 5t$ (b) $2ab - 3bc + 5ac - 6$

(c) $-37mpr^2$ (d) $\frac{2}{5}hk$

(e) $-3x^3 + 2x^2 - x$ (f) $\frac{2}{3} + \frac{5x}{121}$

(g) $\frac{5}{6}m + \frac{4}{5}n - \frac{3}{4}mn - \frac{2}{3}$ (h) $-a$

(i) 186 (j) $a^2 - 2ab + b^2$

(k) $m^3 - 3m^2 + 3m - 1$ (l) $x^2 + 2xy + y^2$

6. Classify each polynomial according to the number of its terms.

(a) $2r$ (b) $-8mn$

(c) $h - 1$ (d) $2x + 5y$

(e) $a + b - c$ (f) $3x^2 - 5$

(g) $8x^2 - y^2$ (h) $4m^2np$

(i) $2a^4 + a^3 - 7a^2$ (j) $hg^2 - 2h^2$

(k) $x^2y - 2xy + 3xy^2$ (l) $3a^2b^2 + 5ab - 9$

(m) $-59amxt$ (n) $2ab - 3bc + 5d^2 - 9abc$

7. Find the sum for each of the following:

(a) $(4x + 2) + (3x - 5) + (6 - 2x)$

(b) $(3m - 2n) + (5m + 4n) + (6n - 6m)$

(c) $(xy - 4) + (3xy - 1) + (2 - 4xy)$

(d) $(7ab + 3c) + (5c - 9ab) + (ab - c)$

(e) $(3x^2 - 4y^2) + (4x^2 - 3y^2) + (5y^2 - 5x^2)$

8. Find the sum for each of the following:

(a)
$$\begin{array}{r} 3a - 2b + 7c \\ -5b - 8c \\ a \quad\quad -2c \\ \hline \end{array}$$

(b)
$$\begin{array}{r} 5x + 3y - 2 \\ 3x \quad\quad +7 \\ -4y - 5 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 9a^2b^2 - 5a + 3b \\ -4a^2b^2 - 8a \\ 7a - 6b \\ \hline \end{array}$$

(d)
$$\begin{array}{r} 7st^2 \quad\quad +4wx^2 \\ +u^3v - 2wx^2 \\ -2u^3v + 4wx^2 \\ \hline \end{array}$$

9. Add the following polynomials by Method 1.

(a) $(7x + 3y - 2z) + (2x - 7y - 8z) + (-4x - 2y + 5z)$

(b) $(3ab + 4bc - 5ac) + (ab - 3bc + 2ac) + (4ab - bc + 6ac)$

(c) $(2x^2y - 7xy + 3xy^2) + (5x^2y + 6xy - 8xy^2) + (-4x^2y + 3xy - xy^2)$

(d) $(12a^2 - a - 2) + (3a - 7 - 4a^2) + (5 - 3a^2 + 4a)$

10. Add the following polynomials by Method 2.

(a) $(3a - 2b + c) + (2a - 5b - 4c) + (a + 3b - 4c)$

(b) $(5xy - 2yz + 4xz) + (3yz - 3xy) + (7xz - 5yz + xy)$

(c) $(8m^2 - 5n^2) + (2m^2 - 7mn + n^2) + (3mn + 6n^2)$

(d) $(ab - bc - ac) + (bc - ac - ab) + (ac - ab - bc)$

(e) $(2a - 3b + 4c - d) + (5b + 2c - 6d) + (3c + 5d - 2a)$

11. The known oil reserves are unevenly distributed throughout the world. Draw a graph to show the information in the table.

Known Oil Reserves (billions of cubic metres)

North America	8.55
Western Europe	1.85
U.S.S.R.	7.56
Southeast Asia	2.32
Africa	6.57
Middle East Countries except Saudi Arabia	39.35
South America	4.44
Saudi Arabia	24.66
China	2.25

What will we do for a source of energy for transportation when all the oil resources are depleted?

6·4 Subtraction of Polynomials

In subtraction of monomials, only like terms can be subtracted. The distributive property is used in subtraction of monomials.

Example 1

Subtract:
(a) $12m - 5m$ (b) $8x^2 - 3x^2$
(c) $(-2ab^3) - (-8ab^3)$

Solution
(a) $12m - 5m = (12 - 5)m = 7m$

(b) $8x^2 - 3x^2 = (8 - 3)x^2 = 5x^2$

(c) $(-2ab^3) - (-8ab^3) = [-2 - (-8)]\,ab^3$
$= 6ab^3$

Do Question 1 in Exercise 6·4.

Does your hair ever stand on end? Electrical charges can make this happen when the charges are produced by a Van de Graaff generator. Do not try this on your own because ordinary electrical current is dangerous. Electrical charges are one of many things that can be described by mathematical symbols and formulas.

Let us review subtraction for a few moments:

$3 - (4a + 2b) = 3 - 1(4a + 2b)$
(Place the 1 in front of the parentheses.)
$= 3 - 1(4a) - 1(2b)$
(Distributive property.)
$= 3 - 4a - 2b$

We will often use this property of subtraction. A little practice may be needed. If so, do Question 2 in Exercise 6·4.

Example 2

Simplify: $(7a - 5b + 3c) - (3a + 6b - 9c)$

Solution

First remove the parentheses; then collect the like terms using the commutative and associative properties.

$(7a - 5b + 3c) - (3a + 6b - 9c)$
$= (7a - 5b + 3c) - 1(3a + 6b - 9c)$
$= 7a - 5b + 3c - 3a - 6b + 9c$
(Distributive property.)
$= (7a - 3a) + (-5b - 6b) + (3c + 9c)$
(Associative and commutative properties.)
$= 4a + (-11b) + 12c$
$= 4a - 11b + 12c$

Example 3

Find the remainder:
$(4x^2 + 2x - 7) - (9x^2 + 5x - 3)$

Solution

Method 1

$(4x^2 + 2x - 7) - (9x^2 + 5x - 3)$
$= (4x^2 + 2x - 7) - 1(9x^2 + 5x - 3)$
$= 4x^2 + 2x - 7 - 9x^2 - 5x + 3$ (Why?)
$= 4x^2 - 9x^2 + 2x - 5x - 7 + 3$ (Why?)
$= -5x^2 - 3x - 4$

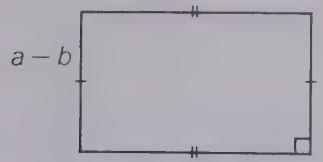
Method 2

Subtract: $4x^2 + 2x - 7$
$9x^2 + 5x - 3$
$-5x^2 - 3x - 4$

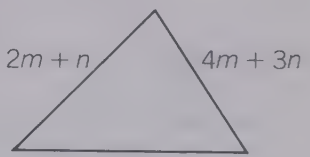
EXERCISE 6·4

1. Subtract as indicated in the following:
   (a) $13a - 7a$ (b) $25y - 9y$
   (c) $-7c - (-5c)$ (d) $14m - (-11m)$
   (e) $-6z - 4z$ (f) $-12h - (-5h)$
   (g) $\begin{array}{r} 18x \\ \underline{13x} \end{array}$ (h) $\begin{array}{r} -8ab \\ \underline{12ab} \end{array}$ (i) $\begin{array}{r} -b^2 \\ \underline{-7b^2} \end{array}$ (j) $\begin{array}{r} 12xyz \\ \underline{15xyz} \end{array}$
2. Remove the brackets in each expression and simplify.
   (a) $4 - (3a - 4b)$
   (b) $6a - (7a + 4b - 2c)$
   (c) $7a - (-2b + 4c - 3d)$
   (d) $3x - (4a - 5c)$
   (e) $7x + 4y - (-7a - 7b + 8c)$
   (f) $-(-7a + 6b - 9m)$
   (g) $-(9m - 18m)$
   (h) $-(9v - 13w^2)$
   (i) $(18a - 3b) - (6x^2 + 8y^2)$
3. Simplify by removing parentheses:
   (a) $(16x + 12) - (6x + 5)$
   (b) $(9m + 13m + 21p) - (4m + 8n + 16p)$
   (c) $(7a + 5b) - (3a - 2b)$
   (d) $(13y - 9z) - (7y - 4z)$
   (e) $(-5x^2 + 2xy) - (3x^2 - 7xy)$
   (f) $(-ab - bc - ac) - (3ab - 4bc + 5ac)$
4. Find the remainder:
   (a) $\begin{array}{r} 3x - 2y + z \\ \underline{x + 2y - 3z} \end{array}$ (b) $\begin{array}{r} 5ab + 8bc - 6 \\ \underline{-2ab - 5bc - 9} \end{array}$
   (c) $\begin{array}{r} x^2y + 7xy - 13xy^2 \\ \underline{-x^2y - 2xy - 6xy^2} \end{array}$ (d) $\begin{array}{r} -5pq + 3qr - rt \\ \underline{12pq + 8qr + 5rt} \end{array}$
   (e) $(4x - 5y + 7z) - (6x + 4y + 2z)$
   (f) $(1.5m - 2.3n + 4.5p) - (3.7n - 2.3p + 2.6p)$
   (g) $(6x^2 - 7y) - (7y + 6x^2 + 8z)$
   (h) $(\frac{1}{3}m^2 - \frac{2}{3}n - \frac{1}{4}p) - (\frac{1}{2}n + \frac{1}{4}n + \frac{1}{3}p)$
   (i) $(6.5x - 7.2y - 4) - (3.5 + 6.8y - 2.3x)$
   (j) $(\frac{2}{3}t - \frac{1}{4}s + \frac{1}{3}y) - (2s - t - \frac{1}{4}y)$
   (k) $(\frac{2}{3}a - \frac{1}{3}b - \frac{1}{4}c) - (\frac{3}{4}a + \frac{3}{4}b - \frac{1}{3}c)$
5. Calculate the perimeter of each region.

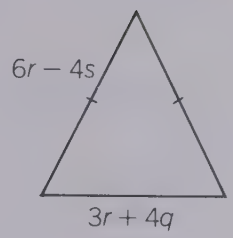
(a)



(b)



(c)



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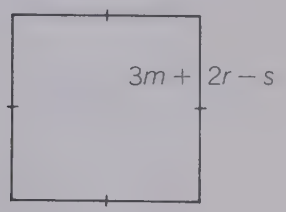

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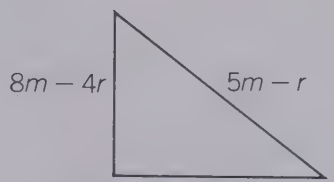
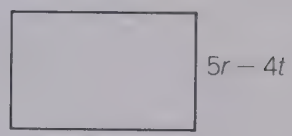
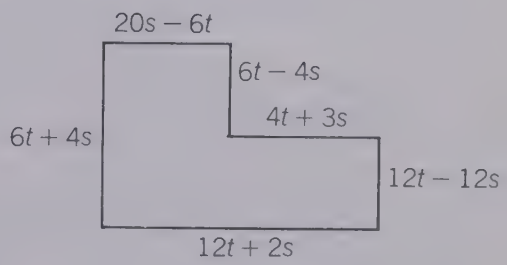


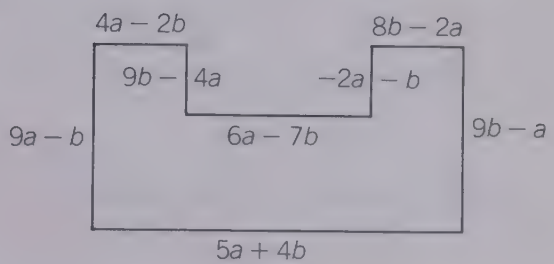
(f)



(g)

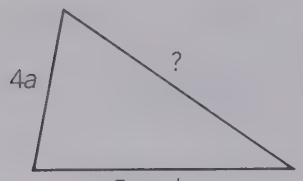


(h)



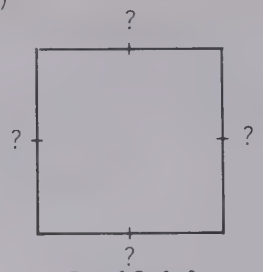
6. Calculate the length of the unknown side(s) in each figure.

(a)



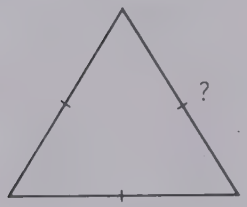
$P = 15a + 4b$

(b)



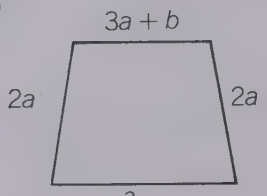
$P = 16p^2q^2$

(c)



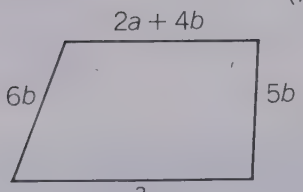
$P = 21r^2$

(d)



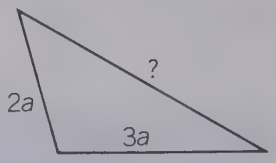
$P = 11a + 3b$

(e)



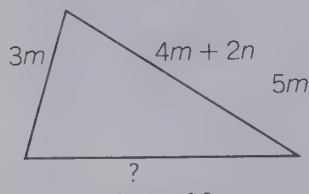
$P = 7a + 22b$

(f)



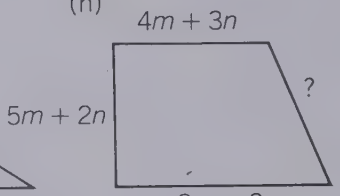
$P = 9a$

(g)



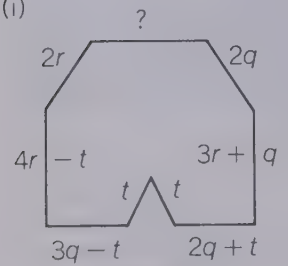
$P = 9m + 10n$

(h)



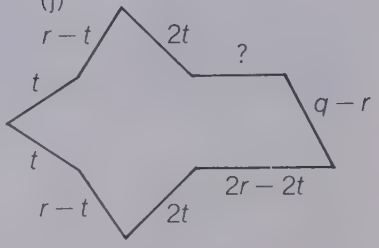
$P = 25m - 4n$

(i)



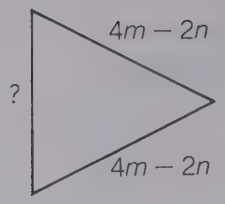
$P = 11q + 11r$

(j)



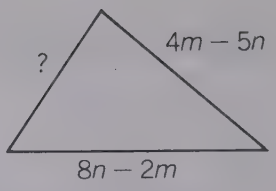
$P = 4t + 2r + q$

(k)



$P = 10m - 3n$

(l)



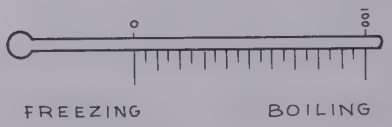
$P = 3m + 2n$

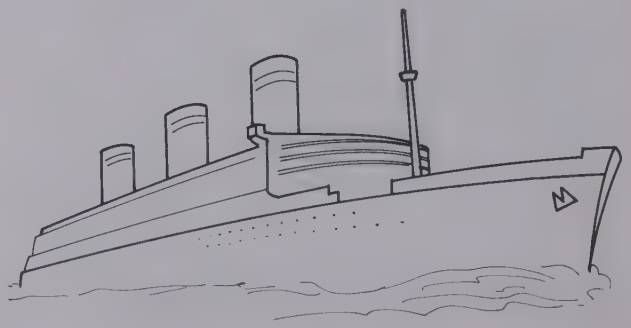
Metric Units

Match the object with the appropriate unit of measure.

1. Distance from Winnipeg to Toronto	(a) mm
2. Area of a wheat field	(b) kg
3. Your mass	(c) cm
4. Gasoline in a car	(d) ℓ
5. Mass of a penny	(e) m
6. Thickness of a dime	(f) km
7. Mass of a ship	(g) g
8. Handspan	(h) °C
9. Length of a pace	(i) t
10. Temperature	(j) ha

Refer to the Appendix if you are not familiar with or are having difficulty with metric units.

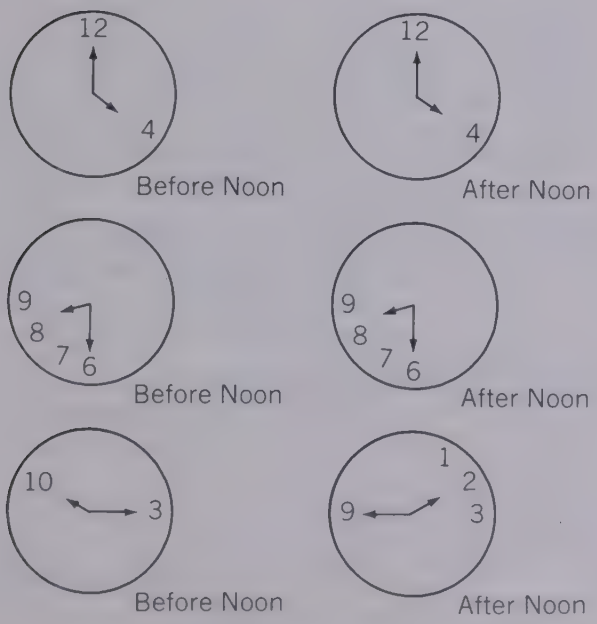




Match the object with the appropriate unit of measure.

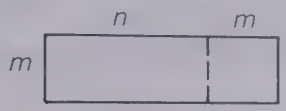
1. Area of the head of a pin	(a) cm^2
2. Volume of a crayon box	(b) ℓ
3. Capacity of a pop bottle	(c) mm^2
4. Area of a skating rink	(d) cm^3
5. Volume of a room	(e) m^3
6. Area of a page of this book	(f) ml
7. Capacity of a bathtub	(g) m^2

Record the time shown on each clock.

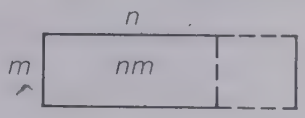


6·5 Product of a Polynomial and a Monomial

The area of a rectangle, such as the one below, can be calculated in several ways.



The area of the first rectangular portion



is $m \times n$, or mn, square units.

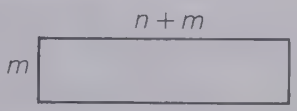
The area of the square portion

is $m \times m$, or m^2, square units.

The sum of the two areas is the total number of square units.

$$\text{Area} = mn + m^2$$

Another method of calculation consolidates the various steps above.



The width of the rectangle is m units.

The length of the total rectangle is $(n + m)$ units.

$$\text{Area} = \text{width} \times \text{length}$$
$$\text{Area} = m(n + m)$$
$$= mn + m^2 \qquad \text{(Distributive property.)}$$

Do Questions 1 and 2 in Exercise 6·5.

We can use the above idea to multiply large numbers mentally.

Example

Calculate the products:

(a) 8×57 (b) 9×452

Solution

(a) $57 = 50 + 7$

$\therefore \quad 8 \times 57 = 8(50 + 7)$

$= 8 \times 50 + 8 \times 7$

$= 400 + 56$

$= 456$

(b) $9 \times 452 = 9(400 + 50 + 2)$

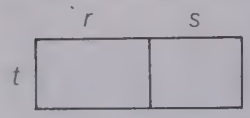
$= 9 \times 400 + 9 \times 50 + 9 \times 2$

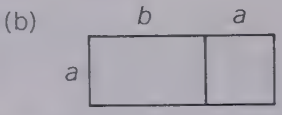
$= 3600 + 450 + 18$

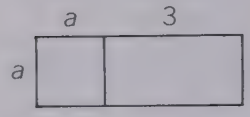
$= 4068$

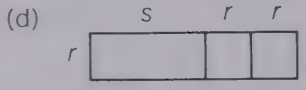
EXERCISE 6·5

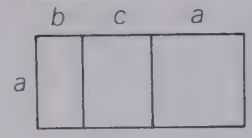
1. Calculate the area of each rectangle. Show each step in your solution.

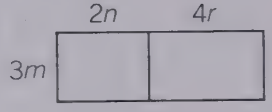
(a) 

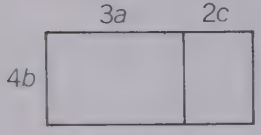
(b) 

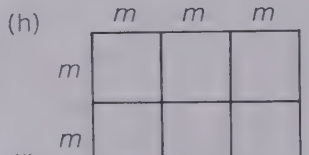
(c) 

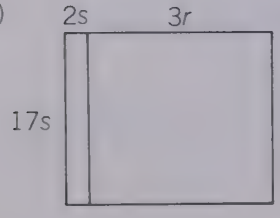
(d) 

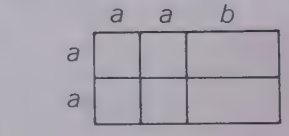
(e) 

(f) 

(g) 

(h) 

(i) 

(j) 

2. Refer to each rectangle in Question 1. For each, state the length and width. Then use $A = \ell \times w$ to find the area of each. Compare your answers with the corresponding answers in Question 1.

3. Draw diagrams to illustrate each of these problems. Then give the answer to each problem.

(a) $a(b + c)$ (b) $d(b + d)$

(c) $x(y + 3)$ (d) $2a(2b + c)$

(e) $3a(2m + 2n)$ (f) $3x(2x + 3y)$

4. Use the distributive property to calculate the products.

(a) 3×21 (b) 4×62 (c) 8×56

(d) 7×38 (e) 9×46 (f) 8×235

(g) 33×67 (h) 48×56 (i) 15×496

(j) 2×463 (k) 5×378 (l) 9×6214

6·6 More Products

In multiplying a monomial by any polynomial, the distributive property is applied. Thus,

$$5(m - 2n) = 5m - 10n.$$

Let us study some examples.

Example 1

Find the products of:

(a) $3a(2x + y - 4z)$ (b) $4a(2b - 3c)$

(c) $2x(x^2 - 2x + 1)$

Solution

(a) $3a(2x + y - 4z) = 3a(2x) + 3a(y) + 3a(-4z)$

(Distributive property.)

$= 6ax + 3ay - 12az$

(b) $4a(2b - 3c) = 4a(2b) + 4a(-3c)$

(Distributive property.)

$= 8ab - 12ac$

(c) $2x(x^2 - 2x + 1) = 2x(x^2) + 2x(-2x) + 2x(1)$

(Distributive property.)

$= 2x^3 - 4x^2 + 2x$

Example 2

Simplify:

$3x(a-2b+3c)-2x(3a+b-2c)-x(4a+b+c)$

Solution

$3x(a-2b+3c)-2x(3a+b-2c)-x(4a+b+c)$

$=3xa-6xb+9xc-2x(3a)-2x(b)-2x(-2c)-x(4a)-x(b)-x(c)$
$=3xa-6xb+9xc-6xa-2xb+4xc-4xa-xb-xc$
$=(3xa-6xa-4xa)+(-6xb-2xb-xb)+(9xc+4xc-xc)$
$=-7xa-9xb+12xc$

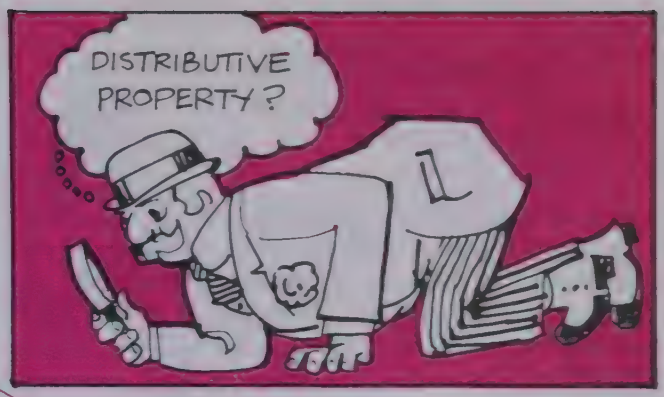
EXERCISE 6·6

Find the products of:

1. $7(2a-b+c)$
2. $3(5x+12y)$
3. $2m(-3x+4y-z)$
4. $6a(3a-4b-2c)$
5. $4b(b^2-2ab-5)$
6. $-2p(-3-5p+7p^2)$
7. $-7x(x^2-3x+2)$
8. $mn(m^2+2mn+3n^2)$
9. $-3a^2(-2a^2+a-1)$
10. $k^3(k^5+5k^3-7k)$
11. $2xy(x^3+y^2)$
12. $-3abc(a^2b+b^2c)$
13. $2a(a^2+b^2+c^2+2ab+2ac+2bc)$

Simplify:

14. $2(4m-n+3p)-3(m-2n+p)$
15. $y(y+1)+2y(y+2)+3y(y+3)$
16. $x(x^2-2x+3)+2x(5+x+3x^2)$
17. $ab(a+b)-2ab(b-a)$
18. $m^2n(5m-3n^2)+3m^2n(2m+n^2)$
19. $2xyz(2x-y+3z)+2xyz(3y-z-2x)$
20. $a(a^2-a+1)+3(a^2+a-2)-2(a^2+2a-3)$



6·7 Product of Two Binomials

Now we shall show the product of a binomial multiplied by a binomial.

In Figure 6-1(a), we note the product of $(a+x)$ multiplied by $(a+x)$.

Figure 6-1 (a)

	a	$+x$
a	a^2	ax
$+x$	ax	x^2

In Figure 6-1(b), we observe the product of $(x+2y)$ multiplied by $(x+y)$.

Figure 6-1 (b)

	x	y	y
x	x^2	xy	xy
y	xy	y^2	y^2

$(x+2y)(x+y)=x^2+xy+xy+xy+y^2+y^2$
(From Figure 6-1(b).)
$=x^2+3xy+2y^2$

Do Questions 1 and 2 in Exercise 6·7.

The diagrams illustrate the product of two binomials, but it is not always convenient or practical to use illustrations for finding such products. Therefore, we need to discover a rapid and convenient method of calculation. Let us use the same binomials as in Figure 6-1(a).

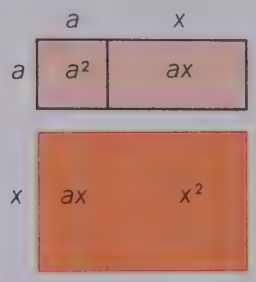
For $(a + x)(a + x)$, first multiply the a from the first binomial by the second binomial $a + x$, and then multiply the x from the first binomial by the second $a + x$.

Thus,

$(a + x)(a + x) = a(a + x) + x(a + x)$ (We are using the distributive property twice.)

$= a^2 + ax + ax + x^2$

$= a^2 + 2ax + x^2$



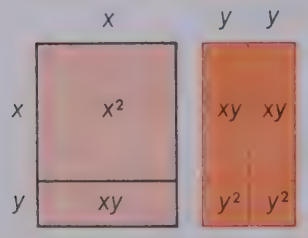
Perform the same operations for $(x + 2y)(x + y)$.

Thus,

$(x + 2y)(x + y) = x(x + y) + 2y(x + y)$

$= x^2 + xy + 2xy + 2y^2$

$= x^2 + 3xy + 2y^2$



Note! Whenever a binomial is multiplied by itself (as in Figure 6-1(a)), this is called **squaring a binomial**. Therefore, instead of writing $(a + x)(a + x)$, we may write $(a + x)^2$.

Example 1

Multiply:

(a) $(x + 3)(x + 7)$ (b) $(x + m)^2$

Solution

(a) $(x + 3)(x + 7) = x(x + 7) + 3(x + 7)$

$= x^2 + 7x + 3x + 21$

$= x^2 + 10x + 21$

(b) $(x + m)^2 = (x + m)(x + m)$

$= x(x + m) + m(x + m)$

$= x^2 + mx + mx + m^2$

$= x^2 + 2mx + m^2$

In the following example, notice how binomials involving subtraction are handled.

Example 2

Expand:

(a) $(2m - 3)(m + 5)$ (b) $(a + 3b)(3a - b)$

(c) $(2x - 5y)(3x - 2y)$

Solution

(a) $(2m - 3)(m + 5) = 2m(m + 5) - 3(m + 5)$

$= 2m^2 + 10m - 3m - 15$

$= 2m^2 + 7m - 15$

(b) $(a + 3b)(3a - b) = a(3a - b) + 3b(3a - b)$

$= 3a^2 - ab + 9ab - 3b^2$

$= 3a^2 + 8ab - 3b^2$

(c) $(2x - 5y)(3x - 2y) = 2x(3x - 2y) - 5y(3x - 2y)$

$= 6x^2 - 4xy - 15xy + 10y^2$

$= 6x^2 - 19xy + 10y^2$

MY DEAR WATSON, GREAT CARE MUST BE TAKEN WHEN MULTIPLYING BINOMIALS. THE F.O.I.L RULE HELPS.
OBSERVE
$(a+b)(a+b)=a^2$
F.
MULTIPLY THE FIRST TERMS...
...THEN MULTIPLYING THE OUTSIDE TERMS...
$(a+b)(a+b)=a^2+ab$
F.O.
$(a+b)(a+b)=a^2+ab+ab$
F.O.I.
... NEXT MULTIPLY THE INSIDE TERMS-
$(a+b)(a+b)=a^2+ab+ab+b^2$
F.O.I.L
AND FINALLY MULTIPLY THE LAST TERMS!
THEY SAY THAT GENIUS IS AN INFINITE CAPACITY FOR TAKING PAINS. IT'S A VERY BAD DEFINITION BUT IT DOES APPLY TO DETECTIVE WORK.

Example 3

Find the value of:

(a) 38×62 (b) $(46)^2$

Solution

Two solutions will be shown for (a).

(a) (i) $38 \times 62 = (30 + 8)(60 + 2)$
$= 30(60 + 2) + 8(60 + 2)$
$= 1800 + 60 + 480 + 16$
$= 2356$

(ii) $38 \times 62 = (40 - 2)(60 + 2)$
$= 40(60 + 2) - 2(60 + 2)$
$= 2400 + 80 - 120 - 4$
$= 2480 - 124$
$= 2356$

Do you see the difference between the two ways for computing the same answer?

(b) Two solutions will also be given here for (b).

(i) $46 \times 46 = (40 + 6)(40 + 6)$
$= 40(40 + 6) + 6(40 + 6)$
$= 1600 + 240 + 240 + 36$
$= 2116$

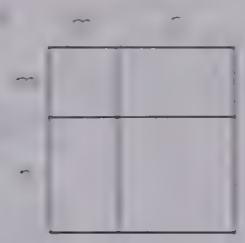
(ii) $46 \times 46 = (50 - 4)(40 + 6)$
$= 50(40 + 6) - 4(40 + 6)$
$= 2000 + 300 - 160 - 24$
$= 2300 - 184$
$= 2116$

Try $46 \times 46 = (50 - 4)(50 - 4)$. Do you get the same answer?

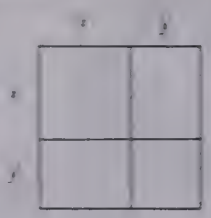
EXERCISE 6·7

1 Calculate the area of each region.

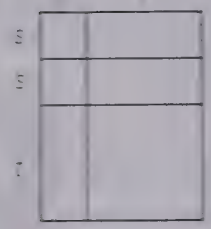
(a)



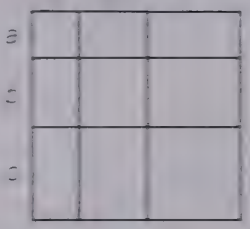
(b)



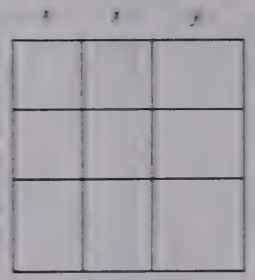
(c)



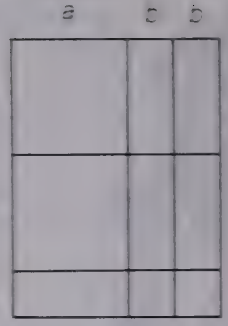
(d)



(e)

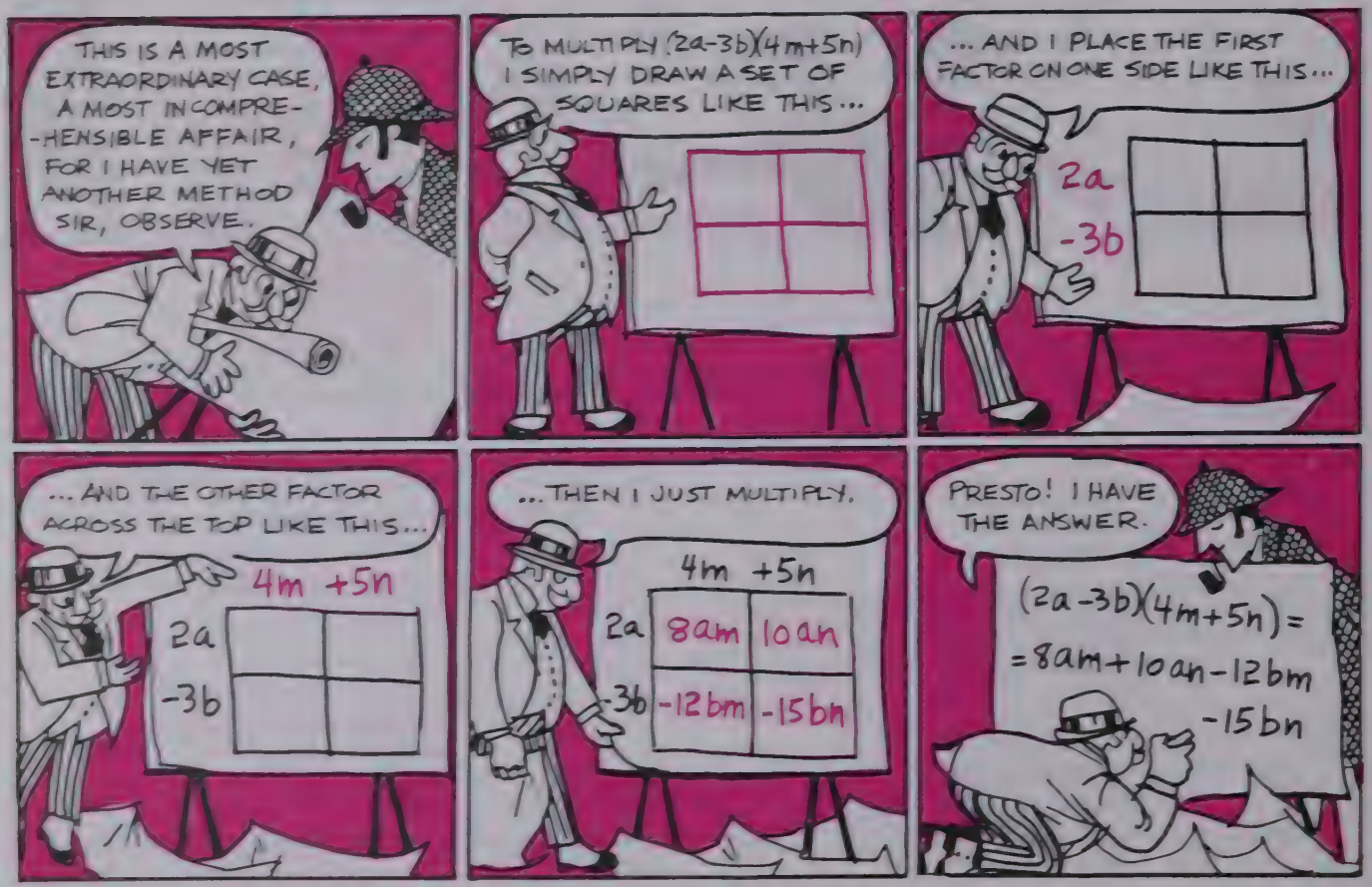


(f)



Write each area as the product of two polynomials and give the answer as indicated.

Example: $(m+n)(m+n) = m^2 + 2mn + n^2$.


THIS IS A MOST EXTRAORDINARY CASE, A MOST INCOMPRE-HENSIBLE AFFAIR, FOR I HAVE YET ANOTHER METHOD SIR, OBSERVE.
TO MULTIPLY (2a-3b)(4m+5n) I SIMPLY DRAW A SET OF SQUARES LIKE THIS...
... AND I PLACE THE FIRST FACTOR ON ONE SIDE LIKE THIS...
2a
-3b
... AND THE OTHER FACTOR ACROSS THE TOP LIKE THIS...
4m +5n
2a
-3b
...THEN I JUST MULTIPLY.
4m +5n
2a 8am 10an
-3b -12bm -15bn
PRESTO! I HAVE THE ANSWER.
(2a-3b)(4m+5n) =
=8am+10an-12bm
-15bn

2. Draw diagrams to illustrate each problem and give the product for each.

(a) $(a+b)(a+b)$ (b) $(2a+b)(a+2b)$

(c) $(2x+y)(x+2y)$ (d) $(3m+2n)(3m+2n)$

(e) $(m+n+t)(m+n+t)$

(f) $(2s+2t)(2s+2t)$

3. Find the product in each of the following:

(a) $(a+1)(a+2)$ (b) $(x-1)(x-2)$

(c) $(r-3)(r+4)$ (d) $(b+3)(b-1)$

(e) $(m+2)(m+2)$ (f) $(d-4)(d+9)$

(g) $(y+4)^2$ (h) $(z-3)^2$

(i) $(z-9)(z+9)$ (j) $(x+6)(x+3)$

(k) $(h-7)(h+2)$ (l) $(m-4)^2$

(m) $(m+5)(m+9)$ (n) $(k-5)(k-4)$

(o) $(m-12)^2$ (p) $(x+13)(x-10)$

(q) $(y-8)(y+5)$ (r) $(y+25)^2$

4. Expand each of the following:

(a) $(2a-1)(a-2)$ (b) $(2x-4)(x-3)$

(c) $(3x-4y)(5x-8y)$ (d) $(2m+3)(m-7)$

(e) $(2y+5)^2$ (f) $(6y-2z)^2$

(g) $(3z-4)^2$ (h) $(5a-2b)^2$

(i) $(3m-4n)^2$ (j) $(3b-1)(2b-5)$

(k) $(6m+2n)(6m-2n)$ (l) $(4k-1)(5k-2)$

(m) $(x-3y)(2x-3y)$ (n) $(9z-2y)(9z+8y)$

(o) $(4a-b)(a+7b)$ (p) $(7h+2k)(3h-k)$

(q) $(11m-12n)(11m-12n)$

(r) $(3h+7)(2h-3)$

5. Multiply by the method of Example 3.

(a) 13×24 (b) 52×81 (c) 69×32

(d) 74×29 (e) 85×38 (f) 58×79

6 Simplify. (First multiply, then add or subtract.)

(a) $(a+3)(a+4)+(a+1)(a+2)$

(b) $(x+3)(x-1)-(x-5)(x+2)$

(c) $3(k+5)(k-4)-(2k-3)^2$

(d) $2(m-3)^2-3(2m-5)(m+6)$

(e) $3(d-5)(d-2)-2(2d+3)(d-3)$

7. Use Sherlock Holmes' FOIL method to multiply these binomials.

(a) $(2a-3b)(2a-3b)$ (b) $(4m-5n)(4m-5n)$

(c) $(a+b)(c+d)$ (d) $(x-y)(n-m)$

(e) $(3a-2n)(4a-5n)$ (f) $(6m-2b)(3m+4b)$

(g) $(6a-3x)(2a+4x)$ (h) $(5b-2y)(6b-4y)$

(i) $(4y-3d)(5y+6d)$ (j) $(3y-6d)(4y-7d)$

8 Use Dr. Watson's method to multiply these binomials.

(a) $(3a-2b)(4a+5b)$ (b) $(7m-n)(6m+n)$

(c) $(3a-2b)(3a-2b)$ (d) $(8m-3y)(8m-3y)$

(e) $(12r+5y)(12r-6y)$ (f) $(9x-4p)(9x+4p)$

(g) $(12g-3h)(12g-3h)$ (h) $(a-b)(a+c)$

9 Can you modify Dr. Watson's method to multiply these?

(a) $(a-b)(2a-3b-4c)$

(b) $(a-b+c)(3a-2b+c)$

10. $(x+3)(x+4)=x^2+7x+12$.

(a) How did we get the "+7" in $+7x$?

(b) How did we get the "+12"?

11. $(x-5)(x-6)=x^2-11x+30$.

(a) How did we get the "−11" in $-11x$?

(b) How did we get the "+30"?

12. $(x-7)(x+4)=x^2-3x-28$.

(a) How did we get the "−3" in $-3x$?

(b) How did we get the "−28"?

13. $(x+3)(x-11)=x^2-8x-33$.

(a) How did we get the "− 8" in $-8x$?

(b) How did we get the "− 33"?

14. $(x-5)(x+7)=x^2+2x-35$.

(a) How did we get the "+ 2" in $+2x$?

(b) How did we get the "−35"?

Project

Project 1

Purpose

To develop experience in statistics.

To use data collected in surveys.

Method

1. Each student conducts a survey and lists the different ways electricity is used in his or her home and community.

2. Working in groups of four, compile one list of the different ways electricity is used. For example, lighting in the home is only one way even though there may be bulbs in various fixtures.

3. Compare the lists made by the various groups. Who has the longest list? Discuss.

4. What use requires the most kilowatts per hour? What use requires the least?

You may need this information.
Wattage = Ampere × Voltage
1 kW (kilowatt) = 1000 W (watts)
A kilowatt-hour is equivalent to 1 kW for 1 h.

5. If you were required to reduce the amount of electricity used in your home, which five uses would you discontinue or reduce first? List them in order. Which five uses, in order, would be the last to be discontinued?

6. Do you think people in your area "waste" electrical energy? If your community had to cut power consumption, what regulations should be enforced?

We can conserve energy-producing natural resources in many ways. These two students at the Ontario Science Centre, Toronto, are producing energy by foot-power. The energy is sufficient to turn the spindle of the lathe. Can you explain how foot-power turns the lathe?

6·8 Product of the Sum and Difference of Two Numbers

Sometimes the product of two binomials produces a binomial. Two such binomials are of a special form.

Example 1

Find the products of:

(a) $(x+1)(x-1)$ (b) $(a+b)(a-b)$

(c) $(5+m)(5-m)$

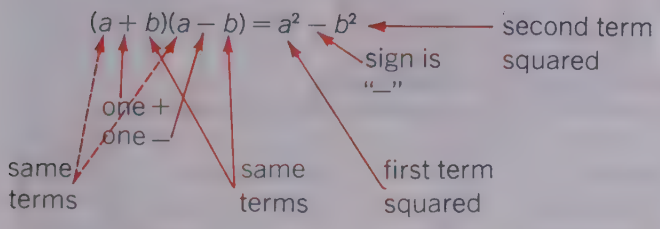
Solution

(a) $(x+1)(x-1) = x(x-1) + 1(x-1)$
$= x^2 - x + x - 1$
$= x^2 - 1$
(Note that the x-term disappears.)

(b) $(a+b)(a-b) = a(a-b) + b(a-b)$
$= a^2 - ab + ab - b^2$
$= a^2 - b^2$
(There is no ab-term in the answer.)

(c) $(5+m)(5-m) = 5(5-m) + m(5-m)$
$= 25 - 5m + 5m - m^2$
$= 25 - m^2$

Notice the form of the two binomials and the product.



Let us use this short cut method from now on.

Example 2

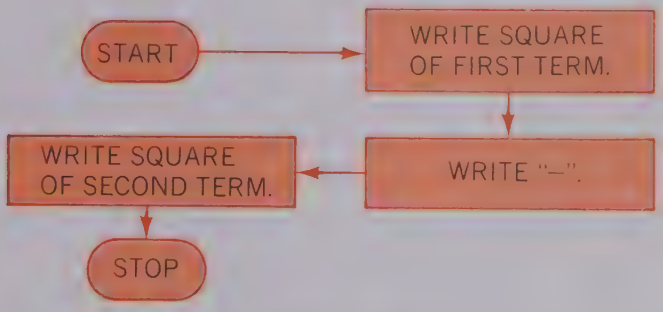
Find the product:

(a) $(2a-3b)(2a+3b)$ (b) $(4x-7y)(4x+7y)$

Solution

(a) $(2a-3b)(2a+3b) = 4a^2 - 9b^2$

(Difference of two squares.)



(b) $(4x-7y)(4x+7y) = 16x^2 - 49y^2$

1st term 2nd term

Example 3

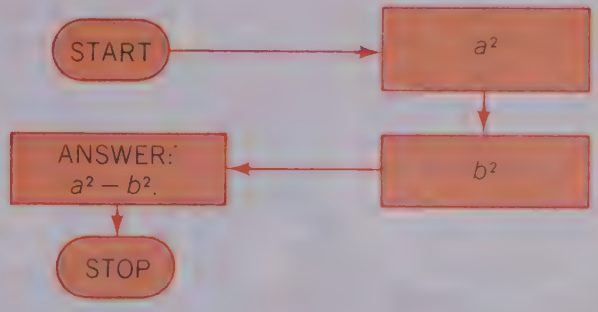
Multiply:

(a) 58×62 (b) 297×303

Solution

(a) $58 \times 62 = (60-2) \times (60+2)$
$= 3600 - 4$
$= 3596$

Check to see that this product is correct.



(b) $297 \times 303 = (300-3)(300+3)$

Use $(a-b)(a+b) = a^2 - b^2$

$297 \times 303 = 90\,000 - 9$
$= 89\,991$

EXERCISE 6·8

Find the products. Use the flowchart method in Example 2 to give the answers for 1 to 10. Write the answers in your notebook or give them orally.

1. $(b+1)(b-1)$
2. $(1+k)(1-k)$
3. $(x+y)(x-y)$
4. $(a-b)(a+b)$
5. $(m-n)(m+n)$
6. $(2x+1)(2x-1)$
7. $(y+2)(y-2)$
8. $(a+3)(a-3)$
9. $(z-8)(z+8)$
10. $(k+2t)(k-2t)$
11. $(a-5b)(a+5b)$
12. $(4s+t)(4s-t)$
13. $(7h-k)(7h+k)$
14. $(10x-y)(10x+y)$
15. $(3a+2b)(3a-2b)$
16. $(4m+5n)(4m-5n)$
17. $(3y+6z)(3y-6z)$
18. $(6p-4q)(6p+4q)$
19. $(9x-7y)(9x+7y)$
20. $(20a+50b)(20a-50b)$

Compute the products by the method of Example 2 above.

21. 99×101
22. 51×49
23. 68×72
24. 87×93
25. 198×202
26. 403×397
27. 1004×996
28. 805×795
29. 697×703
30. 1997×2003

6·9 Factoring: Common Factors

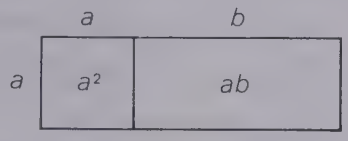
Earlier we multiplied a polynomial by a monomial to obtain a product. In this section, we shall reverse the operation. We shall start with a product and write it as a product of a monomial and a polynomial.

DR. WATSON, THE MURDOCK CASE INVOLVED MURDER, ROBBERY AND ARSON.
THE GREENWOOD CASE INVOLVED BREAKING AND ENTRY, FRAUD AND ARSON.
THE HARTFORDSHIRE CASE INVOLVED GRAND LARCENY, EMBEZZLEMENT AND ARSON.
AH, I SEE SIR! THE COMMON CRIME IN ALL THREE CASES IS ARSON. DO YOU THINK THE THREE CASES ARE RELATED?
IT IS A CAPITAL MISTAKE TO THEORIZE BEFORE YOU HAVE ALL THE EVIDENCE. IT BIASES THE JUDGEMENT. HOWEVER, IN THIS INSTANCE, I FEEL WE ARE JUSTIFIED.
WHAT IS MEANT BY THE COMMON CRIME IN ALL THREE CASES?

If the area of the rectangle is $a^2 + ab$,

Area $= a^2 + ab$

then the lengths of the sides can be as shown.



We can write the factors of the product $a^2 + ab$ as:

$$a^2 + ab = a(a + b).$$

Do Question 1 in Exercise 6·9.

When we factor a polynomial in this way, we say the *common factor* has been factored out.

$5a - 5b = 5(a - b)$ (5 is the factor common to both terms.)

$a^2 - ab = a(a - b)$ (a is the common factor.)

Example 1

Find the common factor of each of the following expressions.

(a) $5x + 5$ (b) $3a - 6$

(c) $a^2 - 2a$ (d) $x^2y + xy^2$

Solution

(a) The common factor for $5x + 5$ is 5.

(b) The common factor for $3a - 6$ is 3.

(c) The common factor for $a^2 - 2a$ is a.

(d) The common factor for $x^2y + xy^2$ is xy.

Do you notice that the common factor divides into each term of the expression? For $x^2y + xy^2$, there are two other common factors. They are x and y. But the greatest common factor is xy.

Example 2

Find the greatest common factor (G.C.F.) of each.

(a) $6m^2 - 9m$ (b) $a^3b + a^2b^2 - ab^3$

(c) $12x^3y^3 - 8xy^2$

Solution

Find the G.C.F. of the numerical coefficients for each; then find the G.C.F. of the literal coefficients.

(a) 3 is the G.C.F. of 6 and 9, and m is the G.C.F. of m^2 and m. The greatest common factor for $6m^2 - 9m$ is $3m$.

(b) a is the G.C.F. of a^3, a^2, and a; and b is the G.C.F. of b, b^2, and b^3. The greatest common factor for $a^3b + a^2b^2 - ab^3$ is ab.

(c) The greatest common factor for $12x^3y^3 - 8xy^2$ is $4xy^2$. Can you explain why?

Example 3

Factor each of the following:

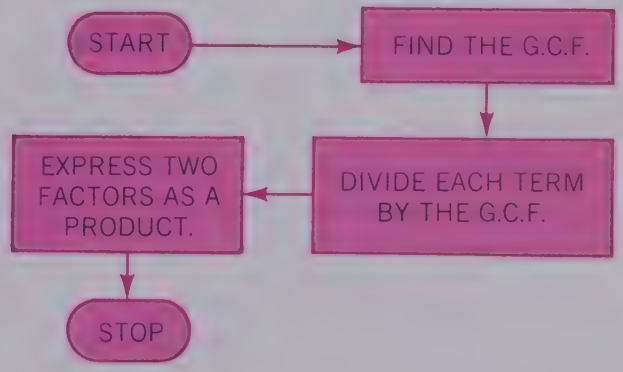
(a) $3 - 6x$ (b) $2x^2 + 8x$ (c) $4a^2b - 6ab^2$

Solution

(a) $3 - 6x = 3(1 - 2x)$

(b) $2x^2 + 8x = 2x(x + 4)$

(c) $4a^2b - 6ab^2 = 2ab(2a - 3b)$

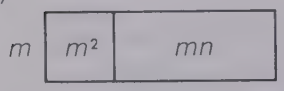


In the solutions above, we first choose the greatest common factor. Then we divide that common factor into the expression and the result is the other factor. Do you see that factoring is the reverse procedure of multiplying?

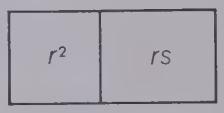
EXERCISE 6·9

1. Find the length and width of each rectangle. Use the common factor method.

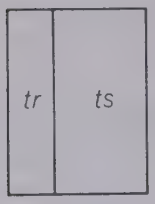
(a)



(b)



(c)



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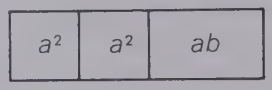

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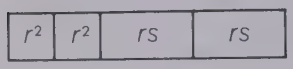

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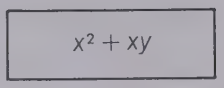
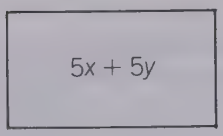
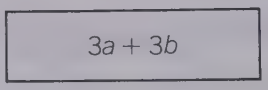


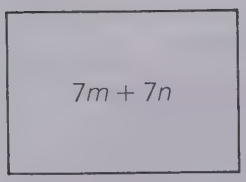
(g)



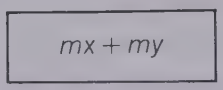
(h)



(i)



(j)



2. Find the common factor of each of the following:

(a) $2b+2$	(b) $6-6m$	(c) $7b+7$
(d) $4x-2$	(e) $3+6k$	(f) $15x-5$
(g) $6y-8$	(h) $9z-6$	(i) $8b+12$
(j) $3a-ab$	(k) $xy+2y$	(l) $4n-3mn$
(m) $2x+4xy$	(n) $3h-6hk$	(o) $5xy-3xyz$

3. Find the greatest common factor of each:

(a) $6x^2-9x$	(b) $4ab+8b$
(c) $12m^2+6m$	(d) $15y-10xy$
(e) x^3-x^2	(f) $3a^2-6ab$
(g) $18m^2+12mn$	(h) $14b^3+21ab$
(i) $16xyz-24x^2y$	(j) $28st^2-8s^2t$
(k) $18h^2k^2+30h^3k$	(l) $26a^3b^2+39a^2b^3$

4. Factor each of the following:

(a) $3h-3$	(b) $4x-8$
(c) $6a-3$	(d) $2m+6$
(e) $5k+10$	(f) $6t+8$
(g) $9b-6$	(h) $8z+12$
(i) $3xy+3x$	(j) $ab-2a$
(k) $4n-3mn$	(l) $4xy+8y$
(m) $3h+9hk$	(n) $7ab-3abc$
(o) x^2-x	(p) $12m^2+9m$
(q) $10y^3-15ay$	(r) bx^2+b^2x
(s) $3x^2-12x^2$	(t) $4m^2n+6mn^2$
(u) $18ab^2c-21ab$	(v) $13t^2x+26tx^2$

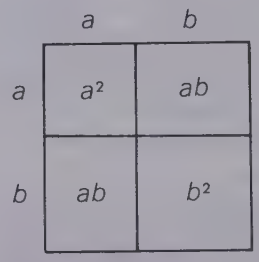
5. Factor each expression in Questions 2 and 3.

6·10 Factoring Trinomials

In this section, we will deal with trinomials that are products of two binomials. Of these, we will choose the simple ones—those which have 1 as the coefficient of the second degree term.

Notice that if $3 \times 7 = 21$

then $21 = 3 \times 7$.



Similarly, the product of the two binomials $(a+b)(a+b)$ is $a^2+2ab+b^2$; and so the factors of the product $a^2+2ab+b^2$ are $(a+b)$ and $(a+b)$.

Note that if the area of the rectangle is $a^2+3ab+2b^2$, we can determine the lengths of the two sides.

What factor is common to each of these columns?

What factor is common to each of these areas?

What factor is common to each of these areas?

a^2	ab	ab
ab	b^2	b^2

We can write the factors of $a^2 + 3ab + 2b^2$ by recording the lengths of each side of the rectangle:

$$(a + b) \text{ and } (a + b + b) \text{ or } (a + 2b)$$

Hence, $a^2 + 3ab + 2b^2 = (a + b)(a + 2b)$.

Do you remember the factors of 6? They are {1, 2, 3, 6, −1, −2, −3, −6}. The pairs of factors whose products are 6 are listed:

$$\{(1,6), (2,3), (-1,-6), (-2,-3)\}.$$

Do Questions 1, 2, and 3 in Exercise 6·10.

Let us look closely at a trinomial and its factors.

$$a^2 + 5a + 6 = (a + 2)(a + 3)$$

We observe that, in $(a + 2)(a + 3)$, the product of 2 and 3 is 6; while the sum of 2 and 3 is 5. This is the clue to the factoring of this type of trinomial.

Let us choose another trinomial and apply the same line of reasoning. Look at:

$$x^2 + 9x + 14.$$

We want two factors of 14 whose sum is 9. We know that 2 and 7 are the factors of 14 whose sum is 9. Therefore, the factors for

$$x^2 + 9x + 14 \text{ are } (x + 2) \text{ and } (x + 7).$$

Observe how we handle a trinomial like

$$y^2 - 3y - 10.$$

Again, we are looking for the two factors of −10 whose sum is −3. We note that −5 and +2 are the factors of −10 whose sum is −3. So we write

$$y^2 - 3y - 10 = (y - 5)(y + 2).$$

To check whether the factoring is correct, all we have to do is multiply one factor by the other to compare the product with the original trinomial.

Example 1

Factor and check:

(a) $m^2 + 6m + 8$ (b) $z^2 + 3z - 18$

Solution

(a)

Factors of 8 are:
(1,8) (−1,−8)
(2,4) (−2,−4)

For $m^2 + 6m + 8$, 4 and 2 are the two numbers which are factors of 8 and whose sum is 6.

Therefore, $m^2 + 6m + 8 = (m + 4)(m + 2)$.

Check: $(m + 4)(m + 2) = m^2 + 4m + 2m + 8$
$= m^2 + 6m + 8$

(b)

Factors of −18 are:
(−1,18) (1,−18)
(−2,9) (2,−9)
(−3,6) (3,−6)

For $z^2 + 3z - 18$, +6 and −3 are the two factors of −18 whose sum is 3.

Therefore, $z^2 + 3z - 18z = (z + 6)(z - 3)$.

Check: $(z + 6)(z - 3) = z^2 + 6z - 3z - 18$
$= z^2 + 3z - 18$

Example 2

Factor:

(a) $x^2 - 5x - 24$ (b) $a^2 - 9a + 20$

Solution

(a) For $x^2 - 5x - 24$, -8 and $+3$ are the two factors of -24 whose sum is -5. Therefore,

$x^2 - 5x - 24 = (x - 8)(x + 3)$.

Factors of -24 are:
$(-1,24)$ $(1,-24)$
$(-2,12)$ $(2,-12)$
$(-3,8)$ $(3,-8)$
$(-4,6)$ $(4,-6)$

(b) For $a^2 - 9a + 20$, -5 and -4 are the two factors of $+20$ whose sum is -9. Therefore,

$a^2 - 9a + 20 = (a - 5)(a - 4)$.

Factors of $+20$ are:
$(1,20)$ $(-1,-20)$
$(2,10)$ $(-2,-10)$
$(4,5)$ $(-4,-5)$

EXERCISE 6·10

1. Determine the total length and width of each rectangle.

(a)

m^2	mn
mn	n^2

(b)

m^2	nm	nm
nm	n^2	n^2

(c)

$a^2 + 2ab + b^2$

(d)

$a^2 + 3ab + 2b^2$

(e)

$x^2 + 3xy + 2b^2$

(f)

$m^2 + 4mb + 3b^2$

2. List all the pairs of factors whose product is the number:

(a) 6	(b) 8	(c) 9	(d) 12
(e) -15	(f) -21	(g) 32	(h) 20
(i) -12	(j) 36	(k) -11	(l) -72

3. List the two factors of each number such that their sum is the number shown.

Number	Sum of Factors	Factors
16	10	{8,2}
12	7	?
40	13	?
11	12	?
-11	-10	?
-6	-1	?
-6	1	?
-6	-5	?
-6	$+5$	?
-42	-1	?
-42	$+1$	?
-42	-19	?
-42	$+19$	?
-42	-11	?
-42	$+11$	?

4. Make up 20 questions of the type in Question 3 and solve each. Exchange your questions with a classmate.

Factor each of the following and check your answer.

5. $m^2 + 6m + 9$	6. $h^2 - 5h + 6$
7. $s^2 + 4s + 4$	8. $x^2 - 14x + 49$
9. $z^2 + 16z + 64$	10. $a^2 - 2a + 1$
11. $x^2 - 4x - 21$	12. $b^2 - b - 20$
13. $m^2 + 7m + 10$	14. $x^2 - 7x + 12$
15. $y^2 + 10y + 24$	16. $x^2 + 8x + 15$
17. $z^2 + z - 30$	18. $a^2 - 18a - 40$
19. $c^2 - 4c - 32$	20. $k^2 + 6k - 40$
21. $b^2 + 12b + 35$	22. $x^2 + 12x + 36$
23. $a^2 - 20a + 100$	24. $h^2 - 12h + 11$
25. $c^2 - 15c - 54$	26. $y^2 - 9y - 70$
27. $m^2 + 14m - 72$	28. $t^2 - 2t - 3$
29. $x^2 - x - 72$	30. $h^2 + h - 42$
31. $a^2 + a - 90$	32. $c^2 - 3c - 54$

6·11 Factoring: Difference of Two Squares

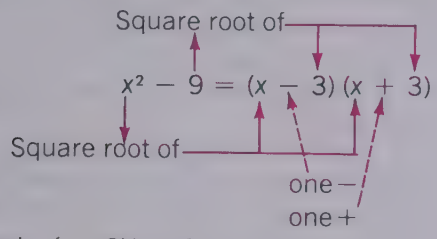
Let us review the short method for multiplying binomials that have the same terms but opposite signs.

$(x-3)(x+3)$ = 1st term squared − 2nd term squared
$= x^2 \qquad - 9$

To factor the binomial composed of the difference of two squares,

$$x^2 - 9,$$

we only need to find the square root of the 1st term and of the second term, and then arrange these answers in the parentheses as indicated.



Check: $(x-3)(x+3) = x^2 - 9$.

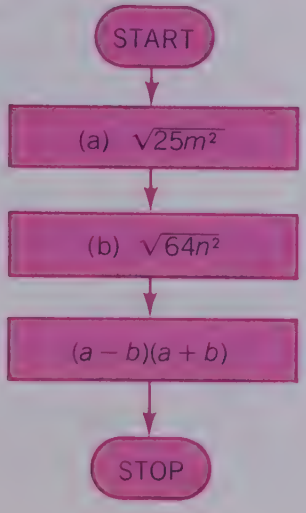
Let us try another.

Example 1

Factor $25m^2 - 64n^2$.

Solution

First: What is $\sqrt{25m^2}$?
Answer: $5m$.
What is $\sqrt{64n^2}$?
Answer: $8n$.



Hence,
$25m^2 - 64n^2$
$= (\quad)(\quad)$
$= (5m \quad)(5m \quad)$
$= (5m \quad 8n)(5m \quad 8n)$
$= (5m - 8n)(5m + 8n)$

We must be careful that we use this method only when factoring problems of the type called the *difference of two squares*.

Example 2

Factor the following:

(a) $b^2 - 25$ (b) $9x^2 - 64$ (c) $36 - 25t^2$

Solution

(a) $b^2 - 25 = (b-5)(b+5)$

(b) $9x^2 - 64 = (3x-8)(3x+8)$

(c) $36 - 25t^2 = (6-5t)(6+5t)$

EXERCISE 6·11

Factor where possible. Write the answer directly in your notebook (or give the answer orally) to the first 12.

1. $x^2 - 4$
2. $m^2 - 9$
3. $h^2 - 16$
4. $a^2 + 64$
5. $m^2 - n^2$
6. $x^2 - 49$
7. $1 - h^2$
8. $25 - b^2$
9. $z^2 - 49$
10. $k^2 - 100$
11. $t^2 + 1$
12. $p^2 - 81$
13. $25a^2 - 49b^2$
14. $16r^2 - 9s^2$
15. $4k^2 - 1h^2$
16. $36x^2 - 100y^2$
17. $64x^2 - 81y^2$
18. $144a^2 - 121b^2$
19. $m^2 - 225n^2$
20. $400b^2 - 2500a^2$
21. $81z^2 - 49y^2$
22. Make up five *difference of squares* questions and factor each. Exchange your questions with a classmate.

Weeds Cost the Farmer Money

1. In a normal barley crop, there are 150 barley plants per square metre. If 50 wild oats grow in that area, the barley yield is 14% less than normal. If barley in a weed-free field yields 1.60 t/ha, what would the crop yield be if wild oats had been in this proportion in the field?

2. If barley is $115/t, what is the loss in dollars per hectare due to wild oats in the proportion given in Question 1?

3. If a farmer has a barley field of 260 ha, what is the loss in dollars to the farmer at the above rate of loss due to wild oats?

4. Wild oats spoils wheat even more than it spoils barley. Fifty wild oats per square metre causes a 22% loss in production of wheat. If a normal wheat crop without weeds is 2.47 t/ha, what is the loss due to wild oats at this rate? If wheat sells for $142/t, what is the loss in dollars per hectare? Many Prairie farmers have 520 ha of wheat. At the above rate, what would be the loss to such a farmer due to wild oats?

5. The Canadian Grain Commission reports that the average cereal grain delivered to the elevator included $2\frac{1}{2}$% weed seeds. Some farmers have as much as 25% weed seeds in their grain. If a farmer delivers 650 t of "dirty" grain to an elevator, what is the measure of grain and what is the measure of weeds, in tonnes, if the delivery is:

   (a) $2\frac{1}{2}$% weeds? (b) 25% weeds?

6. The dirtiest rapeseed ever delivered was reported to be 48% weed seeds. What was the measure of rapeseed if the load delivered was 2.4 t in total?

7. Japan discounts Canadian barley at 2% from the price of Australian barley because the Canadian barley contains wild oats. On a shipment of Canadian barley grossing $2.43 million, what is the net sale price?

8. Russia suffered a crop failure in 1972. The Russian wheat harvest was only 62 Mt. (The symbol M is for *mega* which means million. Hence, Mt is read *megatonnes* and means *million tonnes*.) The harvest was 24% below the expected production. What was the expected production in 1972?

9. In the same year, Australia suffered a 25% drop in wheat production due to severe drought. The production that year was 6.5 Mt. What would have been the production had there been a full crop in 1972?

10. Due to food crop failures around the world (Asian rice production fell 12%), the export price of Canadian top-quality hard wheat rose from $68.50/t to $266.40/t. What was the percent increase in price?

11. The same year, the price of Durum wheat rose from $67.50 to $360/t. What was the percent increase in price?

12. Canada exported 15 Mt of wheat in 1971 and 17 Mt in 1972. What was the percent increase.

13. The expected total grain harvest in Russia for 1973 was approximately 187 Mt. The previous year's production—a disastrous year—was only 168 Mt. Russia's target for 1972 was 197 Mt. What percent was their production off the target for 1972? What percent was their production in 1973 above that of the crop failure in 1972?

14. Due to a world meat shortage in 1973-74, the price of beef in Canada jumped from $28.50 per 45 kg to $52.60 per 45 kg. What was the percentage increase?

15. Do you think a country with a surplus should raise its prices when the rest of the world is suffering a shortage? Consider the world food shortage of 1973-74 and the world oil shortage of 1973-74.

REVIEW EXERCISE 6

Perform the indicated operations.

1. $5x + (-3x) + (-6x) + 2x$
2. $8b^2 + 4b + (-7b) + (-6b^2)$
3. $(3h^2 - 7h + 12) + (h^2 + 2h - 5)$
4. $(6x^2 - 4xy + 3y^2) + (-8x^2 + 7xy - 10y^2) + (5x^2 - 2xy + 9y^2)$
5. $(12a - 3b) - (7a - 6b)$
6. $(8m + 7mn - 6n) - (5m - 2mn + 4n)$
7. $(-3t^2 + 2t - 1) - (-5t^2 - 3t + 4)$
8. $(a^3) \times (a^5)$
9. $b^7 \div b^4$
10. $(-mn^5)(m^2n^3)$
11. $18b^3x^7 \div 3bx^2$

Simplify:

12. $5x(a + 3b) - 3x(2a - b) + 2x(3a - 7b)$
13. $4m(7p - 2q + r) + 3m(-6p + 5q - 9r)$

Expand and simplify:

14. $(3x - 5)(x + 2)$
15. $(2a + 7b)(6a - 3b)$
16. $(4m - 3)^2$
17. $(3x + 7y)^2$
18. $4(a - 2)^2 - (3a + 1)(5 - a)$
19. $(4t - 9)(4t + 9)$
20. $(5x + 3y)(5x - 3y)$

Factor each of the following:

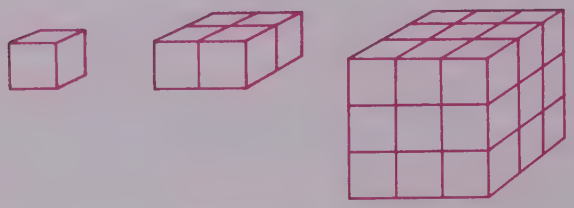
21. $3m^2 - 12m$
22. $15x^3y^5 + 6x^2y^3$
23. $16x^2y - 12xy^2$
24. $a^2 - 5a - 6$
25. $x^2 + 11x + 28$
26. $x^2 - 16x + 39$
27. $4 - t^2$
28. $h^2 - 36$
29. $81 - 4s^2$

Using the Library

1. Read pages 18-20 in *Mathematics: Man's Key to Progress*, Book B, Gerstine, Richard A. Franklin Publication Inc. (1972).
   (a) What is the Fibonacci Sequence? Generate several applications of the sequence.
   (b) Do Question D on page 50 of the same book, or use the Lattice method for multiplying.
   326×478 961×5207 7978×7264
   Make up and solve three more problems like these.
   (c) Do Question E on page 30 of the same book, or make a set of Napier's Rods, or Napier's Bones. Use your set of Napier's Rods to multiply:
   384×6 5207×9 473×98 8954×76
   Make up and solve eight more problems like these.
   (d) Do Questions F and G on page 30 of this book.
2. Read pages 77-83 of the book listed above. Do Questions A, B, C, D, E, F, and H on pages 83-85.
3. *Times And The Shape Of Times*, Regents A. V., 300 Watts, K.S.O. Doubleday and Company Inc. (1964).
   (a) Page 3: Perform the indicated multiplication problem. What happened to the first 1?
   (b) Name six (6) ways we can (a) have the number 1 (b) keep (or be) the identity or unit (multiplication unit).
   (c) Page 40: Use a rule for multiplying by nines (9) on hands. Write answers for 3×9, 7×9, and 9×9.
4. Circles and Curves, Regents A. V., and Watts, K.S.O. Doubleday and Company Inc. (1964).
   (a) Read pages 14-16 and the method shown on page 15 to find π to 707 decimal places.
   (b) What is the area and shape of the largest square you could place in the circle formed during the first five steps?

Do You Remember . . .

. . . the fun you had as a youngster with some of your favourite toys? Let us look at cubes made from two-centimetre blocks. The edge of the first cube is one block long, the edge of the second cube is two blocks long, the third is three blocks long, and so forth.



Note! Consider each cube to be on a glass surface so that the bottom is open to view.

For each cube:

(a) how many blocks have four sides facing to the outside?

(b) how many blocks have just three sides facing to the outside?

(c) how many blocks have just two sides facing to the outside?

(d) how many blocks have just one side facing to the outside?

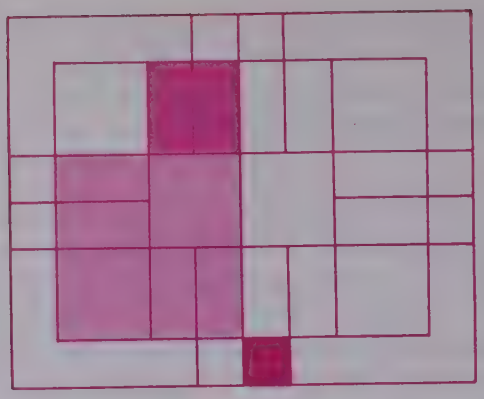
(e) how many blocks are on the inside, completely hidden by outside blocks?

Prepare a chart to show your answers.

Something to Think About

How many squares can you find in the top right diagram?

Hint! Some squares are shaded to give you a clue. There are large squares and small squares. There are squares within squares. Some squares overlap each other.



Can you prepare an organized approach to this question?

Have fun. (The answer is more than 30.)

Have You Discovered that . . .

. . . when two dice are tossed, there are 36 different possible outcomes? Let us choose a black die and a red die. When we write down a pair of numbers, the first of the pair will be the outcome on the black die and the second of the pair will be the outcome on the red die. Study the pairs on this chart and complete the chart in your notebook.

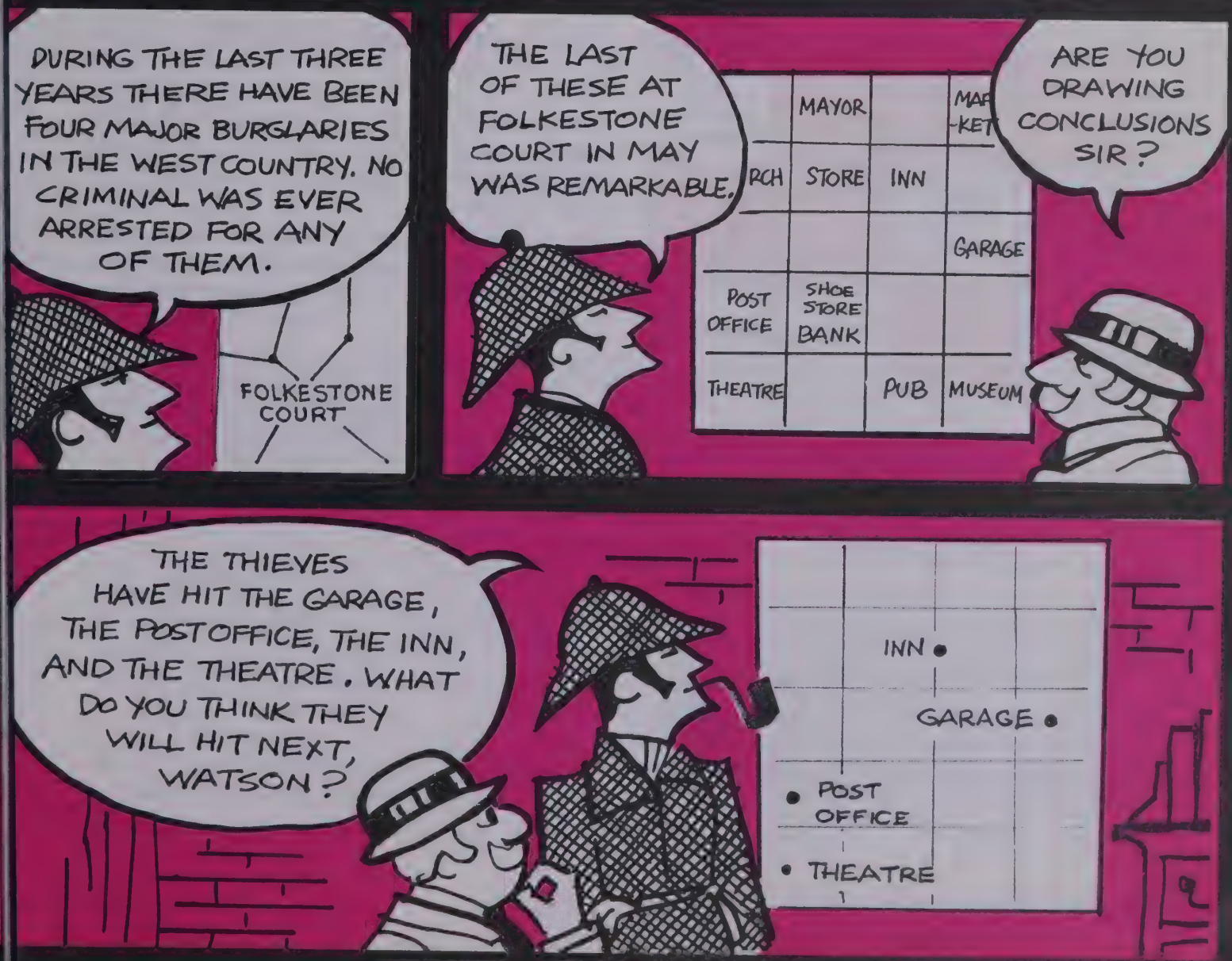
1 1	2 1	3 1	4 ?	5 ?	6 1
1 2	2 2	3 2	4 ?	5 2	6 2
1 3	2 3	? ?	? ?	? ?	? ?
? 4	? ?	? 4	? ?	? ?	6 4
? 5	? ?	? ?	? 5	5 ?	? ?
? 6	? ?	? ?	4 ?	? 6	? ?

After you have completed the chart, find how many pairs have totals of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12.

How many pairs give a total of 2? of 12? of 4? of 11?

Which total is represented by the most spaces? When two dice are tossed, which total is likely to come up most often? This study relates to *probability*.

Chapter Seven
IT'S THE RELATIONS THAT COUNT
Graphing Equations


DURING THE LAST THREE YEARS THERE HAVE BEEN FOUR MAJOR BURGLARIES IN THE WEST COUNTRY. NO CRIMINAL WAS EVER ARRESTED FOR ANY OF THEM.
FOLKESTONE COURT
THE LAST OF THESE AT FOLKESTONE COURT IN MAY WAS REMARKABLE.
MAYOR
MAR-KET
RCH
STORE
INN
GARAGE
POST OFFICE
SHOE STORE
BANK
THEATRE
PUB
MUSEUM
ARE YOU DRAWING CONCLUSIONS SIR?
THE THIEVES HAVE HIT THE GARAGE, THE POST OFFICE, THE INN, AND THE THEATRE. WHAT DO YOU THINK THEY WILL HIT NEXT, WATSON?
INN
GARAGE
POST OFFICE
THEATRE

7·1 Graphing Relations

Read the cartoon strip on the previous page. Study the map of Folkestone. Locate the garage at Folkestone Court, the Inn, the Post Office, and the Theatre.

Figure 7-1

EXERCISE 7·1

1. What places do you think the thieves intend to rob? Which one might be next?

   Explain the basis for your conclusion.

2. The thieves intend to "hit" locations B4 and B1. What places will they "hit"? How is this type of map (Figure 7-1) limited with respect to locating places accurately?

   Discuss.

3. Obtain a set of road maps for your class from a gasoline service station or from your local tourist bureau (or use atlases from the library).

   (a) Learn how to locate a place listed in the index of place names.

   (b) Choose five or more places from the index you have never heard of before. Find them on the map. Give their approximate location by stating their direction and distance from some well-known city.

   (c) For small group competition, devise a contest on locating places on the map.

7·2 Ordered Pairs and Their Graphs

In the previous section, we used a method to locate business places on a map. Now we will study a system to locate points on a graph grid—a method used by mathematicians.

Did you observe in Figure 7-1 that the code for each place was made up of a letter and a number? What does the letter designate? And what does the number designate?

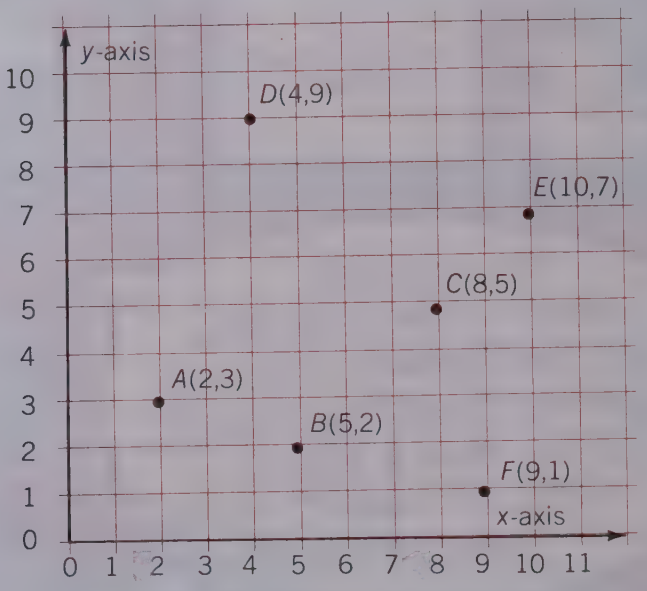
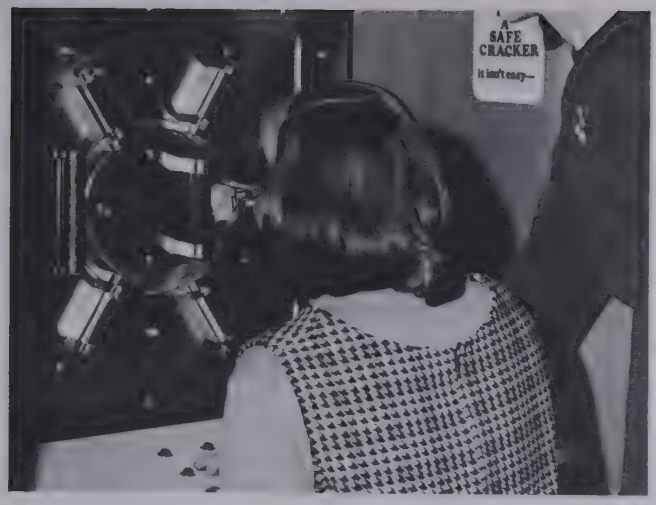


Figure 7-2



There are many types of safes. Some types have tumblers which must drop into slots in a specific order. The study of the combinations and permutations in which the tumblers might drop is part of a special branch of mathematics. Look up combinations and permutations in a mathematics book in your library.

Study Figure 7-2. Two line segments are drawn on graph paper. The horizontal line segment is labelled as the x segment and the vertical line segment as the y segment. The x and y segments (referred to as axes) usually meet at the bottom left. The graph in this case is to the right and up from the point of meeting, which is called the origin.

Study how each point from A to F is labelled on the graph. Each label has a pair of numbers. Because the first number of each pair relates to the horizontal or x segment and the second to the vertical or y segment, there is an order established for each pair. Thus, we call such a pair of numbers an *ordered pair*.

Example 1

With reference to Figure 7-2:

(a) where is (5,3) with respect to point B?

(b) where is (7,5) with respect to point C?

(c) where is (4,3) with respect to point A?

Solution

(a) (5,3) is one division above B.

(b) (7,5) is one division to the left of C.

(c) (4,3) is two divisions to the right of A.

Example 2

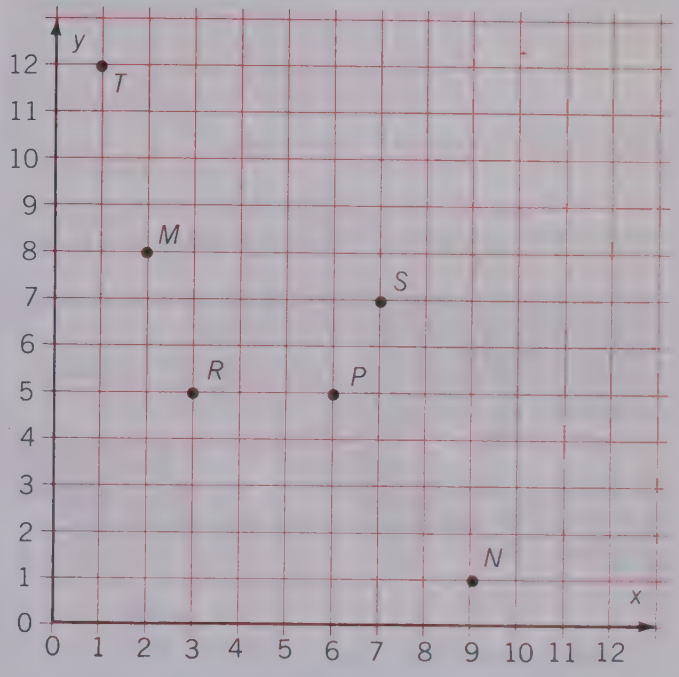
On a graph locate the points $M(2,8)$, $N(9,1)$, $P(6,5)$, $R(3,5)$, $S(7,7)$, and $T(1,12)$.

(a) Which of the points on the graph is farthest to the right?

(b) Which point is higher than the others?

Solution

Locate all the required points on a graph as shown.



(a) N is farthest to the right.

(b) T is higher than the others.

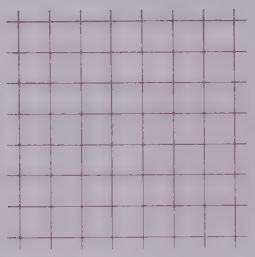
Do Questions 1 to 6 in Exercise 7-2.

In the graphs illustrated so far, the scales on the x-axis and on the y-axis have been the same. This is not always the case. It is often more convenient to use one scale for the x-axis and another for the y-axis. Remember, the scale on the x-axis must always remain the same on a particular graph. Also, the scale must always remain the same on the y-axis for a particular graph.

Example 3

If the following set of points is to be graphed on this grid, what scale for each axis would be best?

(4,4) (6,30) (14,40) (20,70) (24,20) (30,30)



Solution

Consider the *x*-axis first. The largest value of *x* is 30 and there are 7 divisions we can use.

Dividing 30 by 7, we find that $30 \div 7 > 4$.

Hence, each division must be thought of as greater than 4 units. Will 5 units per division work?
$7 \times 5 = 35$. Yes.

Therefore, we label the *x*-axis 0, 5, 10, 15, ..., 35. Notice that our graph will not extend to the very edge but comes close.

Now consider the y-axis.

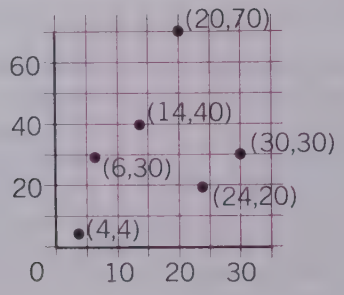
The largest value of *y* is 70 and there are 7 units.

Dividing 70 by 7, we find that $70 \div 7 = 10$.

Hence, each division is counted as equal to 10 units.

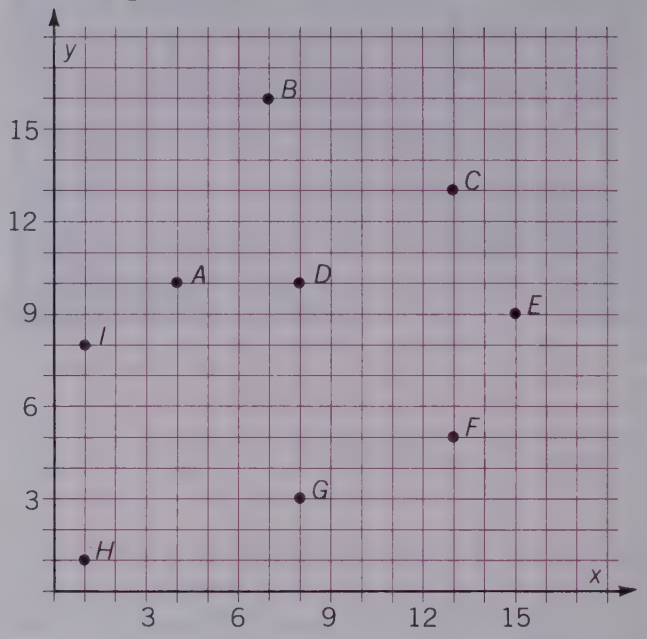
We note in this example that the *x*-scale is different than the *y*-scale.

Next, the points are plotted.



EXERCISE 7·2

1. Give the ordered pair for each of the points *A* to *I* on the graph as shown.



2. On grid paper, mark off a horizontal or *x* segment and label it from 0 to 16. Mark off a vertical or *y* segment and label it from 0 to 16 also. For each of the following ordered pairs, locate its corresponding point on the graph. Then join the points by drawing lines in the sequence the ordered pairs are given.

   (4,1) (16,10) (0,10) (12,1) (8,16) (4,1)

   What figure do you get?

3. Mark these points on a graph. Give the ordered pair of the midpoint between the given points. In each case, locate the points on the graph to determine the midpoints. Can you identify a rule for the ordered pair of the midpoint?

   (a) (3,2) and (3,8)
   (b) (5,1) and (5,5)
   (c) (8,6) and (8,12)
   (d) (7,3) and (9,3)
   (e) (3,3) and (11,3)
   (f) (12,1) and (12,13)

4. Mark these points on a graph. How far apart are the following pairs of points? Plot the points on the graph and count the divisions.

   (a) (3,8) and (3,19) (b) (5,11) and (17,11)

   (c) (5,2) and (5,13) (d) (2,6) and (13,6)

   (e) (15,4) and (15,13) (f) (11,11) and (4,11)

   Can you identify a rule for determining how far apart the points are?

5. Which of the following points on the graph is farthest to the left?

   (17,6) (12,9) (18,2) (13,7) (15,12)

6. Which of the following points is the highest on the graph?

   (12,5) (8,7) (4,11) (15,10) (3,13) (6,9)

7. The number of divisions available on each axis is given. Select a suitable scale for the set of points to be graphed so that the major portion of the grid will be used.

   (a) *x*-axis – 8 divisions
   y-axis – 6 divisions

   (i) {(0,0), (14,18), (28,24), (35,36), (42,60), (56,72)}

   (b) *x*-axis – 14 divisions
   y-axis – 12 divisions

   (ii) {(0,0), (25,70), (65,90), (95,140), (100,180), (130,220)}

   Graph each set on the grids as indicated.

BUT, SIR, ARE GRAPHS ALWAYS RESTRICTED TO WHOLE NUMBERS?
X ∈ W
CANNOT ORDERED PAIRS BE MADE OF INTEGERS ?
I = {..., -3, -2, -1, 0, +1, ...}
THAT IS A LOGICAL CONCERN MY GOOD MAN...
... LET US READ ON AND INVESTIGATE!
MATH PROBE 1
MATH PROBE 2
MATH PROBE 3
MATH

7·3 Other Ordered Pairs (and Their Graphs)

Let us investigate ordered pairs such as (–3,7), (8,–5), and (–4,–1), and see how to graph them.

In Figure 7-2, the *x* and *y* segments began at zero and moved to the right and upward. Let us replace the *x* segment with an integer line (an *I*-line) and label it with integers. Similarly, replace the *y* segment with an integer line and label it. Observe how it is done in Figure 7-3. Now the horizontal and vertical lines form what we usually call the *x*- and *y*-axes. Notice how the axes divide the graph into four parts which are called quadrants. The quadrants are numbered in counterclockwise order beginning with the one in the upper right (see Figure 7-3).

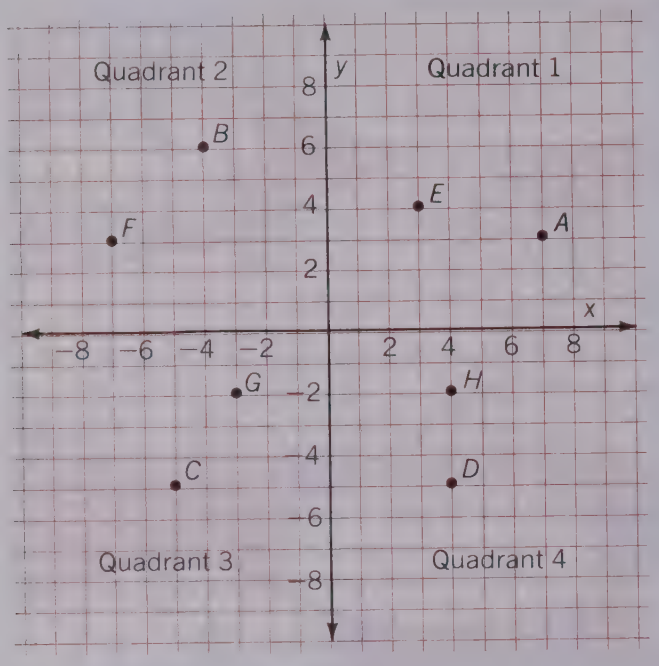


Figure 7-3

Let us draw some conclusions about the points and the associated ordered pairs.

1. Give the ordered pair for each point *A* through *H*.
2. In which quadrant are the points represented by ordered pairs with both numbers negative?
3. Which number of the ordered pairs is negative for points in the 2nd quadrant? Which is negative in the 4th quadrant?
4. In which quadrant is the first number of the ordered pair negative and the second positive?
5. In which quadrant is the first number of the ordered pair positive and the second negative?
6. In which quadrant is the first number of the ordered pair positive and the second positive also?
7. We call the first number of the ordered pair the *x*-value and the second the *y*-value. Can you explain why?

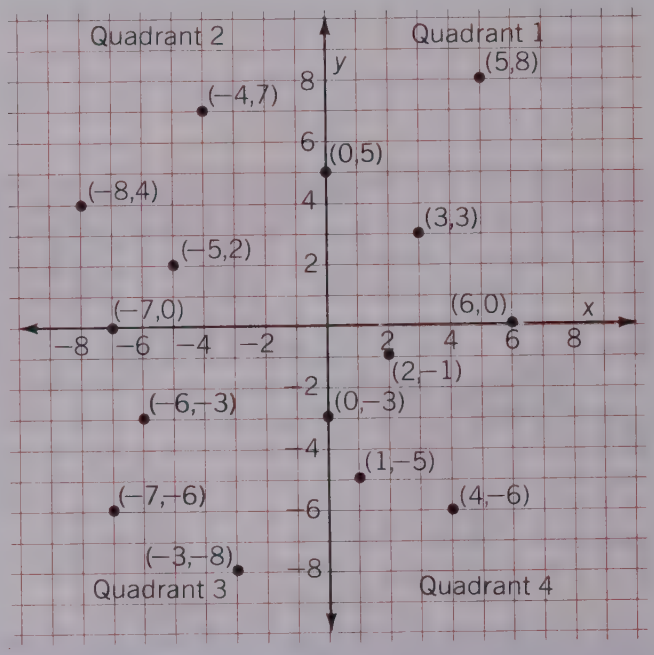
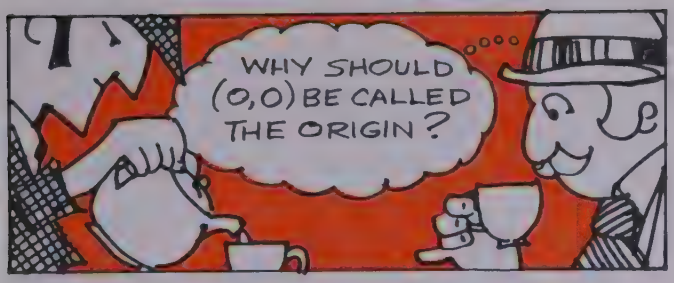


Figure 7-4

Study the graph of Figure 7-4. Also notice the following points on the graph: (0,5), (–7,0), (0,–3), and (6,0). Are they located in any of the quadrants? How do you explain their locations? Where would you locate the point (0,0) on the graph? The point (0,0) on the graph is given the name "the origin". Therefore, where the *x*-axis and the *y*-axis intersect is the point called "the origin".

In Figure 7-4:
– how far from the *y*-axis is each point?
– how far from the *x*-axis is each point?



EXERCISE 7·3

1. Mark off the x- and y-axes on graph paper. Locate the following points on your graph. Join the points in each set in the order they occur. What shape do you get?

   (2,–2), (–13,–2), (–12,5), (–5,12), (2,15), (15,7), (2,–2)

   Start again.

   (2,15), (2,–4), (15,–4), (10,–9), (–7,–9), (–15,–4), (2,–4)

2. In which quadrants (if any) are the following points located?

   (15,–12), (–81,–3), (56,39), (–6,41), (0,29), (25,32), (64,–12), (35,0), (–75,13), (–2,–100), (0,–19), (85,–7), (–47,0), (28,–4), (–88,–8)

3. Locate the following points on the graph and join them in the order in which they are given.

   (a) (–1,–13) (b) (–4,–12) (c) (–7,–12)
   (d) (–8,–11) (e) (–8,–9) (f) (–9,–9)
   (g) (–9,–8) (h) (–7,–7) (i) (–6,–6)
   (j) (–9,–6) (k) (–9,–5) (l) (–8,–4)
   (m) (–11,–3) (n) (–10,0) (o) (–8,2)
   (p) (–9,3) (q) (–8,6) (r) (–9,7)
   (s) (–9,8) (t) (–8,9) (u) (–5,10)
   (v) (–1,11) (w) (2,11) (x) (5,10)
   (y) (7,9) (z) (9,7) (aa) (10,4)
   (bb) (10,0) (cc) (9,–4) (dd) (9,–5)
   (ee) (10,–7) (ff) (9,–8) (gg) (7,–9)
   (hh) (7,–10) (ii) (8,–12)

   What image do you get?

4. On graph paper, join point A to point B and join point C to point D. In each case, make an observation about the two segments AB and CD.

Case	A	B	C	D
1	(1,–3)	(9,5)	(3,3)	(7,–1)
2	(–2,6)	(5,4)	(–1,5)	(6,3)
3	(–4,3)	(2,3)	(–1,–3)	(–1,3)
4	(–6,3)	(–4,–5)	(4,–3)	(–4,–5)
5	(0,–5)	(8,–5)	(6,–3)	(2,–7)

5. Graph the following set of points.
   (–2,–6), (–1,–5), (0,–4), (1,–3), (2,–2), (3,–1), (4,0)
   (a) What observation can you make about this set of points?
   (b) Give the ordered pairs for three more points that would extend this set of points.

6. Make up a question such as Questions 1 to 3 for your classmates.

This woman is typing a program into a teletype terminal connected to a computer. The program is recorded on the cards at the right. There are excellent employment opportunities for key punch operators in the computer and data processing fields. Would you like to work at a job like this?

Project

Project 1

Purpose

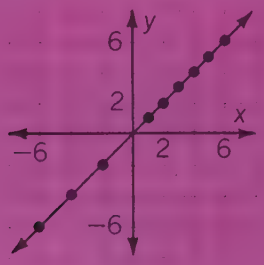
To discover the relationships between the x- and y-values of some relations in graphical form.

Method

Work in groups of four or five students for this project.

Part A

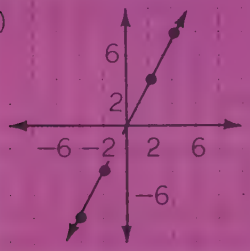
1. List the ordered pairs for each of the points marked on the line.
2. Describe, in as many ways as you can, the relationships between the y-values and the x-values.
3. Record on the chalkboard all the relationships described in Step 2.
4. Discuss the relationships. Are there different ways of expressing the same relationship?
5. Can you express the relation between the x-value and the y-value in algebraic form using x and y?



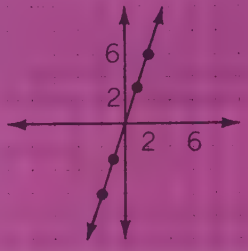
Part B

Repeat Steps 1 to 5 in Part A for each of the following graphs.

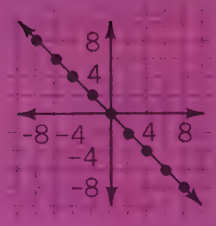
(a)



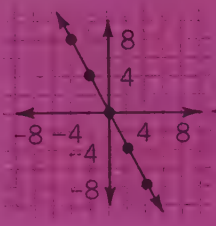
(b)



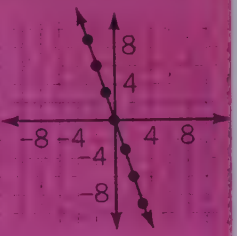
(c)



(d)



(e)



Part C

What conclusions can you draw about the different positions of the graphs with respect to the set of axes? Refer to the graphs of Part B.

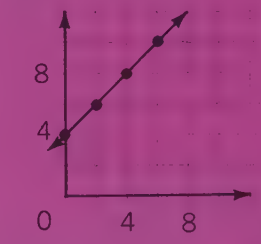
– What one point is common to all graphs?

– In what ways are the ordered pairs of graphs (a) and (b) similar? How are the ordered pairs of (c) and (d) similar?

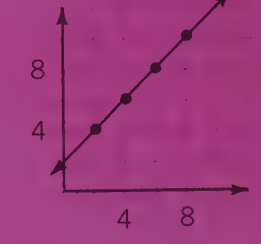
Part D

Repeat Steps 1 to 5 in Part A for each graph.

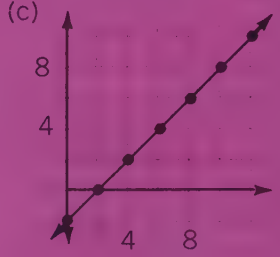
(a)



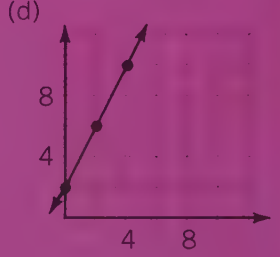
(b)



(c)



(d)



Part E

What conclusions can you draw about the graphs of Part D?

7·4 Graphs of the Form y = ax

What kinds of graphs do the relations such as these make?

$$y = 3x$$
$$y = -5x$$

Let us study the two examples.

Example 1

Graph the relation $y = 3x$ where x is an integer ($x \in I$) and $-6 < x < +6$.

Solution

First, we set up a table of values. This table will give the ordered pairs for the points of the graph.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y											

The x-values are arranged as set out in the question ($-6 < x < +6$ and $x \in I$). Now substitute the values of x into the equation $y = 3x$ and fill in the corresponding y-value in the appropriate blank.

For example:

For $x = -1$

then $y = 3(-1)$

$y = -3$

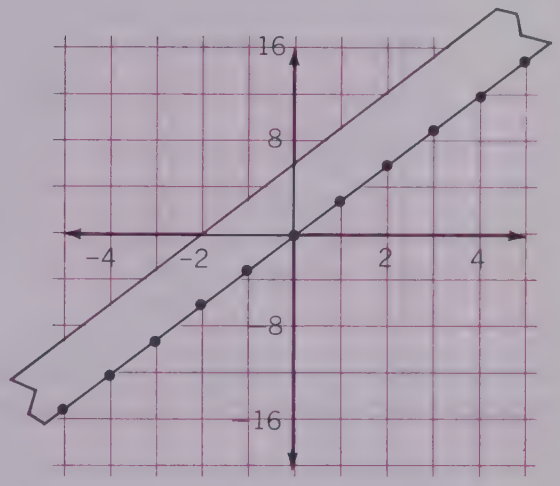
This is the completed table with the values entered for y.

x	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	−15	−12	−9	−6	−3	0	3	6	9	12	15

Do you understand how each y-value was arrived at? If not, ask your teacher for further explanation. The ordered pairs for graphing are

(−5,−15), (−4,−12), (−3,−9), (−2,−6), . . . , (5,15)

The graph of the points looks like this:



If we place a straightedge along the points, we notice that a straight line can be drawn to contain all the points. We say that all the points are on a straight line.

B.C., by permission of John Hart and Field Enterprises, Inc.

I CAME UP WITH A FORMULA TO DISPROVE ALL KNOWN THEORY.
LOSERS ANONYMOUS
WAS IT SUCCESSFUL?
LOSERS ANONYMOUS
I DON'T KNOW, I CAN'T PROVE IT.
LOSERS ANONYMOUS
11-15

Example 2

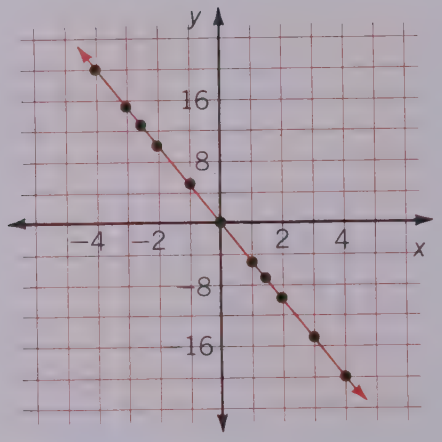
Graph the relation $y = -5x$ where $x \in Q$ and x is from -4 to $+4$.

Solution

We are able to locate only some of the many points on the set of rationals. Another way to show the relation is by means of this mapping diagram.

x	Rule	y
-4	$\longrightarrow$	20
-3	$\longrightarrow$	15
-2.5	$\xrightarrow{-5x}$	12.5
-2	$\longrightarrow$	10
-1	$\longrightarrow$	5
0	$\longrightarrow$	0
1	$\longrightarrow$	-5
1.5	$\longrightarrow$	-7.5
2	$\longrightarrow$	-10
3	$\longrightarrow$	-15
4	$\longrightarrow$	-20

The graph appears as a line since we cannot possibly draw separate dots for all the points represented by the set of rational numbers.

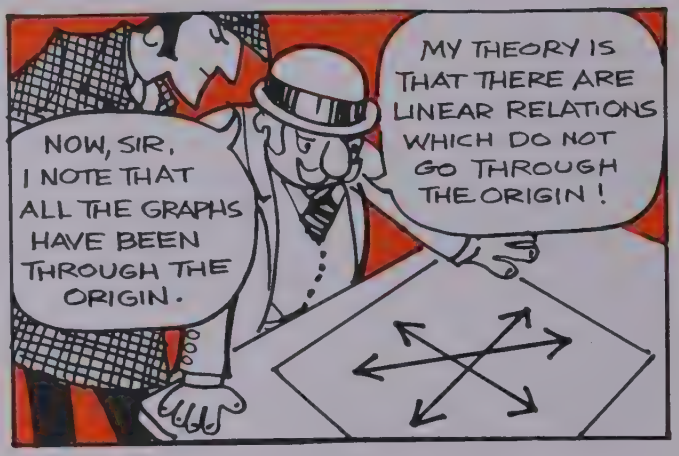


In these two examples, notice that the points are on a straight line. Graphs where points are on a straight line are said to represent a *linear relation*. The graph of the relation $y = ax$ will represent some linear relation when a specific value is given to a in ax.

EXERCISE 7·4

Graph each of the following linear relations on separate grids. Choose about ten ordered pairs for each relation.

1. $y = x$, for $x \in I$ and x is between -11 and $+11$.
2. $y = -2x$, for $x \in I$ and x is between -9 and $+9$.
3. $y = -3x$, for $x \in Q$ and x is between -3 and $+7$.
4. $y = 4x$, for $x \in Q$ and x is between -6 and $+4$.
5. Answer the question at the beginning of Section 7·4.
6. From the graphs you drew, what do you conclude about the direction of the straight line when a is positive? What do you conclude about the direction when a is negative?


NOW, SIR, I NOTE THAT ALL THE GRAPHS HAVE BEEN THROUGH THE ORIGIN.
MY THEORY IS THAT THERE ARE LINEAR RELATIONS WHICH DO NOT GO THROUGH THE ORIGIN!

7·5 Graphs of Other Linear Relations

How do graphs such as $y = 3x - 3$ and $y = -5x + 4$ differ from $y = 3x$ and $y = -5x$?

Did you observe that each of the graphs in Exercise 7·4 was a straight line through the origin? Are there graphs of linear relations which are not through the origin?

Example 1

Graph the linear relation $y = 3x - 3$, for $x \in Q$ and $-4 < x < 6$ (for x between -4 and $+6$).

Solution

The table of values for $y = 3x - 3$ is given to help you.

For $x = -3$,
$y = 3(-3) - 3$
$y = -12$

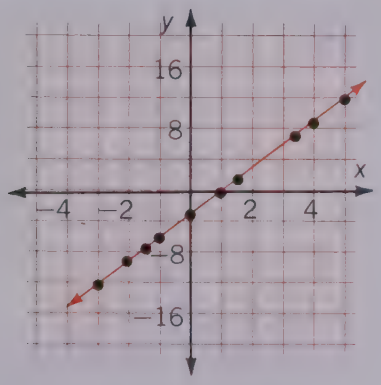
For $x = -2$,
$y = 3(-2) - 3$
$y = -6 - 3$
$y = -9$

For $x = 5$,
$y = 3(5) - 3$
$y = 15 - 3$
$y = 12$

x	Rule	y
-3	→	-12
-2	→	-9
-1.5	$3x - 3$ →	-7.5
-1	→	-6
0	→	-3
1	→	0
1.5	→	1.5
3.5	→	7.5
4	→	9
5	→	12

The graph for the set of values is as shown.

Where does the graph cross the y-axis? the x-axis?



Example 2

Graph the linear relation $y = -5x + 4$, for $x \in Q$ and $-3 \leq x \leq 4$ (for x from -3 to $+4$).

Solution

The table of values for $y = -5x + 4$ is given to help you.

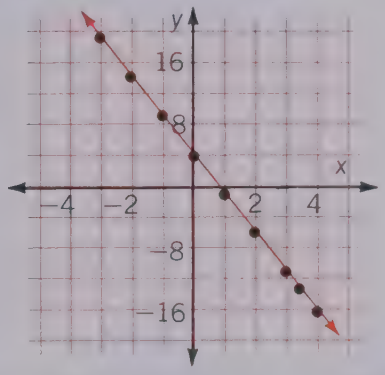
For $x = -3$, $y = -5(-3) + 4$
$y = 15 + 4$
$y = 19$

For $x = 0$, $y = -5(0) + 4$
$y = 0 + 4$
$y = 4$

For $x = 3.5$, $y = -5(3.5) + 4$
$y = -17.5 + 4$
$y = -13.5$

x	-3	-2	-1	0	1	2	3	3.5	4
y	19	14	9	4	-1	-6	-11	-13.5	-16

The graph for the above table of values is as shown.



Where does the graph cross the y-axis? the x-axis?

EXERCISE 7·5

Graph each of the following linear relations.

1. $y = 2x + 1$, for $x \in I$ and $-3 \leq x \leq 3$.
2. $y = -2x - 1$, for $x \in I$ and $-3 \leq x \leq 3$.
3. $y = 2x - 4$, for $x \in Q$ and $-5 < x < 7$.
4. $y = -2x + 7$, for $x \in Q$ and $-4 < x < 9$.
5. $y = 3x + 5$, for $x \in Q$ and $-7 \leq x \leq 4$.
6. $y = 4x - 9$, for $x \in Q$ and $-2 \leq x \leq 6$.
7. $y = -3x - 2$, for $x \in Q$ and $-6 < x < 5$.
8. $y = -5x + 8$, for $x \in Q$ and $-3 \leq x \leq 6$.
9. Answer the question at the beginning of Section 7·5.
10. From the graphs you drew, what conclusion do you draw about the effect the constant part of the relation has on the graph? (The constant is the b in $y = ax + b$.)

7·6 Graphs in Problem Solving

In a biology class, Brian and Muriel took bacteria counts in a culture at regular intervals. In one culture used to test the effect of cold temperatures, the following counts were taken and recorded on a graph.

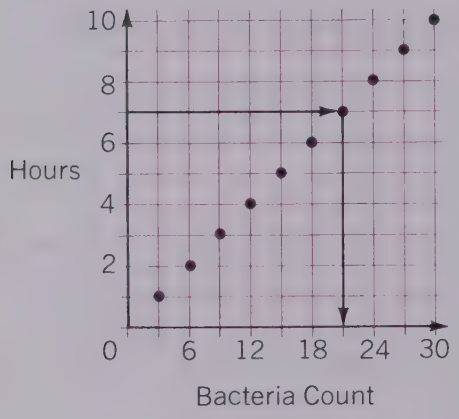


Figure 7-5

Every black dot on the graph is at the intersection of two line segments (one horizontal and one vertical).

To find the bacteria count after seven hours:

(i) Follow the horizontal line from 7 to the dot on it.

(ii) From the dot, follow the vertical line downward to the bacteria count.

1. What was the bacteria count after 7 h?
2. What was the bacteria count after 4 h? After 4.5 h?
3. Are the black dots on the graph in a straight line? What kind of relation do the dots represent?
4. After how many hours would there be a bacteria count of 25?
5. Do you think we should draw a solid line through these points? Why?

Let us look at other ways graphs are of help in solving problems.

Example 1

If a car is travelling at the constant rate of 14 cm/s, graph the distance that car travels for any number of seconds up to and including 20 s.

Solution

The relation between distance and time for a body travelling at a constant rate is:

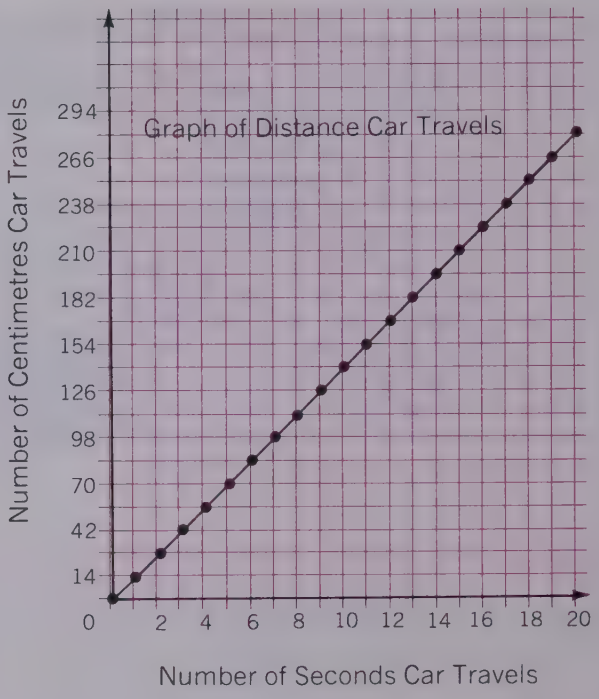
$$D = rt.$$

In this example, $r = 14$ (cm/s).

Part of the table of values is shown.

t	0	1	2		9	10	11	19	20
D	0	14	28		126	140	154	266	280

See the graph.



Do you think we can reasonably draw a line through these points? Why?

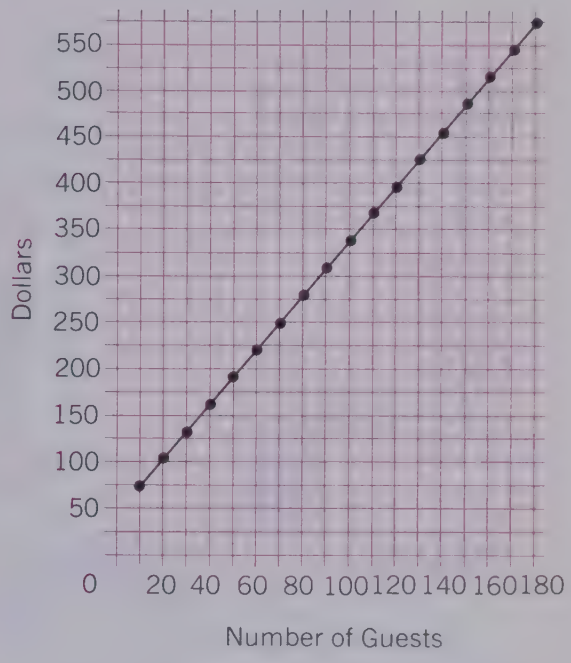
What does it mean when we draw a line through the points?

An Optical Illusion

Look closely at the picture. What do you see? A pretty young lady or an old hag? Keep looking if you can't see both—it may take some time. It is the same in mathematics. It is often necessary to see relationships in a variety of ways. Seeing only one relationship when in fact there are many will restrict your success.

Example 2

A caterer for high school graduation banquets drew up this graph to read quickly the total cost for serving a turkey dinner in the school gymnasium. From his graph, answer the following questions.



(a) Is the graph a linear relation? How can we tell?

(b) What is the approximate cost of serving 90 people? How much does that cost per person?

(c) What is the total cost of serving 130 people? How much does that cost per person?

(d) What does the graph read for 180 people? Is this amount double the cost for 90 people?

(e) Should we draw a line through the points to represent the relation? Why?

Solution

(a) The graph is a linear relation because the points on the graph are in a straight line. (Use a straightedge to check this.)

(b) The approximate cost of serving 90 people is \$310. To find this, go from 90 on the *x*-axis up to the line graph, then go across to the *y*-axis. Here you read the number \$310 (approximately). This amounts to \$3.45 per person.

(c) For 130 people, the approximate total cost is \$425. This works out to \$3.27 per person. Tell how we got this.

(d) The graph reads \$575 for 180 people. This amount is not double that for 90 people. It is less than double. Double the rate for 90 people is:

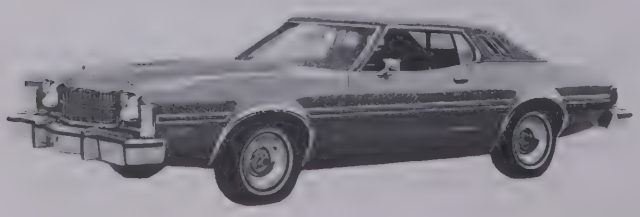
$$2 \times \$310 = \$620.$$

(e) YES. Since we may want to know the cost for 61 guests we will need a line. But we must remember that we do not include the rational numbers such as 52.75. Since 52.75 guests makes no sense in this question, we rule out *Q* and only consider *W*.

EXERCISE 7·6

1. Refer to the graph in Example 2 above.
   (a) Why do you suppose it costs more per person to serve 90 people than 180 people?
   (b) What would be the approximate cost of serving 155 guests? 55 guests? 30 guests? 120 guests? 200 guests?
   (c) Do you think the graph would have any value if it were extended upward to the right? Would it have value if extended downward to the left?

2. Some cars have a device which can be set to maintain a definite speed for as long as needed. Suppose a car is set to travel at 80 km/h on a 240 km trip. Draw a graph which would show the distance travelled for ten-minute time intervals. (See Example 1 for the relation.)
   (a) From your graph, read the distance travelled in 40 min; $1\frac{1}{2}$ h; 2 h 20 min.
   (b) Read your graph to find the time it takes to travel 20 km; 110 km; 160 km.



3. Sandy's class decided to raise some money for charity by selling key chains. They discovered that they can manufacture 6 souvenir key chains per minute. Draw a graph to show how many key chains they produced in 6 h.
   (a) Read your graph to find how many key chains were manufactured in 54 min; 90 min; 144 min; $4\frac{1}{2}$ h.
   (b) Read your graph to find how long it took to manufacture 66 key chains; 120 key chains; 162 key chains.

4. Sandy's class later found they could produce at the rate of 11 souvenir key chains in 18 min. Draw a graph to show how many key chains they produced in 6 h at this rate.
   (a) Read your graph to find how many key chains were manufactured in 54 min; 90 min; 144 min; $4\frac{1}{2}$ h.
   (b) Read your graph to find how long it took to manufacture 66 key chains; 121 key chains; 165 key chains.

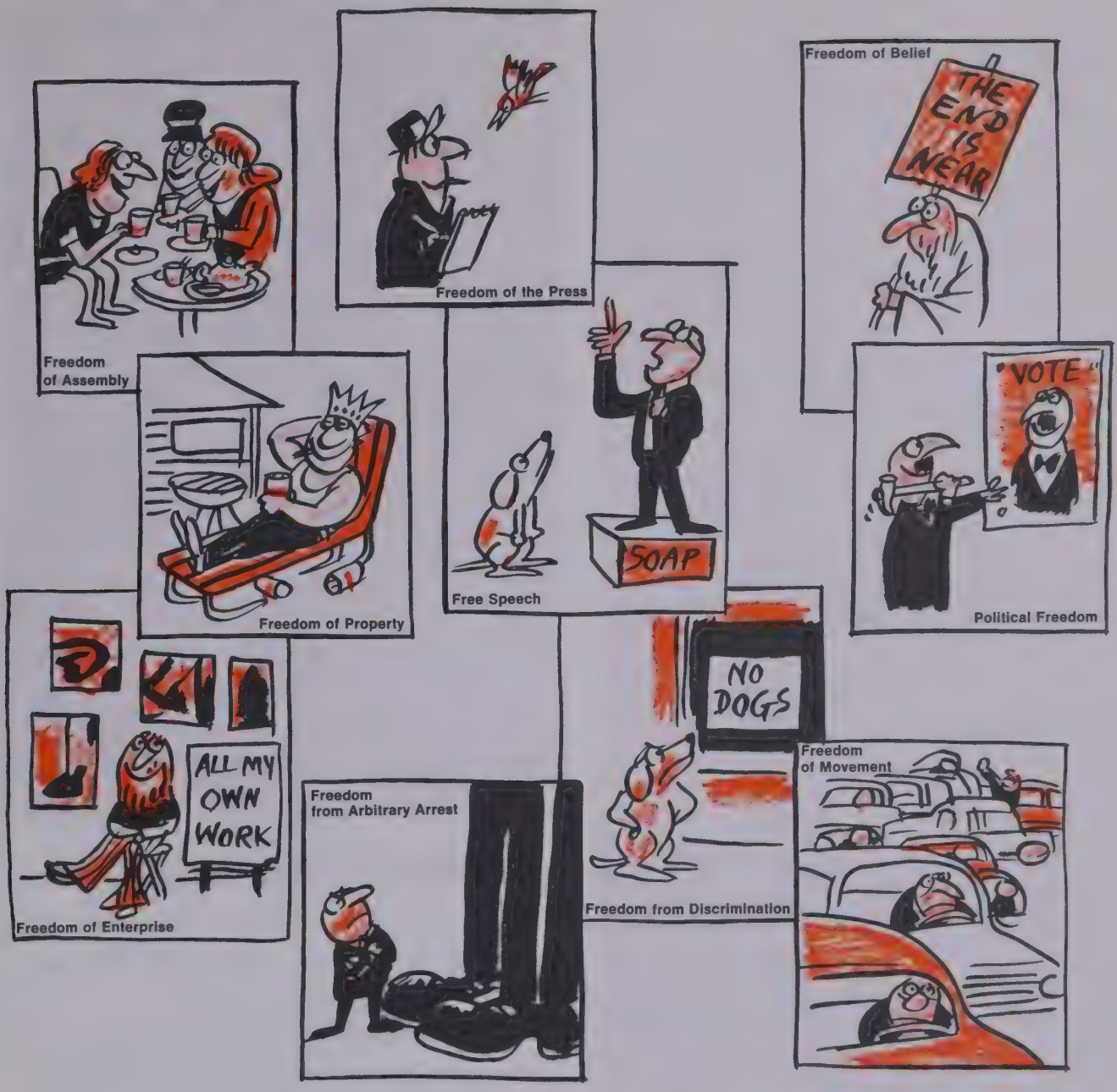
Hint! Set up your table of values like this:

Time in Minutes	0	18	36	54	(Multiples of 18)
Number of key chains	0	11	22	33	(Multiples of 11)

5. A certain quality of art paper sells at 7 sheets for 25¢. Draw a graph to show the selling price of multiples of 7 sheets up to and including 105 sheets. First, write a formula for the relation between cost and the number of sets of 7 sheets.

6. Water running out of a garden hose fills a 3 ℓ container in 2 min. Draw a graph to show this relation up to and including 30 min of flow.
   (a) From your graph, read how long it would take that flow to fill a barrel of 45 ℓ.
   (b) How much water flows from that hose in 14 min? in 22 min?

Freedom under the Bill of Rights



Reproduced by permission from "How Canadians Govern Themselves", by Joseph Schull, Information Canada, Ottawa, 1971.

Rights ⟷ Responsibilities

What is indicated by the Rights ⟷ Responsibilities relationship?

The Bill of Rights assures us of certain freedoms. With each of these rights, there are certain responsibilities. What are our responsibilities to our country?

Project

Project 2

Purpose

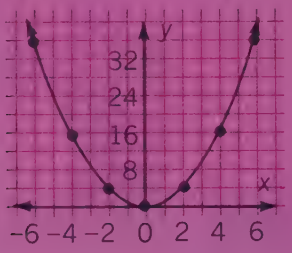
To discover the relationships between the x- and y-values of more relations expressed in graphical form.

Method

Work in groups of four or five for this project.

Part A

1. List the ordered pairs for each of the points marked on the graph.

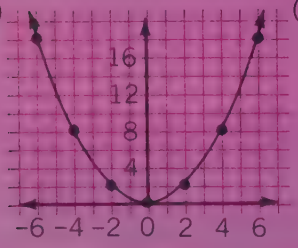


2. In as many ways as you can, describe the relationships between the y-value and the x-value.
3. Record the relationships on the chalkboard.
4. Discuss the relationships in class.
5. Express the relationships in algebraic form using x and y.

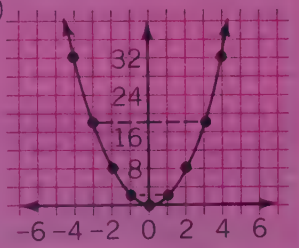
Part B

1. Repeat Steps 1 to 5 of Part A for each of the following graphs. Pick suitable points to list the ordered pairs for each.

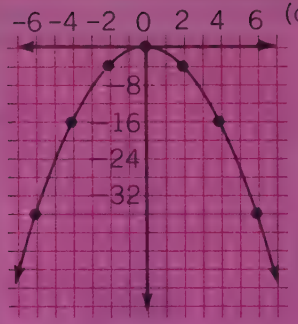
(a)



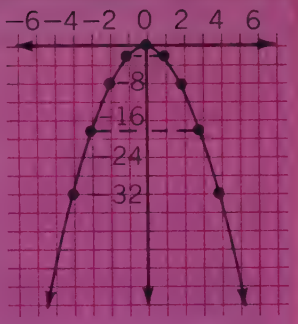
(b)



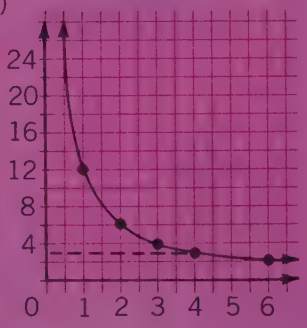
(c)



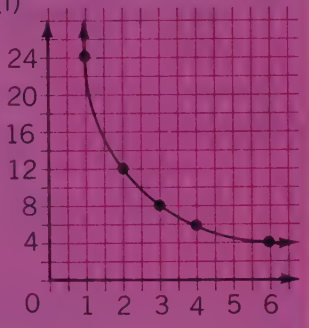
(d)



(e)



(f)



2. How do the graphs (a) to (d) differ from each other?
3. How do the graphs (e) to (f) differ from each other?
4. Use a mirror (or *Reflecto*—see Chapter 8) to determine how many axes of symmetry each of the graphs have.

7·7 Some Other Graphs

If we graph $y = 3x^2$ and $y = \frac{120}{x}$, what kind of lines do we get?

In this section, we will introduce graphs produced by the relations in the forms: $y = ax^2$ and $y = \frac{a}{x}$. Here a is some constant (a definite number like an integer). There are many situations, especially in the study of physics, where we make use of these graphs to aid us in the discovery of new concepts and in finding solutions to new problems.

We will use examples to present these two forms of graphs.

Example 1

Draw the graph of the relation $y = 3x^2$, for $x \in Q$ and $-4 < x < 4$.

Solution

The table of values for this graph is calculated like this:

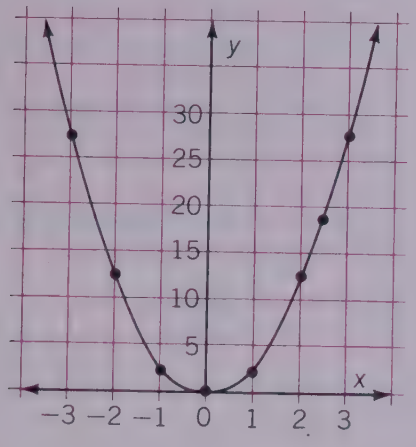
For $x = -3$ $\quad y = 3(-3)^2$, $y = 3(9)$, $y = 27$

For $x = -2$ $\quad y = 3(-2)^2$, $y = 3(4)$, $y = 12$

For $x = 2.5$ $\quad y = 3(2.5)^2$, $y = 3(6.25)$, $y = 18.75$

x	-3	-2	-1	0	1	2	2.5	3
y	27	12	3	0	3	12	18.75	27

After plotting the points on the grid, a smooth curve is drawn through the points.



Do Questions 1 and 2 in Exercise 7·7.

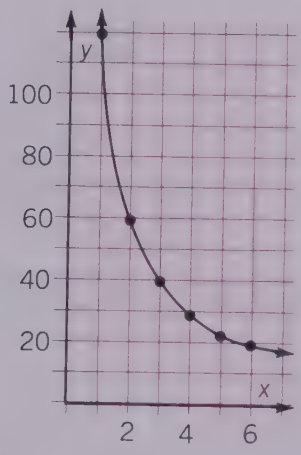
Example 2

Draw the graph of $y = \frac{120}{x}$ for the set x such that $x \in \{1, 2, 3, 4, 5, 6\}$.

Solution

x	1	2	3	4	5	6
y	120	60	40	30	24	20

After plotting the points on the grid, draw a smooth curve through the points.



EXERCISE 7·7

1. (a) Draw a graph of $y = x^2$, for $x \in Q$ and $-5 \leq x \leq 5$.

   (b) Draw a graph of each when, $x \in Q$ and $-5 \leq x \leq 5$.

   $y = 2x^2$ $\quad y = 4x^2$ $\quad y = 5x^2$

   (c) Draw a graph of each when $x \in Q$ and $-5 \leq x \leq 5$.

   $y = -x^2$ $\quad y = -2x^2$ $\quad y = -3x^2$ $\quad y = -5x^2$

2. Discuss the type of graph you obtained for the relation $y = ax^2$.

   (a) What is the "direction" of the graph of $y = ax^2$ when a is positive? when a is negative?

   (b) How do the graphs of $y = ax^2$ vary as a gets larger? when a is greater than zero? when a is less than zero?

3. (a) Draw a graph of $y = \frac{60}{x}$ for the set x such that $x \in \{1, 2, 3, 4, 5, 6\}$. Draw a smooth curve through the points.

   (b) Draw each graph when $x \in \{1, 2, 4, 6, \ldots, 30\}$. Draw a smooth curve through the points.

   (i) $y = \frac{30}{x}$ (iii) $y = \frac{40}{x}$

   (ii) $y = \frac{24}{x}$ (iv) $y = \frac{48}{x}$

   Discuss the types of graphs you get. How do they differ?

4. Graph the following table of values.

x	0	1	2	3	4	5	6	7	8	9
y	0	8	14	18	20	20	18	14	8	0

   (a) Does your graph appear like a representation of the flight of a thrown ball at definite intervals of time?

   (b) Between what two successive intervals is the increase in y the greatest? the least?

   (c) What is the greatest decrease in y for one interval of x?

5. Muriel and Brian made this bacteria count in a second culture to show the effects of medium high temperatures.

Hours	0	2	3	4	5	6	7	8
Bacteria Count	5	20	50	150	400	1100	3000	8000

   Draw this graph. Draw a smooth curve through the points.

   (a) Compare the effect of medium high temperatures with the effect of low temperatures on bacteria growth. Turn to the beginning of Section 7·6 for data on the effect of low temperatures.

   (b) Estimate the bacteria count after 5.5 h.

   (c) When was the bacteria count 5000?

REVIEW EXERCISE 7

1. Join the following points on graph paper in the order given:

   (a) (0,–5) (b) (–3,–4) (c) (–5,–5)
   (d) (–4,–3) (e) (–5,0) (f) (–4,3)
   (g) (–5,5) (h) (–3,4) (i) (0,5)
   (j) (3,4) (k) (5,5) (l) (4,3)
   (m) (5,0) (n) (4,–3) (o) (5,–5)
   (p) (3,–4) (q) (0,–5)

   (i) Give an original name to this design.

   (ii) On the same graph, make big dots at (–2,2), (2,2), (2,–2), and (–2,–2).

   (iii) Also on the same graph, join (–1,1) to (1,–1), and then join (1,1) to (–1,–1).

   (iv) Cover the bottom half of the graph; what do you think you see? Can you see the same image by covering another part? How many ways can it be done?

2. Draw a segment on a graph joining (–6,4) and (6,–2). Find the midpoint by measurement of the segment. Give the ordered pair of the midpoint.

3. On the same graph as Question 2, draw the segment joining (0,–2) and (8,0). At what point do the two segments intersect?

4. On the same set of axes, construct the following graphs:

   (a) $y = x - 5$, for $x \in Q$ and $-2 \leq x \leq 8$.

   (b) $y = -2x + 4$, for $x \in Q$ and $-2 \leq x \leq 5$.

   Is there a point on one graph that is also a point on the second graph?

5. Graph $y = -2x^2$, for $x \in Q$ and $-3 \leq x \leq 3$. Which point of this graph is above (higher than) the others?

6. Graph $y = \frac{100}{x}$, $x \in \{1, 2, 4, 5, 10, 20, 25\}$. Draw a smooth curve through the points.

SHOP TALK

by Mary BINER

Saving energy can be good for your bank balance.

Just back from New York city where the topic of conversation is saving energy.

People I talked to, from writers attending the American Designers Group spring openings to householders, are being extremely conscientious about saving energy as are retailers who have not gone all-out this year in Christmas lighting.

Saving energy also saves money, of course, and in these days of high prices we can save money here and there by using our electrical appliances more prudently and by learning to turn off light switches.

- Save 20*l* of hot water by showering instead of bathing and set the hot water heater at 60°C to 70°C. Use warm-wash, cold-rinse cycles in the washing machine.
- No small loads in the wash machine; pack the laundry in.
- As for the dryer, don't bake the clothes. It takes 10 to 15 minutes for synthetics, 20 to 25 minutes for permanent press and 30 to 40 minutes for towels and similar materials. And, of course, keep your appliances clean.
- Apparently, "you cannot save the world by throwing away your electric toothbrush." Gadgets don't use that much power; it is the appliances that produce heat or cold that eat up the wattage.
- You can save on your stove bill by pre-heating the oven for baking not broiling; thaw frozen foods before cooking; turn off a roast 30 minutes before serving because it can roast on retained heat; and stop peeking in the oven. Each look into the oven drops the temperature 15 degrees.
- Pack the most perishable foods as milk and butter on top back shelf of the refrigerator and keep the freezer full. You may be able to use a lower setting in both appliances if you do this.
- Frost-free refrigerators use twice the power and cost $45 a year more to run. Close the refrigerator door on a piece of paper. If it slides out easily, you need a new gasket.
- One 100-watt bulb burning constantly for one year adds $28 to the electric bill. A 40-watt fluorescent bulb gives more light than a 100-watt bulb, Long Island Lighting reports. It lasts 10 times as long and uses half the energy.
- Your black and white television uses about 60 percent less power than color sets.

The Electric Energy Association has compiled a list of the estimated power consumed by home appliances in a year. I would suggest you clip this and tape it to the kitchen bulletin board. Then do peer at it once in a while just to make sure you are using your appliances as economically as possible.

	Average wattage	Estimated kilowatt hours consumed annually
Food preparation		
Blender	386	15
Broiler	1436	100
Carving knife	92	8
Coffee maker	894	106
Deep fryer	1448	83
Dishwasher	1201	363
Egg cooker	516	14
Frying pan	1196	186
Hot plate	1257	90
Mixer	127	13
Oven, microwave	1500	300
Oven, self-cleaning	4800	1146
Range	8200	1175
Roaster	1333	205
Sandwich grill	1161	33
Toaster	1146	39
Food preservation		
Freezer (0.41 m^3)	341	1195
Freezer (frostless 0.41 m^3)	440	1761
Refrigerator (0.33 m^3)	241	728
Refrigerator (frostless 0.33 m^3)	321	1217
Refrigerator/Freezer (0.38 m^3)	326	1137
(frostless 0.38 m^3)	615	1829
Laundry		
Clothes dryer	4856	993
Iron (hand)	1008	144
Washing machine (automatic)	512	103
Washing machine (non-automatic)	286	76
Water heater (standard)	2475	4219
Water heater (quick-recovery)	4474	4811
Comfort conditioning		
Air cleaner	50	216
Air conditioner (room)	1566	1389
Bed covering	177	147
Dehumidifier	257	377
Fan (attic)	370	291
Fan (circulating)	88	43
Fan (rollaway)	171	138
Fan (window)	200	170
Heater (portable)	1322	176
Heating pad	65	10
Humidifier	177	163
Home entertainment		
Radio	71	86
Radio/record player	109	109
Television (black and white)	237	362
Television (color)	332	502
Housewares		
Clock	2	17
Floor polisher	305	15
Sewing machine	75	11
Vacuum cleaner	630	46

Discuss this article taken from a newspaper. Be critical—what is good and what is poor?

Discuss each recommendation, made or implied, in those sections indicated by the dot•.

How much electrical energy, in kilowatt hours, can you save by adopting various recommendations made by Mary Biner? Which would you be willing to adopt in your home? How much money could you save then?

Using the Library

1. *Reading in Mathematics*. Book 2. Adler, I. Ginn and Company (1972).

   (a) On page 184 a game "As If" is described. For each picture, complete the statement "It looks as if ___." Discuss this with your friends, for you all agree?

   Make up and post on the bulletin board several "as if" pictures for your friends – the funnier the better. Challenge your friends with some different ones.

   (b) Page 185. For each picture, make up a reason or reasons that will explain the conclusion you reached when you looked at the picture. The reason tells what happened in the picture. Discuss your reasons with your friends. Draw a picture and let others in your class make up reasons that will explain the conclusion from your picture. Make up reasons for these.

2. *Story of Mathematics for Young People*. Powers, J.T. Random House Canada Limited (1966).

   Read pages 90-95.

   (a) What effect did artists (and the art world) have on mathematics?

   (b) Use the method on page 93 to project different shadows of a square on the floor. Also try a cube skeleton made from straws and pipe cleaners. Trace each shadow.

3. *The Universe of Galileo and Newton*. Horizon Magazine and Bixby, W. American Heritage Publishing Company Limited (Harper and Row) (1964).

   (a) Read pages 76-77 to learn of the events leading to the "Inquisition" before which Galileo appeared. Why was Galileo required to appear? What was the result?

   (b) What picture in the first half of the book (page 1-84) appeals to you most? Choose one picture and make a few notes that relate to or explain the picture.

   (c) Repeat (b) for the second portion of the book.

   (d) Study the picture on page 115. Describe the Mathematical Bridge.

   (e) What disagreement arose between Robert Hooke and Newton? Who was Halley and what role did he play?

   (f) How were Galileo and Newton alike?

Do You Remember . . .

. . . the metric system?

1. How many marshmallows (or candy mints) are there in one tonne?
2. If one tonne of marshmallows covered a surface one layer deep, what area would be covered?
3. What would be the mass of a cubic metre of marshmallows?
4. What would be the mass of the contents if your classroom were filled with:
   (a) marshmallows?
   (b) salt? (c) water?
5. What does each of these prefixes mean? Give the symbol for each.
   (a) mega (b) micro (c) milli
   (d) kilo (e) hecto (f) deci
   (g) centi (h) deca

Refer to the Appendix if you are not familiar with or are having difficulty with metric units.

Have You Discovered that . . .

. . . it takes time for you to react?

Project

Project 3

Purpose

To look at the speed of human response – mathematically.

Method

Part A

1. Arrange a group of ten people in a circle so that each person's left arm is fully extended and is raised about 10 cm above the shoulder of the person on his left.

2. Student Number 1 starts the stopwatch and simultaneously touches the shoulder of Number 2; Number 2 then quickly touches the shoulder of Number 3; and so on.

3. Student Number 1 notes the time until Number 10 touches his shoulder.

   (Care must be taken that a student does not anticipate the touch and move before he is actually touched. It may be necessary for the students, except Number 1, to close their eyes before starting.)

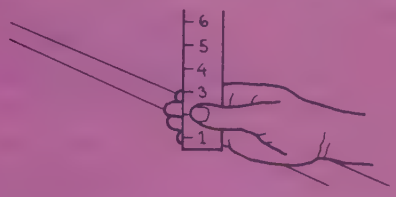
4. Calculate the average reaction time per person.

$$\text{average reaction time} = \frac{\text{time}}{\text{number of persons}}$$

5. Draw a graph to show the time of reaction for 10, 20, 30, . . . , 100 people.

Part B

1. Student A places his hand on his desk with fingers and thumb over the edge – fingers and thumb should be about 1 cm apart.



2. Student B holds a ruler vertically above Student A's hand so that zero of the lower end of the ruler is just between the finger and thumb.

3. Student B releases the ruler.

4. Student A closes his fingers and thumb to catch the falling ruler.

5. Record the number of centimetres the ruler fell before Student A caught it.

6. Use this formula to determine the approximate time for the human response:

   t in seconds; s in centimetres

$$t^2 = \frac{s}{490}$$

Example: If $s = 16$, we get:

$$t^2 = \frac{16}{490}$$

$$t^2 = 0.033$$

$$t \simeq 0.18$$

Check:

$$\begin{array}{r} 0.18 \\ \times\ 0.18 \\ \hline 1\,44 \\ 18 \\ \hline 0.0324 \end{array}$$

Reaction time is about 0.18 s.

7. Graph the reaction time for ten tries or for ten different students. Discuss this graph. What does it tell you? Who reacts the fastest in your class?

8. What does "reaction time" or "speed of human response" have to do with driving a car at high speeds?

9. The relation of $t^2 = \frac{s}{490}$ can also be written $490t^2 = s$. Draw a graph for this relation when $0.5 \leq t \leq 5$. In your own words, write what this graph tells you.

Have You Discovered that . . .

. . . a globe of the earth is marked in a grid consisting of lines of latitude and lines of longitude? Obtain a globe. Learn which are lines of latitude and which are lines of longitude. Locate a city and identify its position by the ordered pair (latitude, longitude). Is there more than one place on the globe with this ordered pair? What else will you need to add for positive identification? Will you have to identify North and South of the Equator? Will you have to identify East or West of the Prime Meridian or International Date Line? This will be an ordered quartet now:
(latitude, North or South of Equator, longitude, East or West of Prime Meridian).

Identify 20 cities with populations over 300 000 by this ordered-quartet method.

Example:

London, England = (50, North, 0, on Prime Meridian)
New York, U.S.A. = (40, N, 75, W)
Sydney, Australia = (35, S, 150, E)

Something to Think About

Let us investigate what happens to the sequence of areas of squares when we keep increasing the side by one unit. Complete the following table of values, where S is the side of the square and A is the area of the square.

S	1	2	3	4	5	6	7	8	9	10
A										

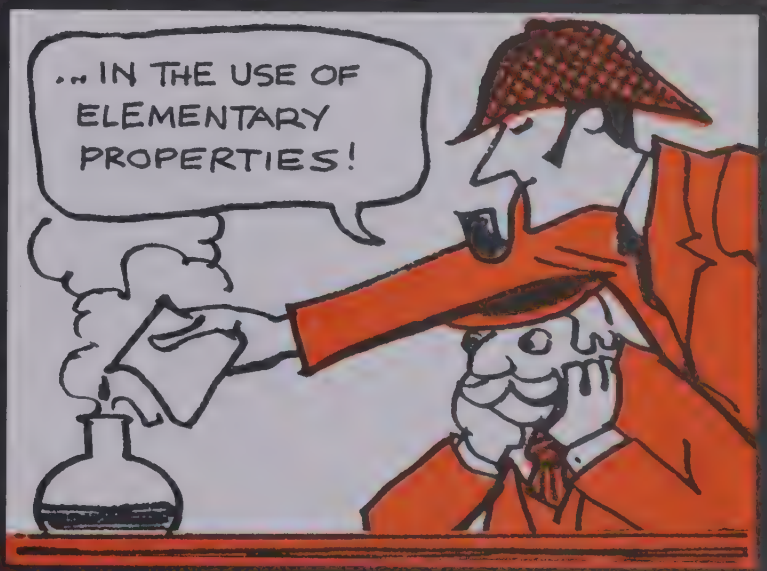
After you have completed a table of values, place the ordered pairs on a graph. Let the S values be on the x-axis and the A values on the y-axis.

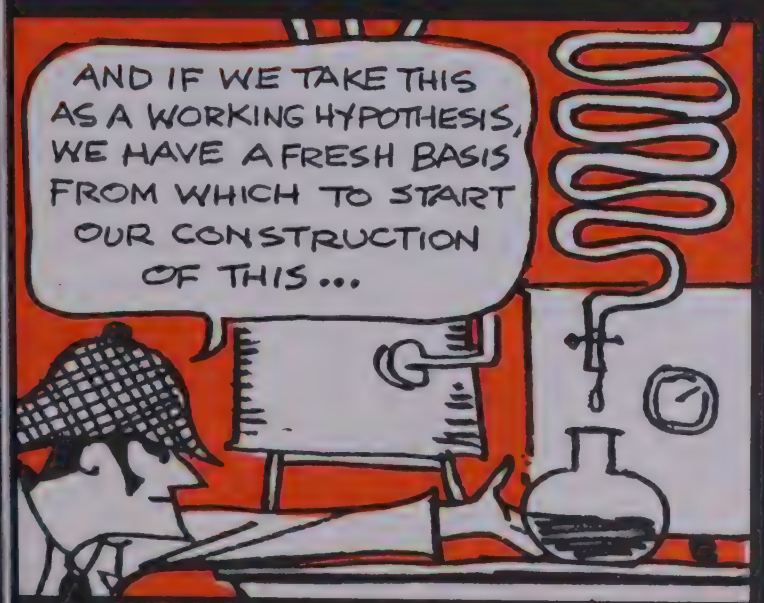
Is the graph an example of a linear relation? Explain.

Find the differences between the successive areas from the table of values. For example, from 1 to 4 the difference is 3; from 4 to 9 the difference is 5.

What do you observe about the differences when taken in order?

Chapter Eight
A NEW LOOK HELPS
Transformations

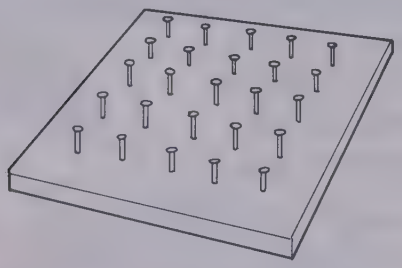




8·1 Geo-boards and Geo-paper

Are you acquainted with the geo-board? Those of you who aren't should refer to Figure 8-1. Figure (a) shows a square board of 1 cm plywood about 20 cm by 20 cm. Finishing nails hammered in at 3 cm intervals form a 25-nail square, 5 nails to a side. A square pegboard with pegs in the holes can also make a geo-board (see Figure (b)). Colored elastic bands complete the parts necessary for geo-board study.

(a)



(b)

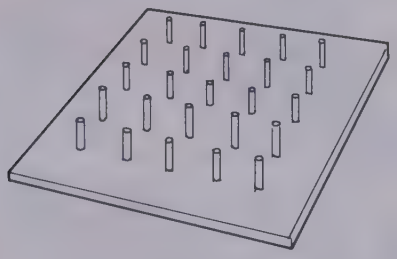


Figure 8-1

If you do not have geo-boards to work with, prepare a stencil of geo-paper from which you can duplicate the required number of sheets. A picture of geo-paper is in Figure 8-2.

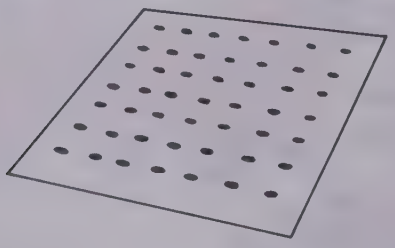


Figure 8-2

EXERCISE 8·1

1. Construct segments of varying lengths by using different colored elastic bands on a 5 by 5 geo-board (or by using colored pencils on 5 by 5 geo-paper). How many segments can you construct?

   The following illustrations (Figure 8-3) show how two different segments are constructed.

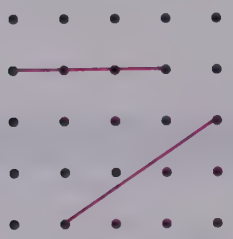
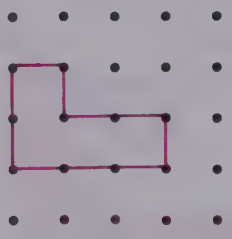


Figure 8-3

2. Construct squares of different sizes on a 5 by 5 geo-board or on geo-paper. How many different sizes of squares can be constructed? There seem to be 8 squares. Is this correct? If you can't get 8, try this question again after completing Questions 6, 7, and 8.

3. Construct rectangles of different sizes on a 5 by 5 geo-board or on geo-paper. How many rectangles can be constructed? (Is a square a rectangle?)

4. Construct a figure like the one in the illustration. How many figures of the same size can be constructed on a 5 by 5 geo-board so that no figure overlaps another?



5. Copy this figure on a 5 by 5 geo-board. Construct similar figures in each corner of the board so that the point is always in the corner.

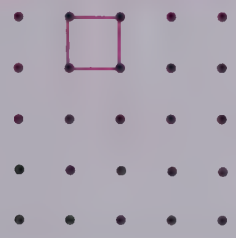
(a) What shape is the figure in the middle?

(b) What are its dimensions?

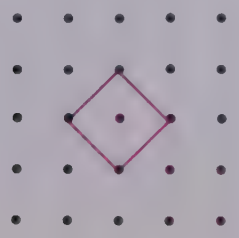
(c) What is its area?

6. Construct a square that is twice the area of the one in the diagram by using elastics on a geo-board or by joining points on geo-paper.

Hint! Draw a diagonal for the given square. Each triangle is then half the square. Put four such triangles together to form a square.

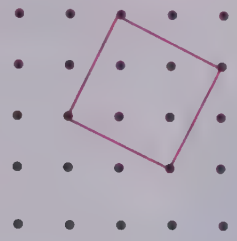


7. Repeat the instructions for Question 6 but use the square in this diagram. Use the Pythagorean Relationship to determine the length of each side of the square. Can you calculate the area of the square by some means other than counting?



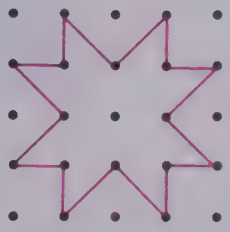
8. Using the square unit of the geo-board as the unit of area, find the area of the square in the diagram by counting. Be prepared to explain how you arrived at your answer.

Hint! A diagonal of a rectangle divides the rectangle into two halves. What is the length of each side of the square? What is the area of the square (by calculation, not by counting)?



9. (a) What is the area of the figure in the diagram?

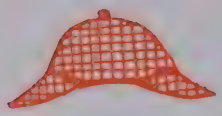
(b) What is the perimeter of the figure? Do you recall how to find the hypotenuse of a right triangle?



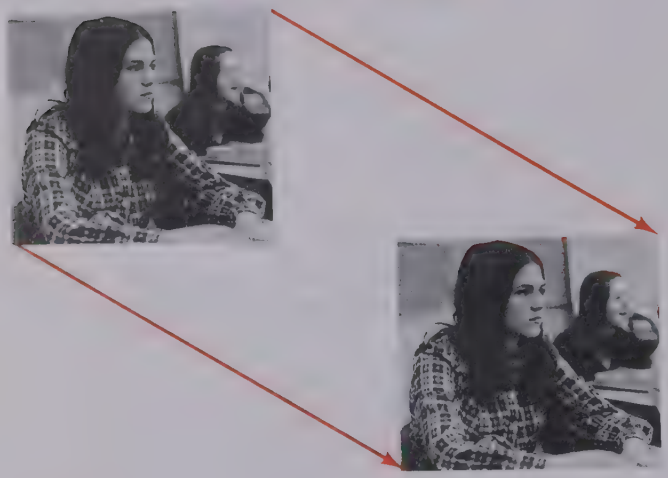
10. Construct a square with an area of ten square units on a geo-board or on geo-paper.

8·2 Translations

Have you observed how one moves a checker or chess piece from one square of the board to another square? Were you ever asked to move your desk or chair forward, sideways, or backward in the classroom? In opening or closing a sliding window or sliding door, how did the window or door change its position? The above are some of the many examples of what are called *translations*.

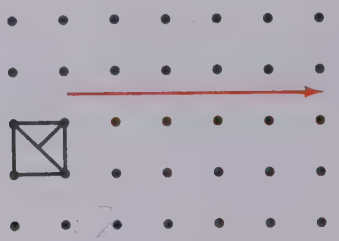


A *translation* is a shift of an object along a straight line for a certain distance without turning the object during the move.

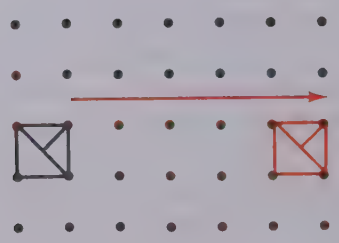


Example 1

Given the position of the object in the diagram, show a translation of five units to the right.

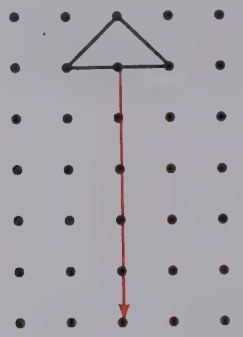


Solution

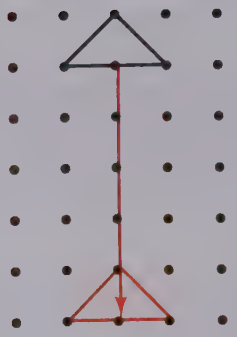


Example 2

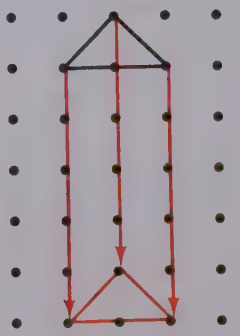
Describe the translation of the object in this diagram. The translation is defined by the length and direction of the arrow.



Solution

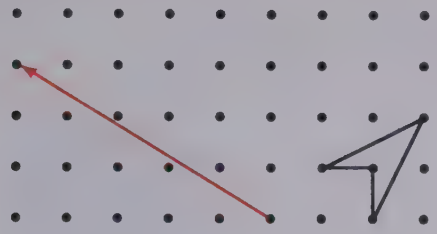


Observe that each vertex of the triangle is shifted in the same direction and by the same distance in this translation. The length of the arrow indicates the distance.

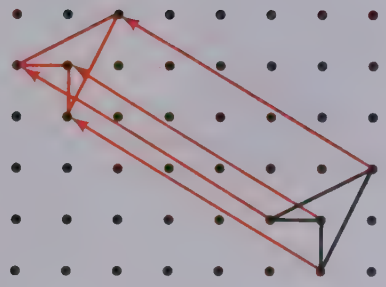


Example 3

Show the position of the object after the indicated translation.



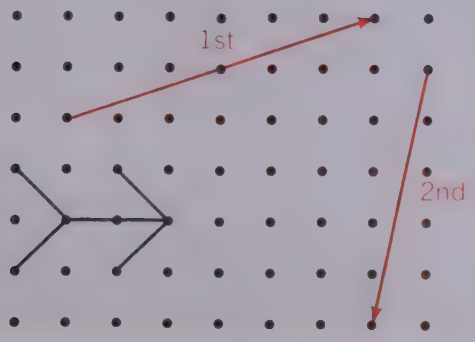
Solution



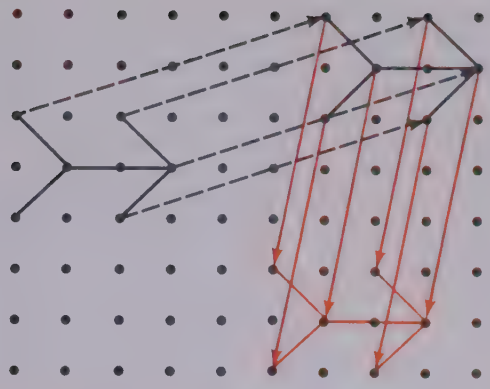
Note! First determine the direction and distance of the shift by counting from the tail to the tip of the arrow. This translation is equivalent to "left 5, up 3". Then translate each point of the object in the same manner. (Would counting the divisions to the left and then up, give the same result?)

Example 4

Perform the two translations indicated in the following diagram.



Solution

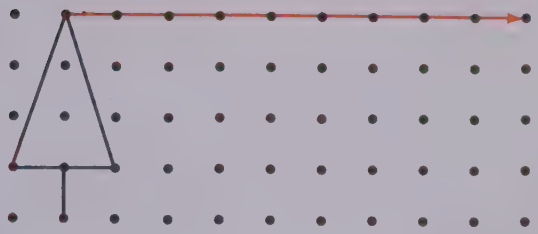


EXERCISE 8·2

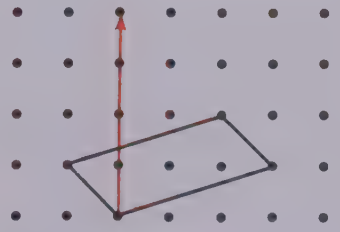
1. Draw each of the following translations on geo-paper.

(a)

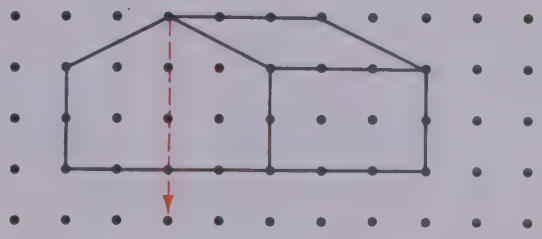
(b)



(c)



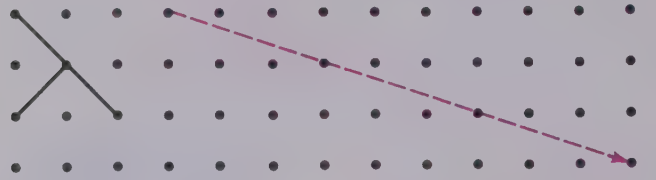
(d)



2. Complete the following translations.

(a)

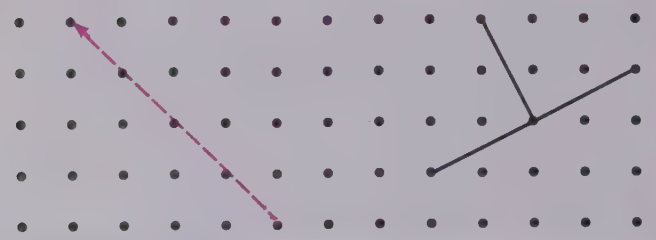
(b)



(c)

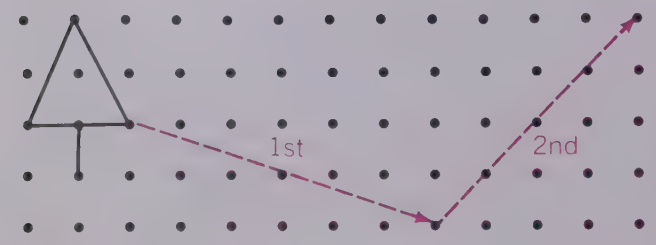
(d)

(e)

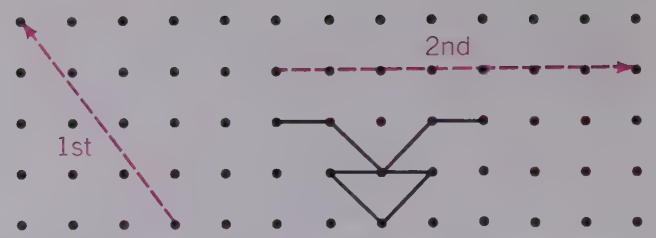


3. Diagram the first indicated translation and then the second.

(a)



(b)



4. For Questions 3(a) and (b), draw a directional arrow to indicate one translation that would equal the sum of the two shown.

5. Compare the size and shape of the object before and after each translation in Questions 1 and 2. What do you observe about the effect of a translation on the shape and size of a figure?

6. Compare the size and shape of each object before and after the two translations in Question 3. Write a statement of your observations. What one word describes the relation of the two shapes?

7. Explain this statement:

"translation + translation → translation."

8. Do you think that three translations can be illustrated by a single translation? Can you illustrate this on geo-paper?

8·3 Reflections

We see reflections when we look into the still water of a pool, or into a shiny chrome-plated surface, or into a plain mirror.

Depending on our position with respect to the reflecting surface, we either see ourselves or we don't. How should a person position himself with respect to a mirror to see his reflection in the mirror?

Project

Project 1

Purpose

To determine:

(a) the image size and shape with respect to object size and shape;

(b) the image distance with respect to object distance;

(c) the image appearance with respect to object appearance in flat-surfaced reflections.

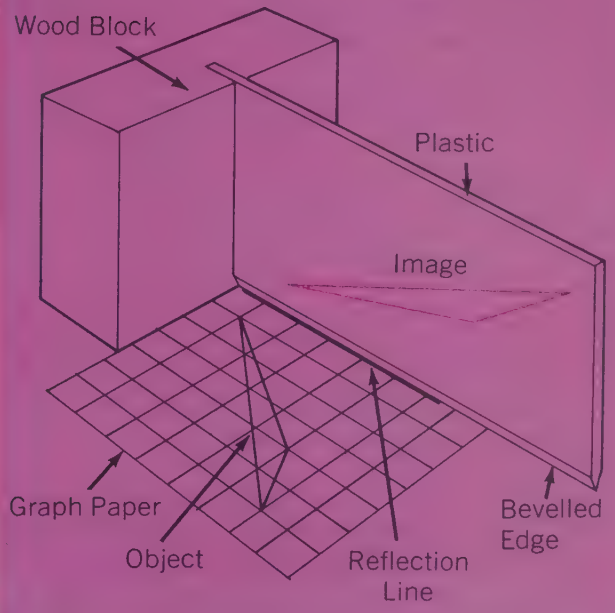
Materials

Obtain a piece of colored transparent plastic about 3 mm thick and 8 cm by 15 cm. Red is the best color. Make a wooden block to hold the plastic *vertical* to the desk. We will call this instrument *Reflecto*.

A similar instrument is known by the commercial name, MIRA. Some department stores have similar items available in their toy departments.

Method

1. Place *Reflecto* on a sheet of graph paper with a scalene triangle drawn on it. Experiment with *Reflecto* and describe the different images of this triangle you can see in *Reflecto*.



A transparent device called a Reflecto *illustrating a reflection. Always work with the bevelled edge towards you.*

Figure 8-4

2. Draw a heavy line on graph paper to divide the paper into two parts. This line will be called the reflection line.

3. Draw an arrow on one side of the paper as in Figure 8-5.

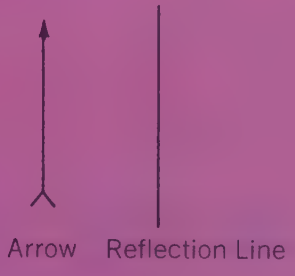


Figure 8-5

4. Turn the paper so that the reflection line runs from side to side in front of you and the arrow is on the side nearest you.

5. Position *Reflecto* with the notched edge on the heavy line.

6. Observe the reflection of the arrow in *Reflecto*. Trace the image with a pencil on the other side of *Reflecto* (as in Figure 8-6).

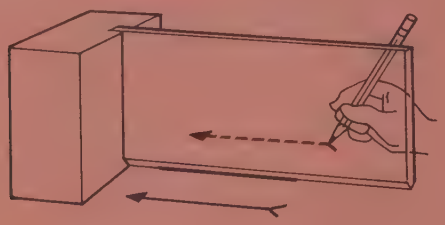


Figure 8-6

7. Compare (a) the size, (b) the shape, and (c) the position of the image with that of the object.

8. How does the distance of the object from the reflection line compare with the distance of the image from the reflection line?

9. Draw a line segment from the tip of the object to the tip of the image. Repeat for the tail. Describe the angle each segment makes with the reflection line.

10. Repeat Steps 2 through 9 but with an object as shown in Figure 8-7. The arrow is now pointing into the reflector.

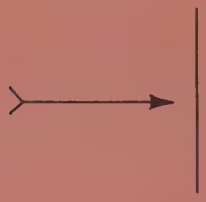


Figure 8-7

11. What do you observe about the direction of the image arrow with respect to the direction of the object arrow for the two sets of drawings? Can you explain the apparent difference?

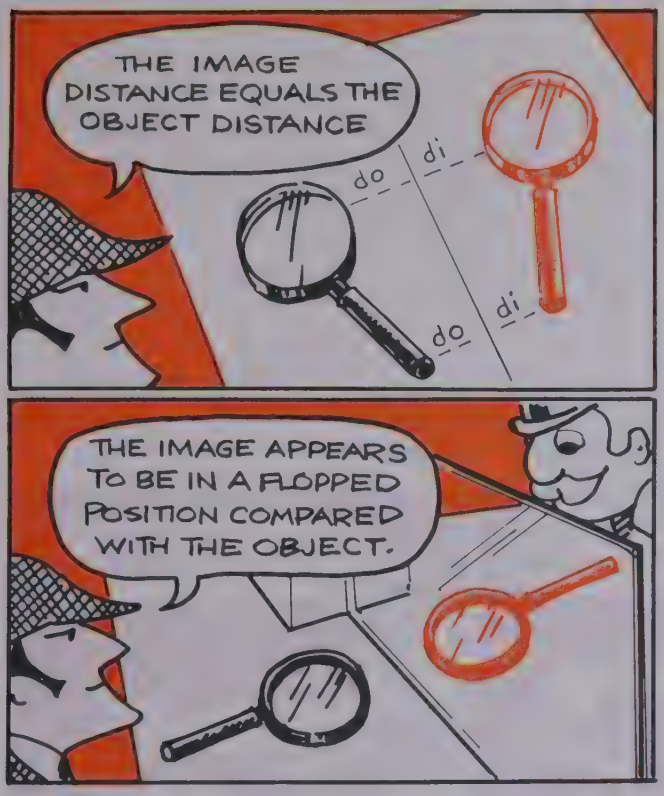
12. Repeat Steps 2 through 9 using five more shapes. Do your conclusions hold true?

Did you come up with the following conclusions?

(a) The image is equal in size and shape to the object. The image and object are congruent.

(b) The image is as far away from the reflection line as the object is. In other words, the image distance equals the object distance. The image is always on the opposite side of the reflection line from the object.

(c) When viewing the drawings of objects and images from left to right, they are the same; but they are inverted when their vertical relationship is compared.


THE IMAGE DISTANCE EQUALS THE OBJECT DISTANCE
do
di
do
di
THE IMAGE APPEARS TO BE IN A FLOPPED POSITION COMPARED WITH THE OBJECT.

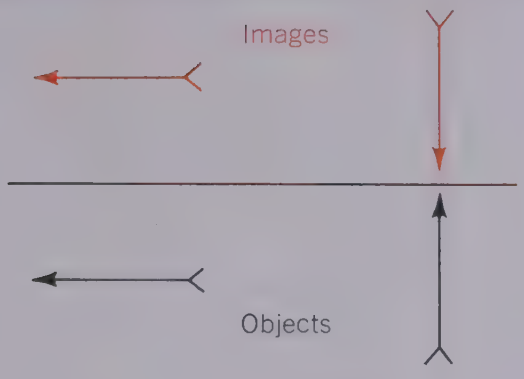
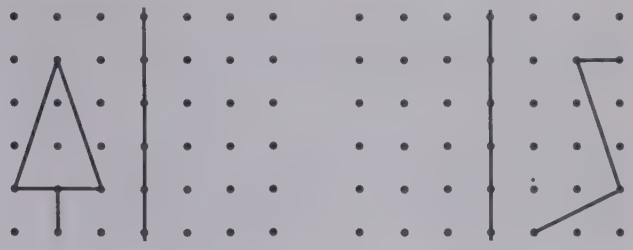


Figure 8-8

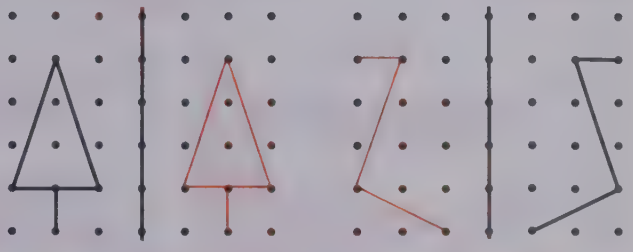
When you see yourself in the mirror, is your image right side up or upside down? When you are in front of a mirror and put your right hand to your right ear, does the image show the right hand touching the right ear? What does your image appear to do in the reflection?

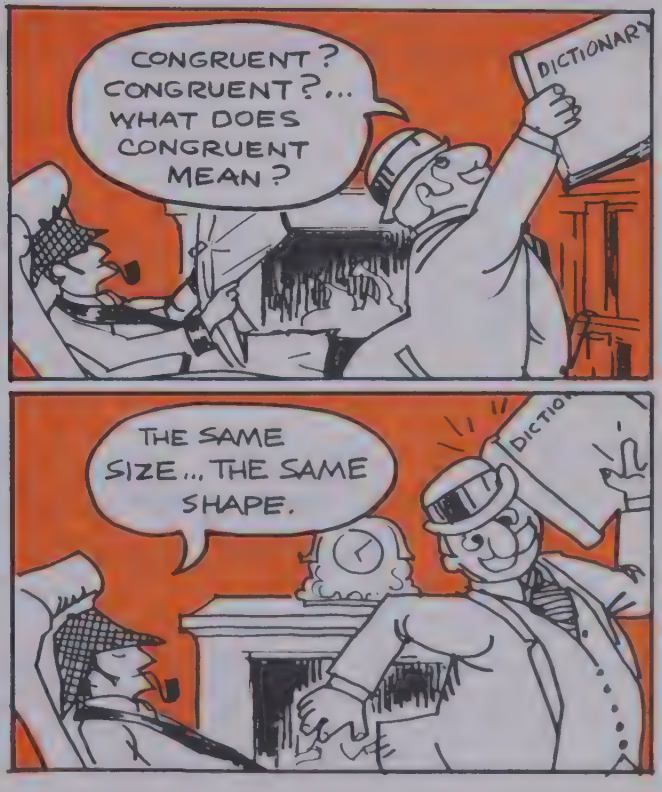
Example 1

Show the image of each of the following objects opposite the indicated line of reflection.



Solution

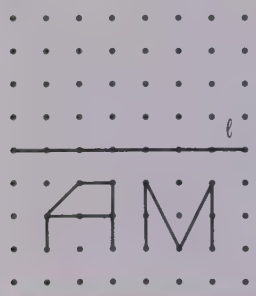




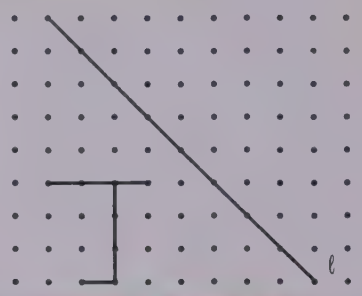
Example 2

Draw the images of the objects opposite each reflection line.

(a)



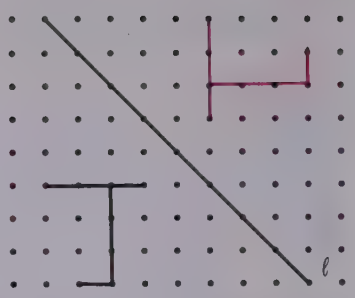
(b)



Solution

(c)

(d)



Check these examples for the three observations and conclusions from page 182.

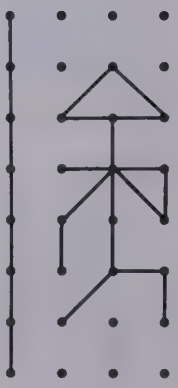
EXERCISE 8·3

1. Copy each of the following (either on geo-paper or by tracing) and complete the drawings to show the images. Use *Reflecto* if necessary.

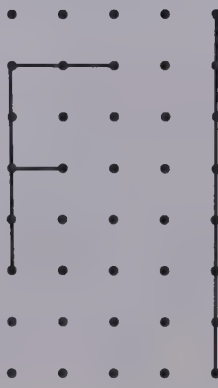
(a)

(b)

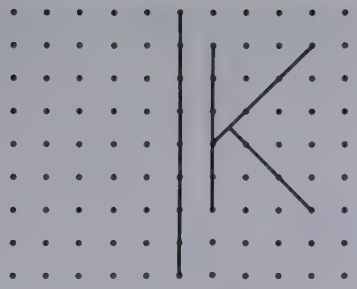
(c)



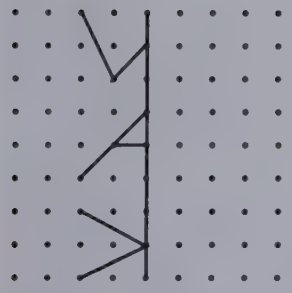
(d)



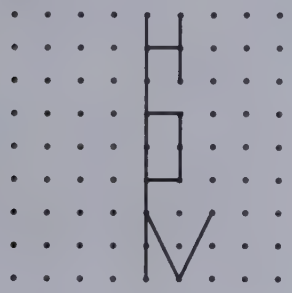
(e)



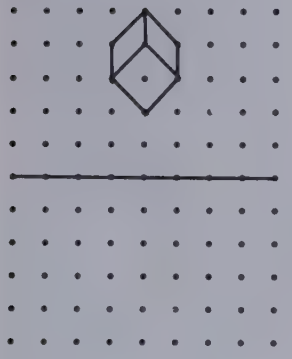
(f)



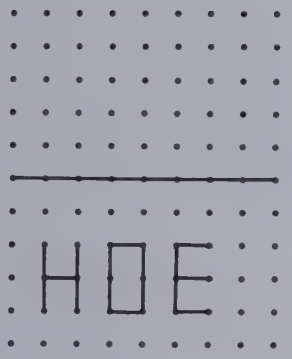
(g)



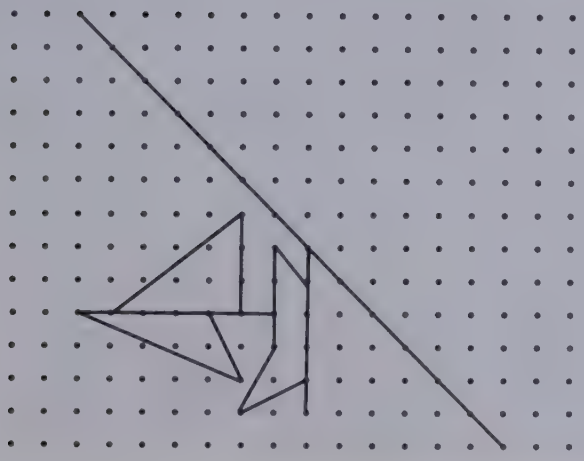
(h)



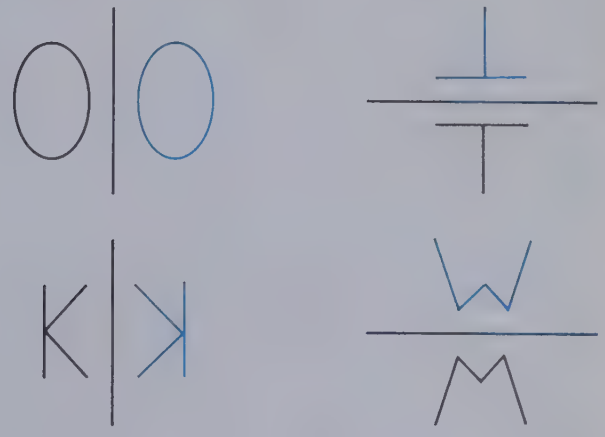
(i)



(j)



2. For which letters of the alphabet do the images appear the same as the objects when the mirror (reflection) line is on the right of the object? Which appear the same when the reflection line is above the object?
3. For which letters of the alphabet do the images appear inverted when the mirror line is above the object?
4. For which letters of the alphabet do the images appear reversed when the mirror line is on the right of the object?
5. In which cases do the images appear as different letters of the alphabet when the mirror line is:
   (a) above the object?
   (b) on the right of the object?
6. Discuss the ten digits in the same way we discussed the letters of the alphabet in Questions 2 to 5.



8·4 Using Reflections and Reflecto

Do you ever have need to find the midpoint of a segment, or to draw a line perpendicular (at right angles) to another line, or to bisect an angle? We will now learn some easy ways to do these things.

Projects

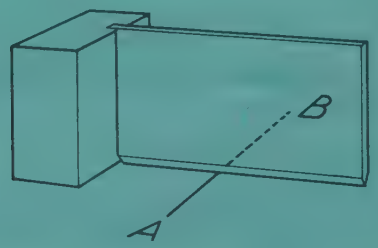
Project 2

Purpose

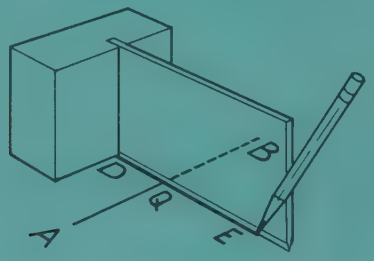
To locate the midpoint of a segment *AB*.

Method

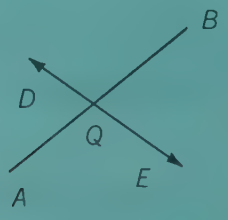
1. Draw segment *AB*.
2. Place *Reflecto* across the segment.



3. Move *Reflecto* until point *A* is mapped onto point *B* (this means that the reflection of point *A* must appear to fit right on point *B*).
4. Draw a line *DE* along the bevelled edge to cut segment *AB* at *Q*.



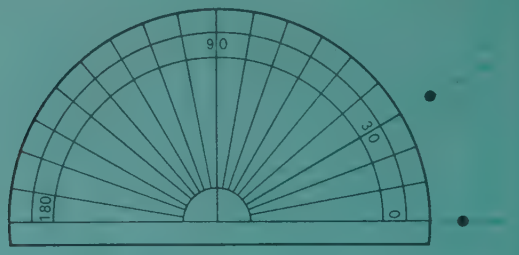
5. Line *DE* bisects segment *AB* at *Q*. Use a ruler to check your work.



STOP

Do Questions 1 to 4 in Exercise 8·4.

We use a protractor to measure an angle.



The measure of $\angle ABC$ is 30°.

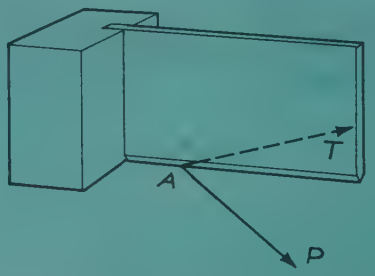
Project 3

Purpose

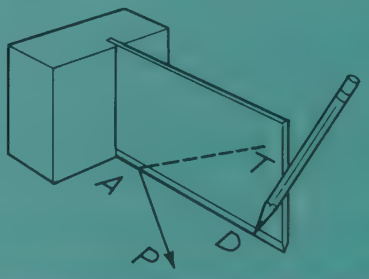
To bisect an angle *TAP*.

Method

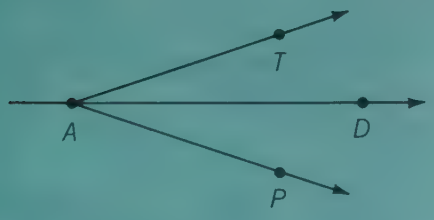
1. Draw an angle *TAP*.
2. Place *Reflecto* across the angle approximately where the bisector will be.



3. Move *Reflecto* until the arm *AP* maps onto the arm *AT* (that is, until the image of arm *AP* appears to lie on top of arm *AT*).
4. Draw a line *AD* along the bevelled edge.



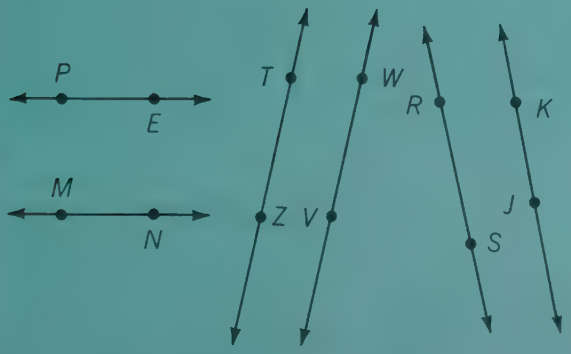
5. Line *AD* bisects the angle. Use your protractor to check your work.



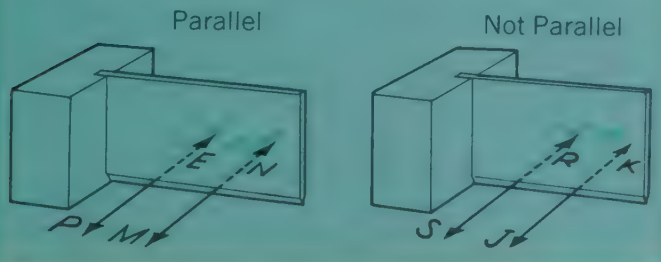
STOP

Do Questions 5 to 9 in Exercise 8·4.

Parallel lines are the same distance apart—they never meet—no matter how far the lines are extended.



We can easily test whether or not lines are parallel by using *Reflecto*. Place *Reflecto* across a pair of lines. Map one line onto itself. Does the other line map onto itself? If it does, the two lines are parallel. If it does not, the lines are not parallel.

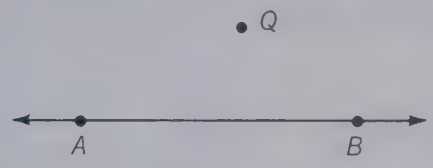


EXERCISE 8·4

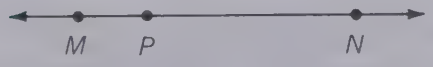
1. Draw three different line segments.
   (a) Use *Reflecto* to locate the midpoint of each.
   (b) What does *bisect* mean?

2. Refer to the constructions in Project 2 (page 186) and to Question 1 immediately above.
   (a) Describe the angle formed by the segment and the line which bisects the segment.
   (b) What does *perpendicular* mean? Is the line *DE* in Project 2 a perpendicular of line *AB*? Explain.

3. Draw a line *AB* with a point *Q* not on the line. Construct the line through *Q* that is perpendicular to line *AB*. Check your work.



4. Draw a line *MN* with a point *P* on the line. Construct a line perpendicular to line *MN* through point *P*.



5. Draw each of these angles.
   (a) $\angle ACE$ with a measure of 50°.
   (b) $\angle DOT$ with a measure of 84°.
   (c) $\angle PAM$ with a measure of 124°.

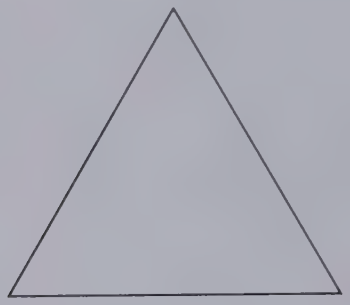
6. Use *Reflecto* to construct the bisector of each angle in Question 5. Check your constructions with a protractor.

7. Draw a right angle and bisect it. What is the measure of each new angle?

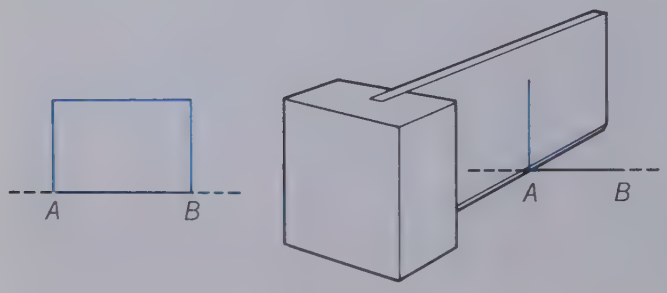
   Now bisect one of the new angles. What are the measures of the newest small angles?

8. Trace this equilateral triangle (*equilateral* means that all sides are of equal length). Measure each angle. What do you notice?

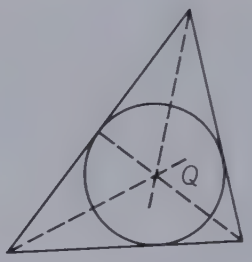
   Now bisect each angle. What is the measure of each new angle? How many congruent triangles have been formed?



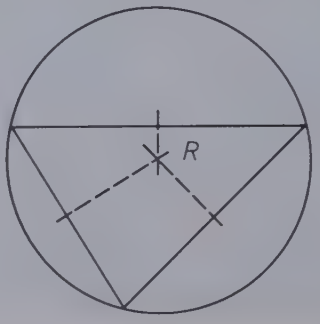
9. Draw a segment *AB*.
   (a) Use *Reflecto* to draw a rectangle having the segment *AB* as its length.
   (b) Draw a second segment *AB*. Use *Reflecto* to draw a square with sides the length of segment *AB*.



10. Draw a triangle.
    (a) Bisect each angle. If the bisectors meet at one point *Q*, your work has been done well.
    (b) Use your compass and the centre *Q* to draw a circle that just touches each of the three sides.



11. Draw a triangle.
    (a) Construct the perpendicular bisector of each side. If the bisectors meet at one point *R*, your work has been done well.
    (b) Use your compass and the centre *R* to draw a circle that just touches each vertex of the triangle.

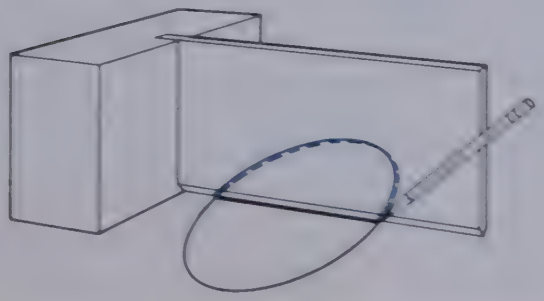


12. Use *Reflecto* to determine which of these lines are parallel.

13. Draw a line ℓ_1. Use *Reflecto* to draw four other lines perpendicular to ℓ_1. Label them ℓ_2, ℓ_3, ℓ_4, and ℓ_5. Describe the relationship of the four lines to each other.

14. Draw two parallel lines ℓ_1 and ℓ_2. Draw a third line, ℓ_3, so that it cuts across ℓ_1 and ℓ_2. Describe the angles formed by ℓ_3 with ℓ_1 and ℓ_2. (Use your protractor.)

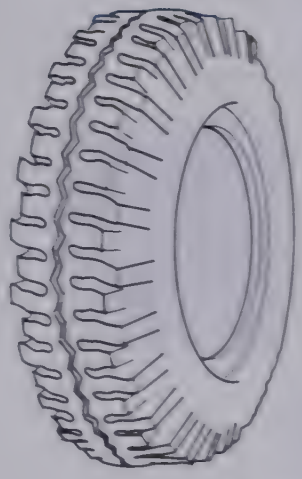
15. Use a compass to draw a circle.
    (a) Place *Reflecto* across the circle.
    (b) Map one half of the circle on the other half.
    (c) Draw a segment across the circle along the bevelled edge.
    (d) Move *Reflecto* to a new position and repeat steps (b) and (c).
    (e) Tell all you can about what you observe.

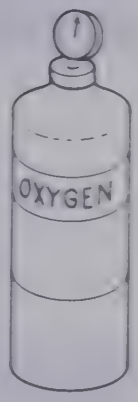


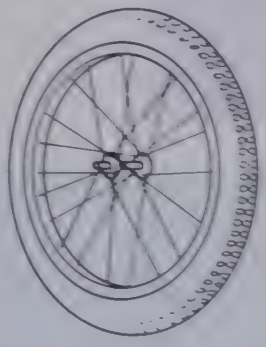
16. Repeat Question 15 for two other circles. Does it always work?

17. Carefully trace this arc. Find the centre of the circle of which this arc is a part. Complete the circle using a compass.

Pressure







Normal atmospheric pressure at sea level is 100 kPa.
Pressure in a bicycle tire is about 200 kPa.
Pressure in an oxygen bottle is 25 kPa.
Pressure in a big truck tire is about 600 kPa.
Pressure due to water alone at the bottom of a pool 3 m deep is about 30 kPa.

8·5 Rotations

Rotation in mathematics means the same as the *rotation* we speak of in everyday life. We may simply say that a rotation is a circular movement around a centre point. Various examples of this can be found in ordinary situations; for example, the movement of the hands of a clock, the movement of a merry-go-round, or the spinning of a top.

Project

Project 4

Purpose

To study the properties of rotations (quarter, half, three-quarter and full rotations).

Method

1. Prepare a strip of cardboard (or stiff paper) about 20 cm long and 3 cm wide.
2. With a pin or thumbtack, pin one end of the strip of paper over graph paper using a drawing board or other suitable backing (see Figure 8-9). The location of the pin is called the point of rotation.

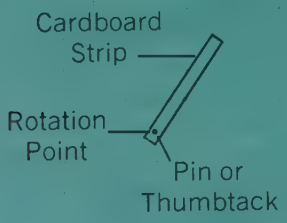


Figure 8-9

3. Fasten a cutout of an arrow to the other end of the strip by using masking tape or a staple. Mount the arrow so that it points to the right when the strip is straight up (see Figure 8-10). Now the paper strip and the arrow are free to rotate like a hand of a clock.
4. Let the starting position be as shown in Figure 8-10. Sketch the direction of the cutout arrow on a sheet of paper.

Figure 8-10

5. Rotate the paper strip $\frac{1}{4}$ turn clockwise. On the sheet of paper, sketch the cutout arrow in that position.
6. Rotate the paper strip another $\frac{1}{4}$ turn clockwise. This is now a $\frac{1}{2}$ turn from the starting position. Sketch the cutout arrow in that position.
7. Rotate the strip still another $\frac{1}{4}$ turn clockwise. This should be $\frac{3}{4}$ of a turn from the start. Sketch the cutout arrow in this position.

Questions

1. Which way is the arrow pointing after a clockwise rotation of $\frac{1}{4}$ turn from the starting position? Which way is it pointing after a rotation of $\frac{1}{2}$? After a rotation of $\frac{3}{4}$? After a rotation of $\frac{4}{4}$?

2. Suppose that the arrow was rotated $\frac{1}{4}$ turn counterclockwise from the starting position. How much of a clockwise rotation would this be equivalent to?

3. Is a $\frac{1}{2}$ rotation clockwise equivalent to a $\frac{1}{2}$ rotation counterclockwise?

4. What rotation counterclockwise is the same as a $\frac{1}{4}$ rotation clockwise?

5. Pin the tail of the arrow at the rotation point and place the head of the arrow at the starting position as in Figure 8-11. In what direction will the arrow point after a $\frac{3}{4}$ counterclockwise rotation? After a $\frac{1}{2}$ counterclockwise rotation? After a $\frac{1}{4}$ clockwise rotation? After a $\frac{3}{4}$ clockwise rotation?

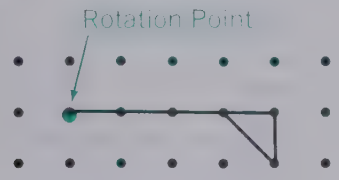
Figure 8-11

6. Is the result of five full rotations the same as two full rotations? Is it the same as one rotation? Does it matter whether the rotations are clockwise or counterclockwise, as long as they are full and not partial rotations?

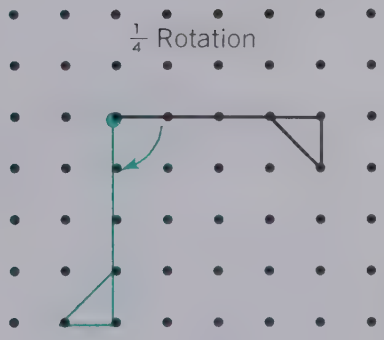
7. To what clockwise rotation is each of these equivalent?
   (a) $3\frac{1}{2}$ clockwise rotations
   (b) $1\frac{1}{4}$ clockwise rotations
   (c) $2\frac{1}{4}$ counterclockwise rotations
   (d) $5\frac{3}{4}$ counterclockwise rotations

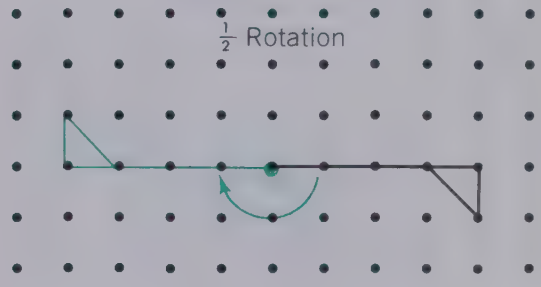
Example 1

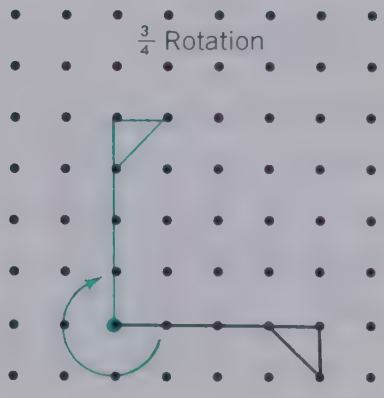
Draw the result of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ clockwise rotations for the following:



Solution

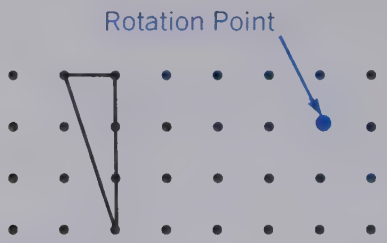




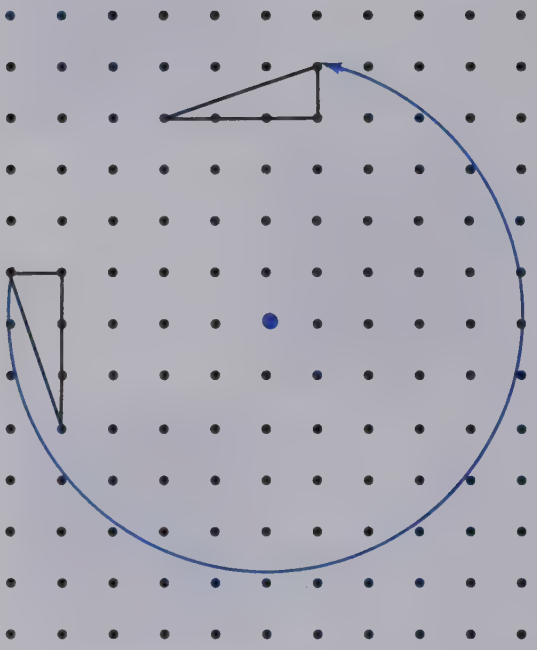


Example 2

Show the result of a $\frac{3}{4}$ counterclockwise rotation for the following:

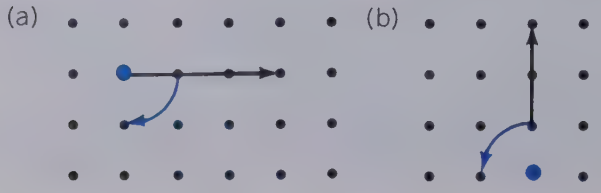


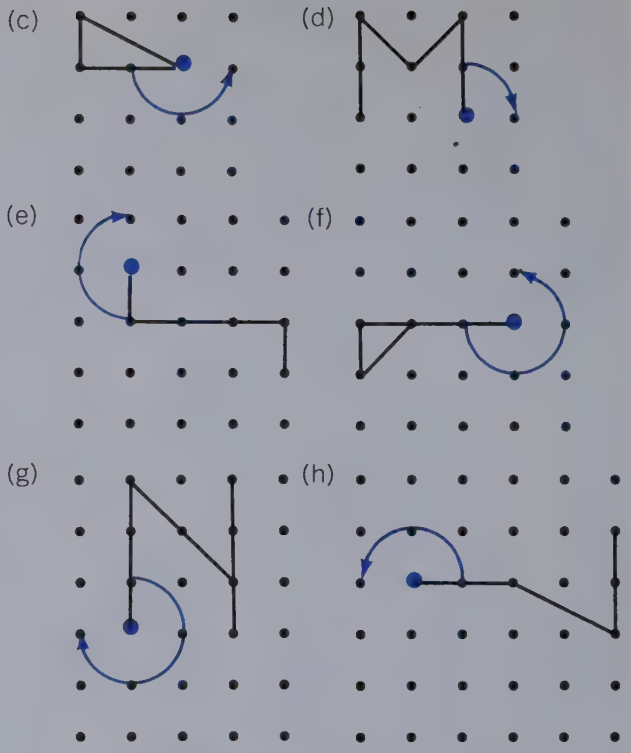
Solution



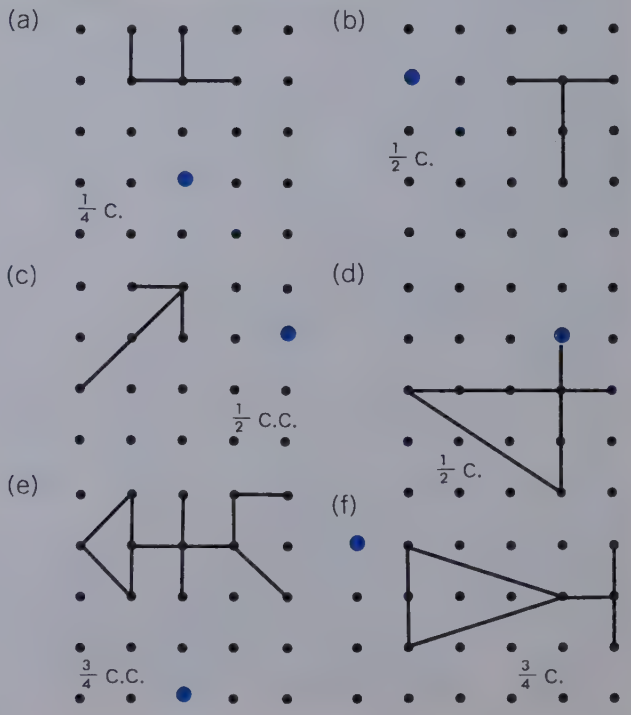
EXERCISE 8·5

1. Copy each of the following on geo-paper and show the result of each indicated rotation.





2. Copy each of these diagrams and draw the result of the required rotation (*c.* means clockwise and *c.c.* means counterclockwise).



8·6 Motion Geometry in the Co-ordinate Plane

Let us recall the study of *two-dimensional graphing* from the previous chapter, especially Section 7·3. It is with respect to ordered pairs in co-ordinate geometry that we wish to relate reflections, translations, and rotations. Note how we do this in the following examples.

Example 1

Choose a point (5,3) in the co-ordinate plane. Give the co-ordinates of the image of the point after:

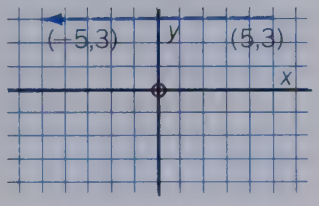
(a) a reflection using the y-axis as the reflection line;

(b) a translation of five units vertically downward (parallel to the y-axis);

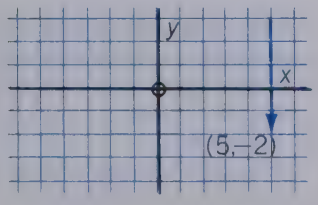
(c) a $\frac{1}{2}$ turn rotation about the origin. (Does the direction of rotation matter?)

Solution

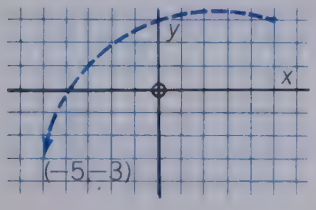
(a)



(b)



(c)



(a) Point (−5,3) is the reflection of (5,3). (Use your *Reflecto* to check this.)

(b) Point (5,−2) is the result of a five-unit downward translation of (5,3).

(c) Point (−5,−3) is the $\frac{1}{2}$ turn rotation of (5,3). Either c. or c.c. would give the same result.

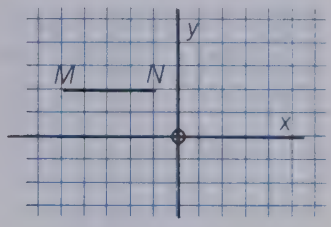
Example 2

For the line segment MN in the diagram, show the image after:

(a) a reflection using the x-axis as the reflection line;

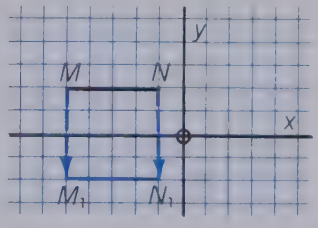
(b) a translation of six units to the right and three units down;

(c) a $\frac{1}{4}$ clockwise rotation about the origin.

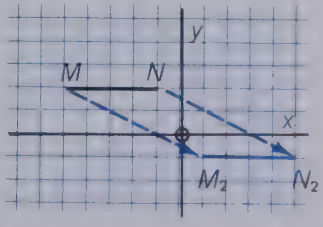


Solution

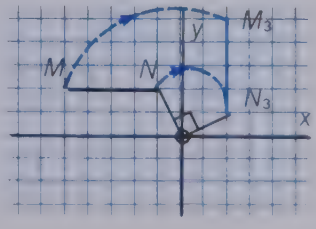
(a) $\overline{M_1N_1}$ is a reflection using the x-axis. Counting from M to the x-axis, M_1 is as far from the x-axis as M. Repeat for N.



(b) $\overline{M_2N_2}$ is a translation six units to the right and three units down.



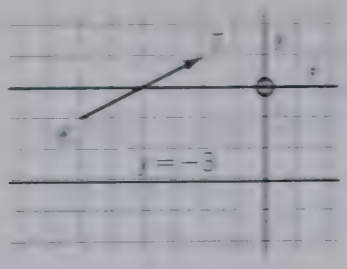
(c) $\overline{M_3N_3}$ is a $\frac{1}{4}$ c. rotation. Draw $\overline{NO}$. Draw ray OP perpendicular to $\overline{NO}$. Use a compass to locate point N_3 so that $\overline{ON}$ is the same length as $\overline{ON_3}$.



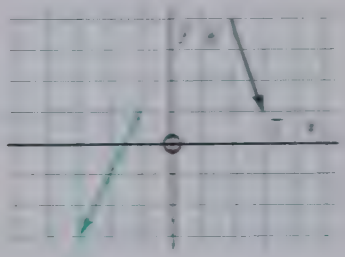
Example 3

Perform the indicated operations on the arrow KT.

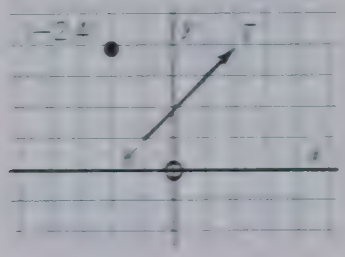
(a) A reflection using the line $y = -3$.



(b) A translation as indicated

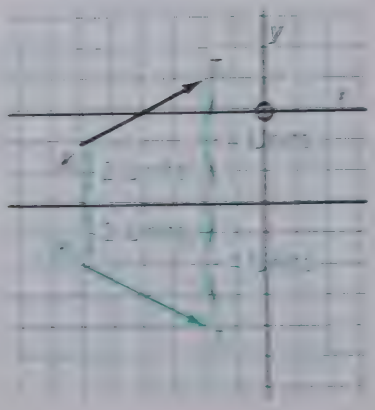


(c) A $\frac{1}{4}$ clockwise rotation about the point $(-2,4)$.

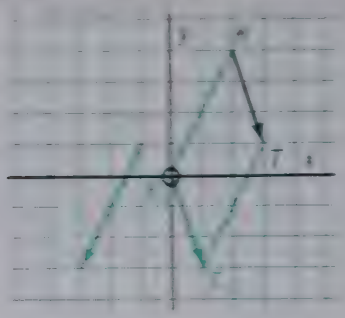


Solution

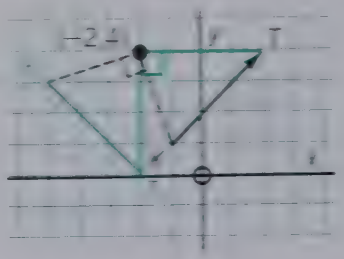
(a)



(b) The arrow indicates that each point, K and T, is translated down four units and left two units.



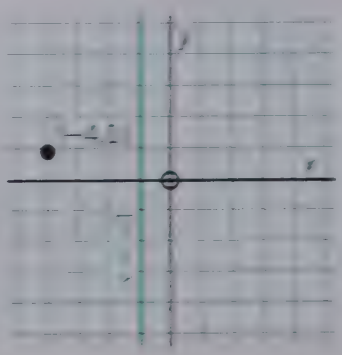
(c) Draw a segment from the rotation point to T.
Draw a perpendicular to this line through the rotation point.
T is four units from the point of rotation $(-2,4)$.
Locate T_1 four units from $(-2,4)$ on the new line.
Repeat for point K.
Join K_1 and T_1.



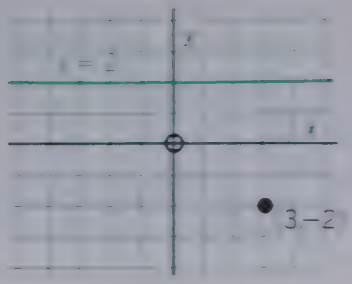
EXERCISE 8·6

1. Copy each onto graph paper. Show the image of each given point for a reflection in the indicated colored line. Give the ordered pair of each reflected point.

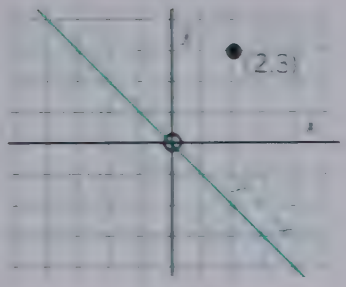
(a)



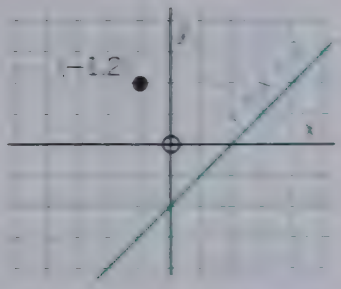
(b)



(c)

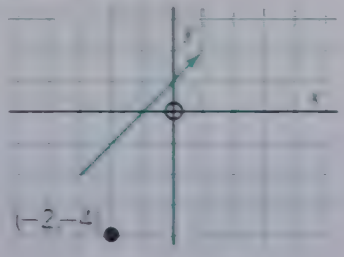


(d)

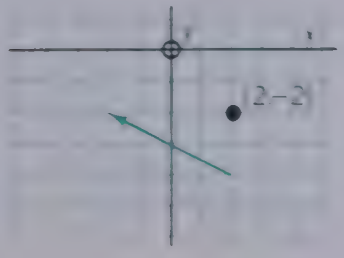


2. Copy each onto graph paper. Perform the translation for each point. Give the ordered pair of the resulting point.

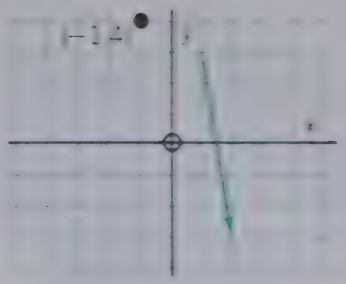
(a)



(b)

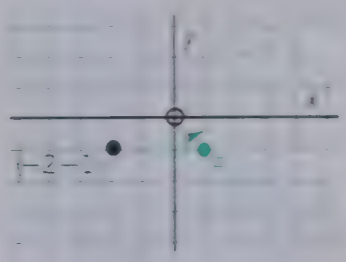


(c)

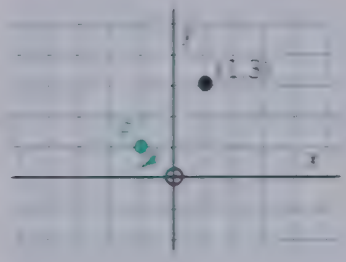


3. Copy each onto graph paper. Perform the indicated rotation about P for each point and then give its ordered pair.

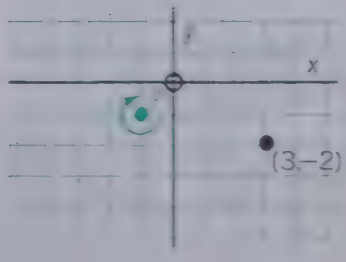
(a)



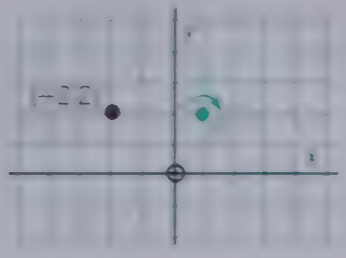
(b)



(c)



(d)



Use graph paper for Questions 4 to 9.

4. Draw a triangle ABC where the ordered pairs of its vertices are $A(2,5)$, $B(3,8)$, and $C(7,1)$. Draw the triangle images for each of the following reflections. What are the ordered pairs of the vertices of each image?
   (a) Reflection in the x-axis.
   (b) Reflection in the y-axis.
   (c) Reflection in the line $y = -x$.

5. Draw $\triangle KMN$ where $K(-3,2)$, $M(-1,-4)$, and $N(2,-2)$. Translate $\triangle KMN$ seven units to the right and four units up. Give the ordered pairs of the vertices of the image triangle.

6. Triangle XYZ is rotated $\frac{1}{4}$ turn clockwise about the origin. Give the ordered pairs of the image of the rotated triangle if $\triangle XYZ$ is at $X(-3,1)$, $Y(0,5)$, and $Z(5,-2)$.

7. Draw the line segment GH, where $G(-2,-7)$ and $H(1,-3)$. Rotate the segment a $\frac{3}{4}$ turn counterclockwise about point $P(0,2)$. Give the ordered pairs of the endpoints of the image.

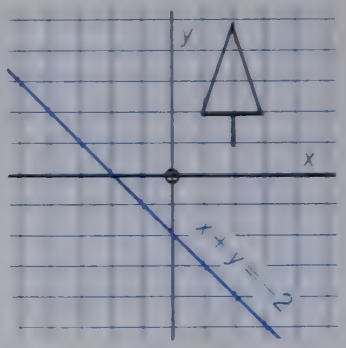
8. Draw the line $y = x + 1$. Draw $\triangle QRS$ where $Q(5,-3)$, $R(3,-7)$, and $S(1,-2)$ and show its reflection in line $y = x + 1$. Give the ordered pairs of the image after the reflection.

9. Draw segment DE, where $D(-4,-1)$ and $E(-1,5)$. Translate $\overline{DE}$ eight units to the right and five units down. Give the ordered pairs of the endpoints of the image after the translation.

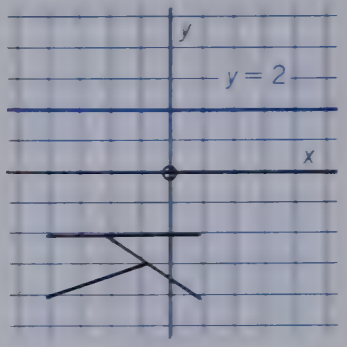
What do you see in the upper left—a vase or two people? We must train ourselves to see all possibilities. Mathematics helps us to do this in problem situations.

10. Copy each onto graph paper. Perform each reflection in the indicated colored line.

(a)

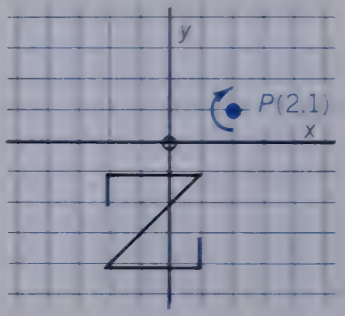


(b)

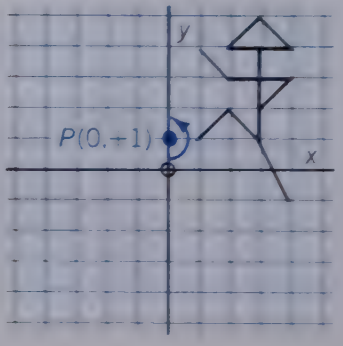


11. Perform each indicated rotation about the given point P.

(a)



(b)

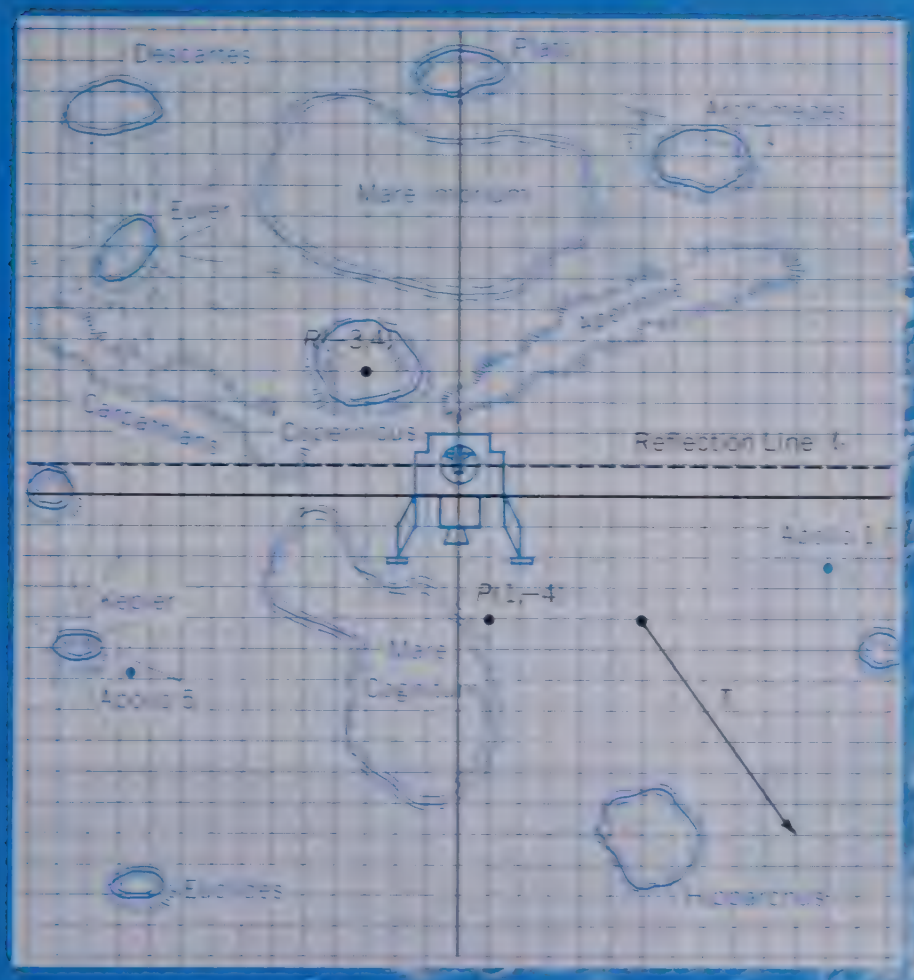


12. A space pirate drew this moon map of a hidden treasure. The clues are given by:

(a) translation T;

(b) reflection in l_1;

(c) rotate $\frac{3}{4}$ turn c. about R.

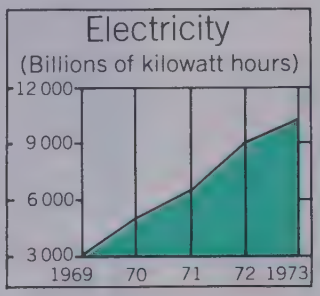


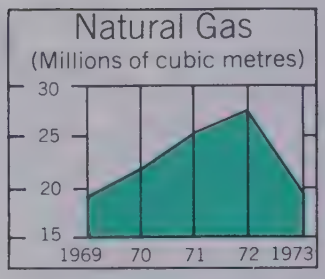
13. Make up a puzzle as in Question 12. Exchange puzzles with your classmates.

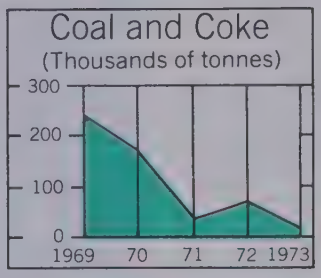
Canada, and many other parts of the world, suffered an energy crisis in 1973 and 1974. It appeared that an acute shortage of energy would continue for some time.

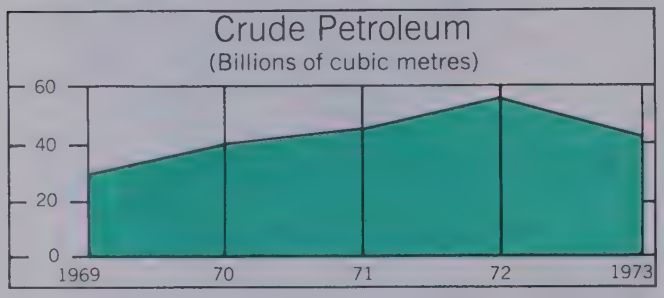
This chart appeared in a newspaper.

Canadian Energy Exports to United States









Discuss the following:

1. Interpret the four graphs. What trends do you see from the graphs?

2. "Canada should discontinue energy exports to the United States." Do you agree or disagree with this statement?

3. "Canada has plenty of energy still in the ground. Greater production rates would solve the energy shortage crisis." Do you agree or disagree?

80 kg GARBAGE MAKES TEN LITRES GAS.

JACKSON, Mich. – Floyd Wallace, a former chemistry teacher, feeds garbage, leaves, coal and other miscellany into a furnace and withdraws gasoline. "Anybody can build one," Wallace said of his refinery.

"All it takes is a vat, a bonfire, coiled tubes, a few gadgets and a little time. You put the garbage in at the top and get the gasoline out of the bottom." Wallace said he's been distilling his own gasoline for 12 years and a fuel specialist told him it has higher octane than aviation gas.

"Just apply a little heat and pressure to organic materials and you get crude oil, much like the stuff that comes out of the ground. Then I fractionate it and I can get methane, propane, axle grease, kerosene, diesel fuel or gasoline." Wallace says he gets 10 ℓ of gas from 80 kg of garbage.

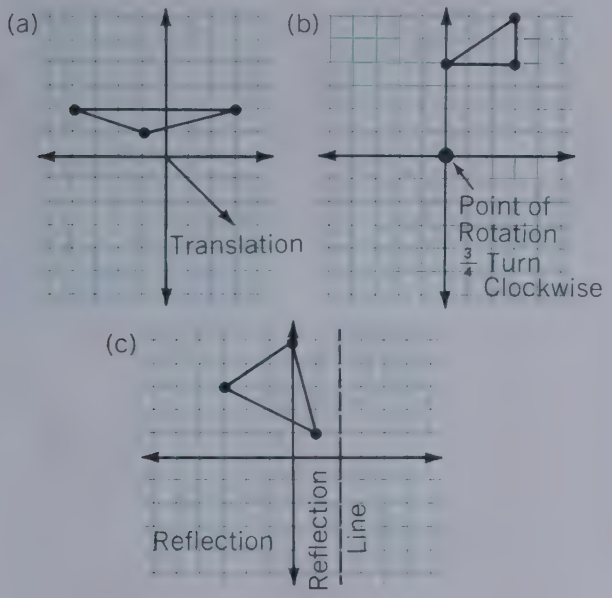
4. Discuss the solution Floyd Wallace of Michigan has to the energy crisis. Do you think it is a practical one? Do you think everybody should make their own gasoline and oil products?

   Discuss:

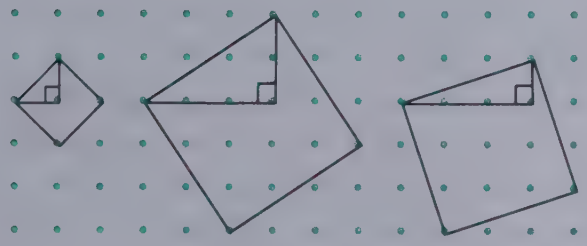
   What does one do with the waste products?
   What about air pollution?
   How many kilograms of garbage would it take to produce a year's supply of gasoline (2000 ℓ per person) for a city of 100 000 population?

REVIEW EXERCISE 8

1. Copy each onto graph paper. Perform the indicated transformation. Give the co-ordinates of the vertices of each image.



2.



Each of these shapes is a square. Using the right triangle indicated in each, calculate the length of the side of each triangle expressed as $\sqrt{x}$.

Use the squares and the information to determine the value of each:

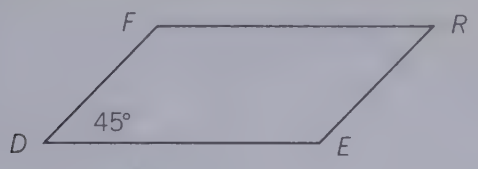
$\sqrt{2} \cdot \sqrt{2}$ $\sqrt{13} \cdot \sqrt{13}$ $\sqrt{10} \cdot \sqrt{10}$

Draw a square on geo-paper to show $\sqrt{8} \cdot \sqrt{8} = 8$.

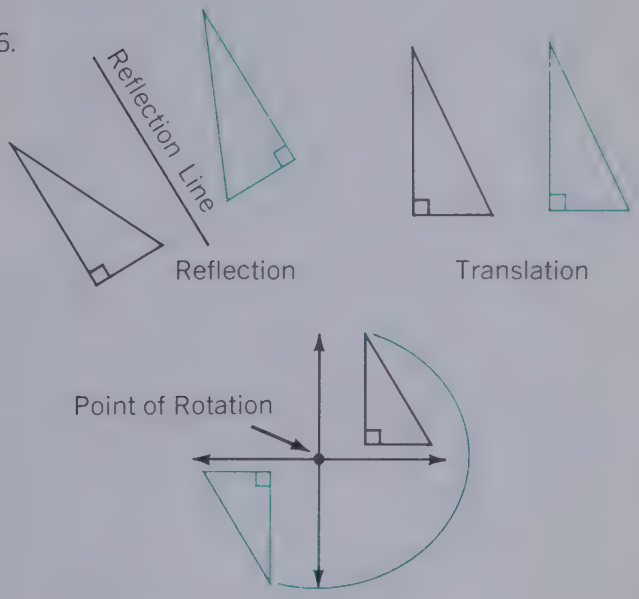
3. Use *Reflecto* to construct a perpendicular bisector of a segment *AB*.

4. Draw an angle and label it *YES*. Bisect the angle using *Reflecto*.

5. Use *Reflecto* to draw the parallelgram *FRED*.



6.



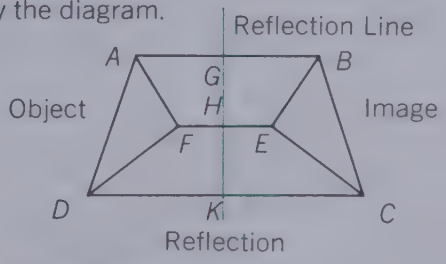
For each transformation, tell what you observe about:

(a) the shape of the object and image;

(b) the size of the object and image;

(c) the position of the object and image (forward, backward, upside down, and so on).

7. Study the diagram.



What is the relationship of:

(a) $\overline{AG}$ and $\overline{GB}$?

(b) $\overline{DK}$ and $\overline{KC}$?

(c) $\angle AGH$ and $\angle BGH$?

What do you know about $\angle AGH$?

Using the Library

1. *Reading in Mathematics* Book 1 Adler I. Dunn and Company (1972). The first half of page 166 poses a question with a choice of three answers. Which do you choose? In the last paragraph on page 166 another question is posed. Try answering this one.

2. *Reading in Mathematics* Book 2 Adler I. Dunn and Company (1972)

   (a) Read page 142. List some big numbers and how to read them. Write one quintillion in numbers using zeros; then write it using exponents. What does the prefix *quint-* mean? Explain its meaning in quintillion.

   (b) Make your own discoveries in mathematics. Read page 162, then carefully read and follow the instructions for Problem 1 on page 163. Continue the steps until you see a pattern. Try the same thing with another four digit number, for example, 5813. What do you observe? Continue with Problems 2 and 3.

3. *Mathematical Entertainments* Fair Spielman Cameron A. J. Pergamon Press (1966). The plate facing page 172 is Dürer's "Adoration of the Magi." Notice how each one of the gifts is positioned in Golden Sections. Dürer was a mathematician and artist. What are some of the things Dürer is famous for in mathematics and art?

4. Use the library to prepare a report on the history of the airplane.

5. Research Stonehenge. Tell what it is, where it is, and what its historical importance is.

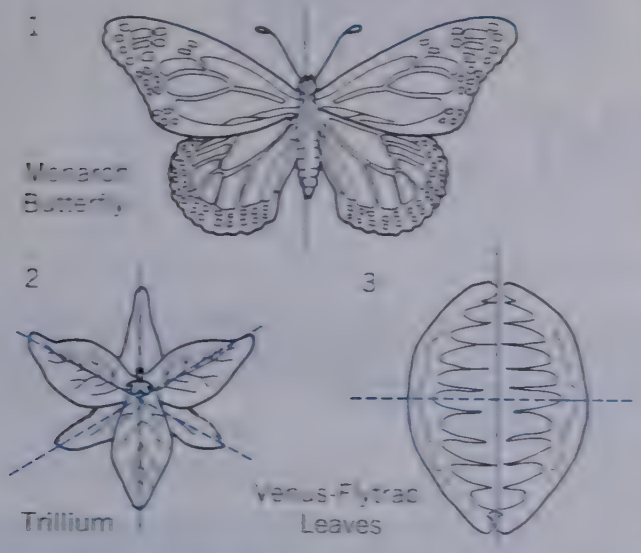
6. *Reading in Mathematics* Book 2 Adler I. Dunn and Company (1972).

   (a) Read pages 33-36. Draw an "Escher" type paradoxical drawing using the trick described. Use your library to learn more about Escher and his art.

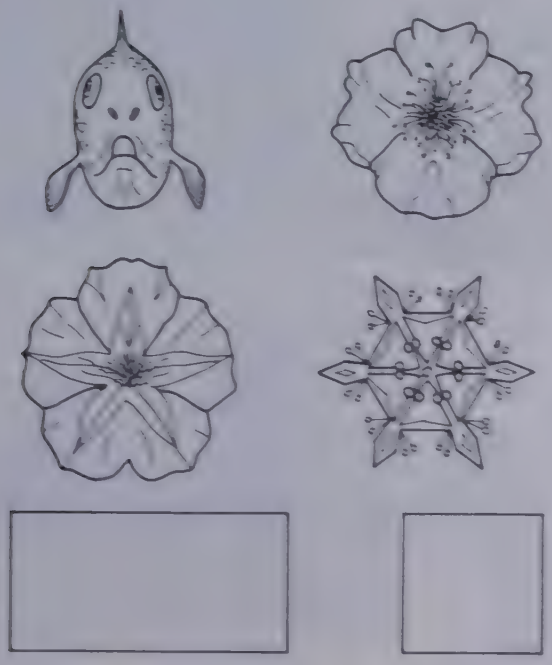
   (b) Read page 102-104. Draw a space filling tessellation.

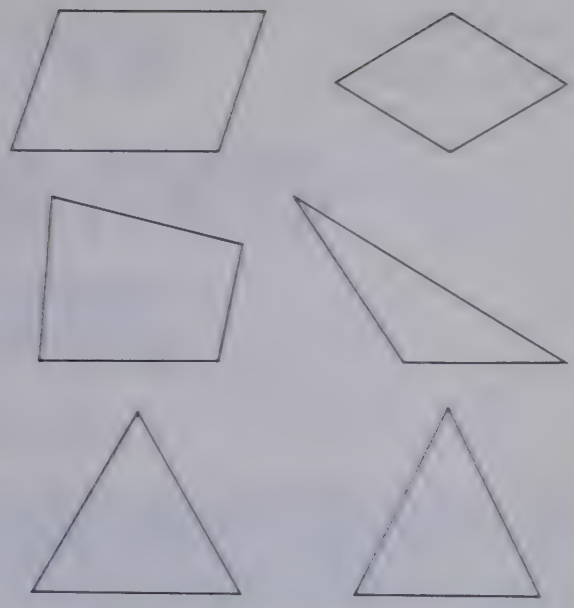
Do You Remember . . .

. . . about symmetry?



This Monarch Butterfly has one axis of symmetry. The Trillium flower has three axes of symmetry. The Venus-Flytrap has two axes of symmetry. Use *Reflecto* to determine how many axes of symmetry these shapes have.





Have You Discovered that . . .

. . . two reflections can equal one translation? On geo-paper or graph paper, construct two lines in which the reflections are to be made (see Figure 8-12).

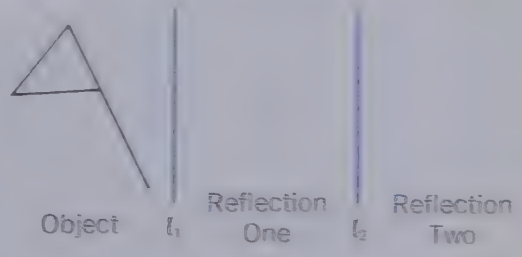


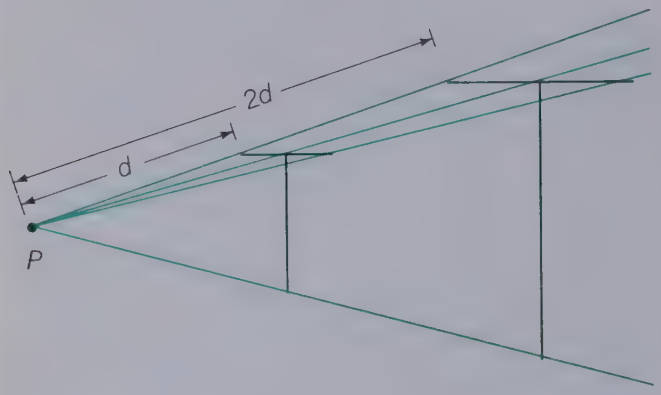
Figure 8-12

To the left of ℓ_1, draw an object to be reflected and then construct a reflection of the object in ℓ_1. This will then be *image one*. Then repeat the process in ℓ_2 by finding the image of image one after the second reflection. Call the second, *image two*. Compare *image two* with the object. Is it a translation of the object?

. . . for each translation, reflection, and rotation, the image and object are congruent? What does *congruent* mean?

Something to Think About

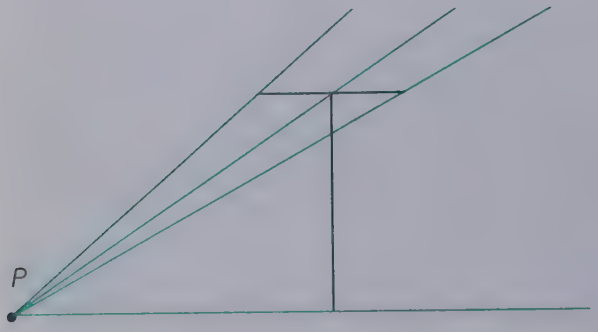
Projection is a form of magnification. Study the following drawing.



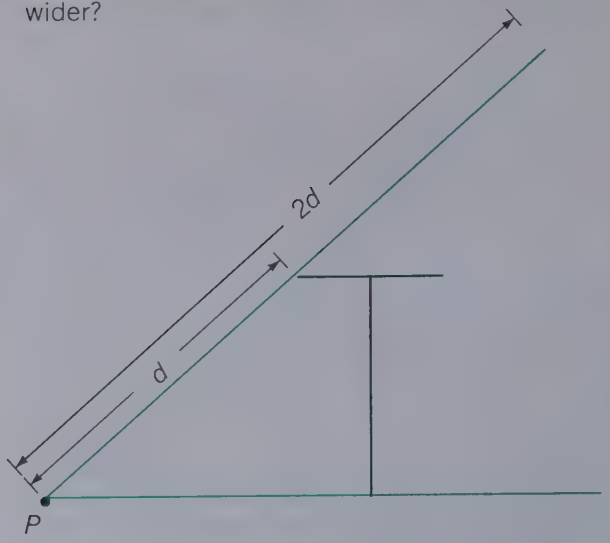
Take a sheet of paper and make a copy of this drawing.

(a) First draw a small T so that it is 3 cm high and 2 cm across the top bar.

(b) From a point *P* on the left, construct straight-line guides, one through every endpoint in the T.

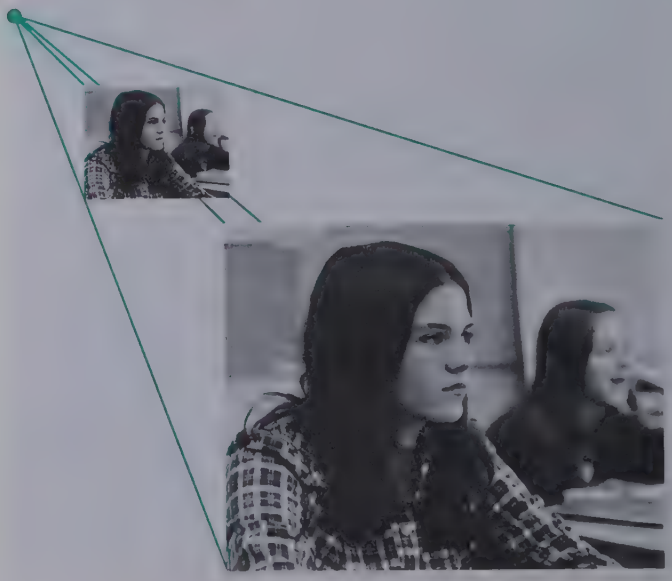


(c) Measure the distance from *P* to every endpoint of the T, then double that distance from *P* along each guideline and mark each position. Join the marked positions to make the large T. How much taller is the large T than the small T? How much wider?



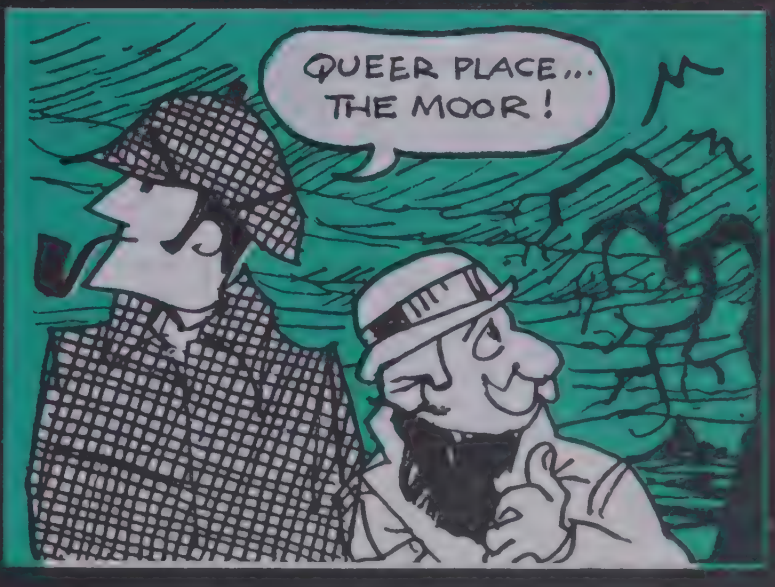
Try another drawing in which the distances along the guidelines are tripled. How does the latter figure compare with the former? Try the procedure in reverse; that is, from the larger figure to the smaller.

Magnification can be one of enlargement or reduction in size.



Chapter Nine
APPLYING THE FACTS TO THE CASE

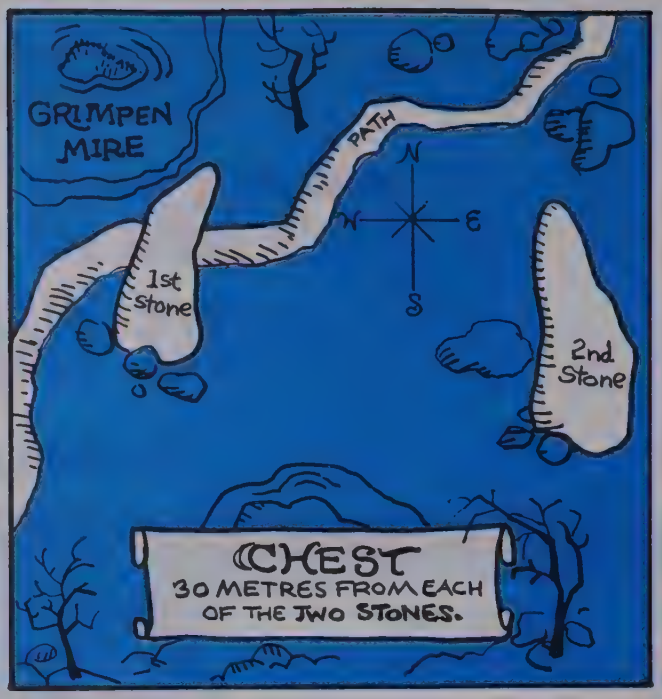
Constructions



9·1 A Basic Construction

Study the map on this page.

We know the evidence being sought is equally distant from the two stones. How would you locate the evidence?



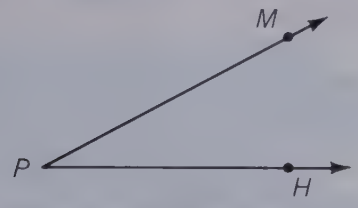
EXERCISE 9·1

1. Draw a diagram to illustrate the method Sherlock Holmes described.
2. How many places would there be to dig if the stones were:
   (a) 40 m apart?
   (b) 60 m apart?
   (c) 70 m apart?

HOW WILL WE KNOW WHERE TO DIG TO FIND THE EVIDENCE?
I HAVE TIED A KNOT THIRTY METRES FROM THE END...
YOU HOLD ONE END WITH YOUR BACK TO THE STONE...
... AND I'LL HOLD THE OTHER END AND MAKE A PATH IN THE SNOW...
WE'LL REPEAT THIS WITH YOUR BACK TO THE OTHER STONE.
Z-Z-
NOW WE KNOW WHERE TO APPLY OUR SPADES!
Z-Z-Z-

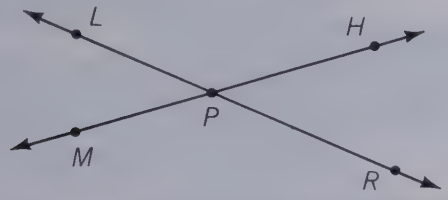
9·2 Measuring Some Angles

The angle illustrated is named angle *MPH*. We abbreviate this by writing $\angle MPH$.



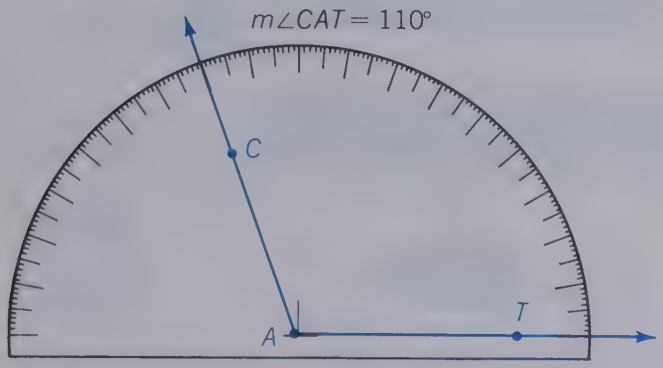
An angle has two arms we call rays. Angle *MPH* has ray *PM* and ray *PH*. The place where the two rays meet, at point *P*, is called the vertex of the angle.

When two lines intersect, four angles are formed. Name the four angles in this diagram.



We measure angles in units called degrees. The instrument used to measure an angle is a protractor.

The measure of angle *CAT* is 110°.

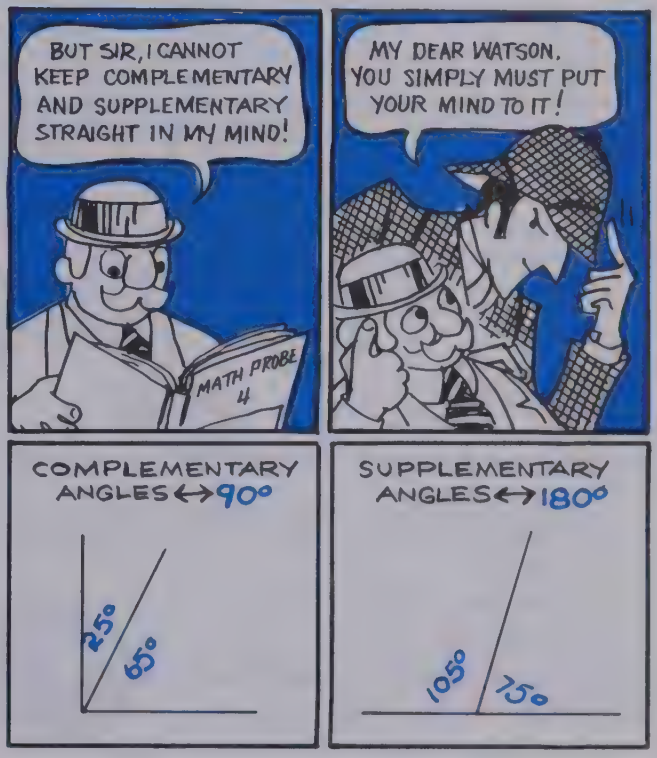


When the sum of the measures of two angles is 90°, the angles are complementary angles.

Two pairs of complementary angles.

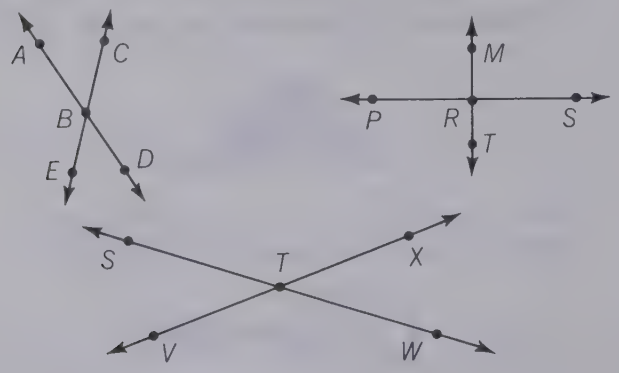
When the sum of the measures of two angles is 180°, the angles are supplementary angles.

Two pairs of supplementary angles.


BUT SIR, I CANNOT KEEP COMPLEMENTARY AND SUPPLEMENTARY STRAIGHT IN MY MIND!
MATH PROBE 4
MY DEAR WATSON, YOU SIMPLY MUST PUT YOUR MIND TO IT!
COMPLEMENTARY ANGLES ↔ 90°
25°
65°
SUPPLEMENTARY ANGLES ↔ 180°
105°
75°

EXERCISE 9·2

1. With ruler and pencil, construct two lines intersecting each other as in the following examples.



(a) Measure $\angle ABE$, $\angle EBD$, $\angle DBC$, and $\angle CBA$. What do you discover about the measures of $\angle ABE$ and $\angle DBC$? of $\angle ABC$ and $\angle EBD$?

(b) Measure $\angle PRM$, $\angle MRS$, $\angle SRT$, and $\angle TRP$. How do the measures of these angles compare?

(c) Measure $\angle STV$, $\angle VTW$, $\angle WTX$, and $\angle XTS$. Which pairs of angles are equal (or approximately equal) in measure?

When two straight lines intersect, (i) how do the measures of the *vertically opposite angles* compare, and (ii) what is the sum of the measures of the four angles?

2. Draw a straight line with another line meeting it as in each of the following examples. Notice this second way of naming angles.

(a) Measure angles p and r. Find the sum of their measures.

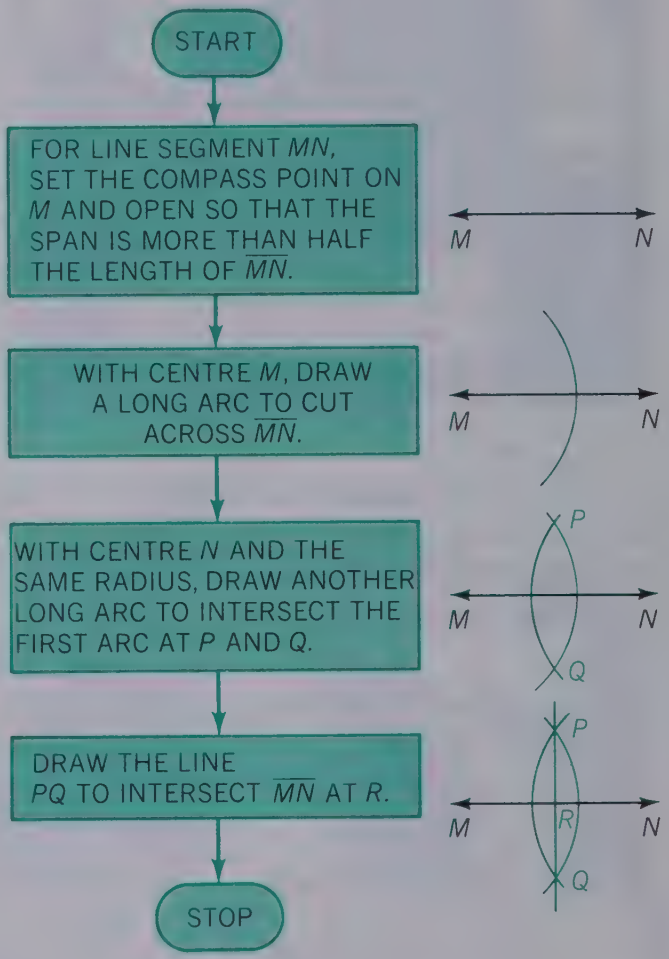
(b) Measure angles s and t. Find the sum of their measures.

(c) Do the same for angles v and w. What conclusion do you draw from these three pairs of measurements?

9·3 Some Constructions

Construction A

To construct the perpendicular (right) bisector of a given line segment:


START
FOR LINE SEGMENT MN, SET THE COMPASS POINT ON M AND OPEN SO THAT THE SPAN IS MORE THAN HALF THE LENGTH OF MN.
WITH CENTRE M, DRAW A LONG ARC TO CUT ACROSS MN.
WITH CENTRE N AND THE SAME RADIUS, DRAW ANOTHER LONG ARC TO INTERSECT THE FIRST ARC AT P AND Q.
DRAW THE LINE PQ TO INTERSECT MN AT R.
STOP
M
N
P
Q
R

Line PQ is the perpendicular bisector of segment MN. That is:

(i) line PQ is perpendicular to segment MN;

(ii) line PQ divides segment MN into two congruent parts.

Do Questions 1 and 2 in Exercise 9·3 before proceeding.

Construction B

To construct the bisector of a given angle:

START

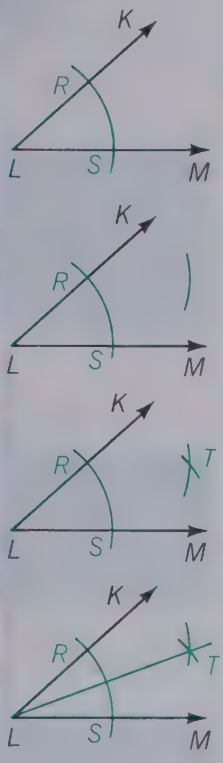
WITH THE COMPASS POINT ON *L* OF ANGLE *KLM*, DRAW AN ARC OF ANY RADIUS TO CUT BOTH ARMS OF THE ANGLE. MARK THE INTERSECTIONS *R* AND *S*.

WITH CENTRE AT *R* AND THE SAME RADIUS, DRAW AN ARC WITHIN THE ANGLE.

WITH CENTRE AT *S* AND THE SAME RADIUS, DRAW ANOTHER ARC TO CUT THE SECOND ARC AT *T*.

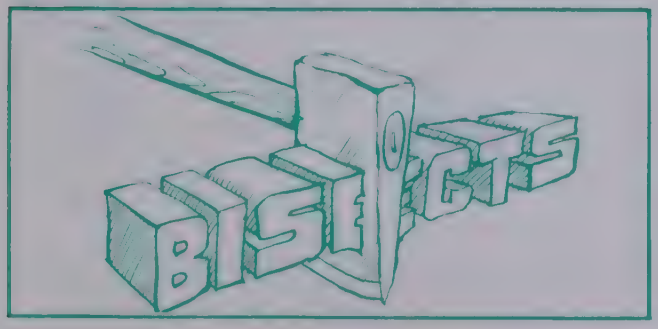
DRAW RAY *LT*.

STOP



Ray *LT* is the bisector of angle *KLM*.
Angle *KLT* is equal in measure to angle *MLT*.

Do Questions 3, 4, and 5 in Exercise 9·3 before proceeding.



Construction C

To construct an angle equal in measure to a given angle:

START

DRAW A RAY *HK* WHICH WILL BE ONE ARM FOR THE REQUIRED ANGLE.

WITH COMPASS ON POINT *B* OF ANGLE *ABC*, DRAW AN ARC TO CUT RAY *BA* AND RAY *BC* AT *M* AND *N*.

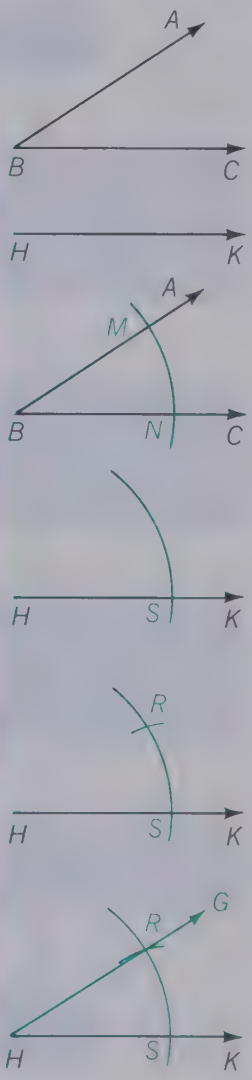
WITH COMPASS POINT ON *H* AND THE SAME RADIUS, DRAW ANOTHER ARC TO CUT RAY *HK* AT *S*.

MEASURE THE ARC *MN* BY MEANS OF A SPAN OF THE COMPASS.

WITH COMPASS POINT ON *S* AND THAT SAME MEASURED SPAN, CUT THE ARC AT *R*.

DRAW RAY *HG* FROM *H* THROUGH *R*.

STOP

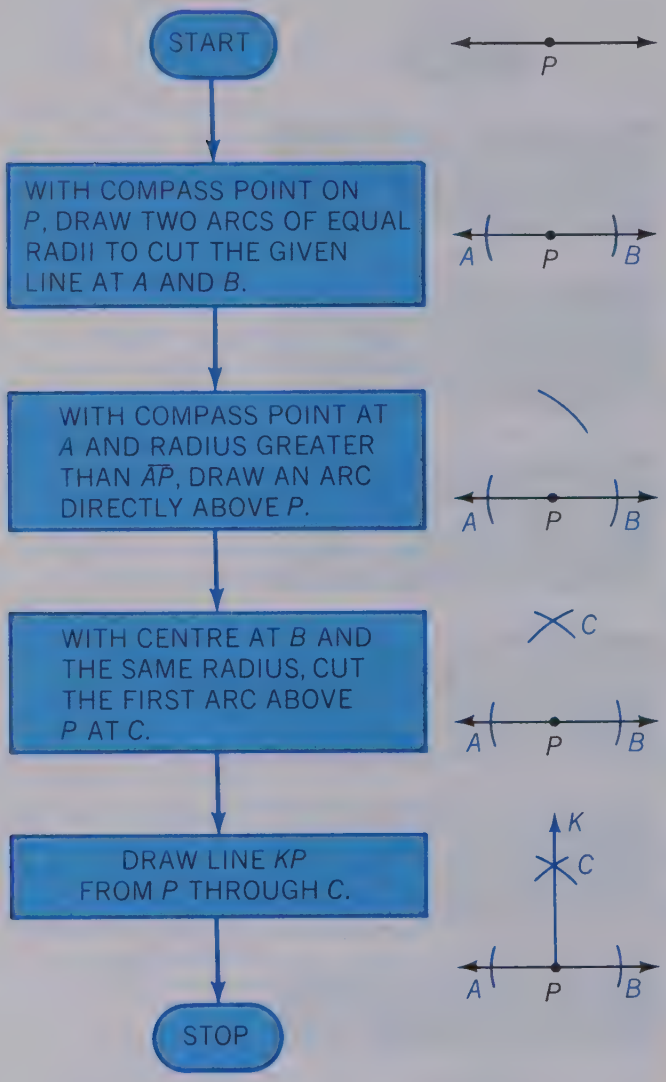


The constructed angle *GHK* is equal in measure to angle *ABC*.

Do Questions 6, 7, and 8 in Exercise 9·3 before proceeding.

Construction D

To construct a perpendicular to a line at a point on the line:


START
WITH COMPASS POINT ON P, DRAW TWO ARCS OF EQUAL RADII TO CUT THE GIVEN LINE AT A AND B.
WITH COMPASS POINT AT A AND RADIUS GREATER THAN $\overline{AP}$, DRAW AN ARC DIRECTLY ABOVE P.
WITH CENTRE AT B AND THE SAME RADIUS, CUT THE FIRST ARC ABOVE P AT C.
DRAW LINE KP FROM P THROUGH C.
STOP
P
A
B
C
K

Line KP is perpendicular to the given line at point P.

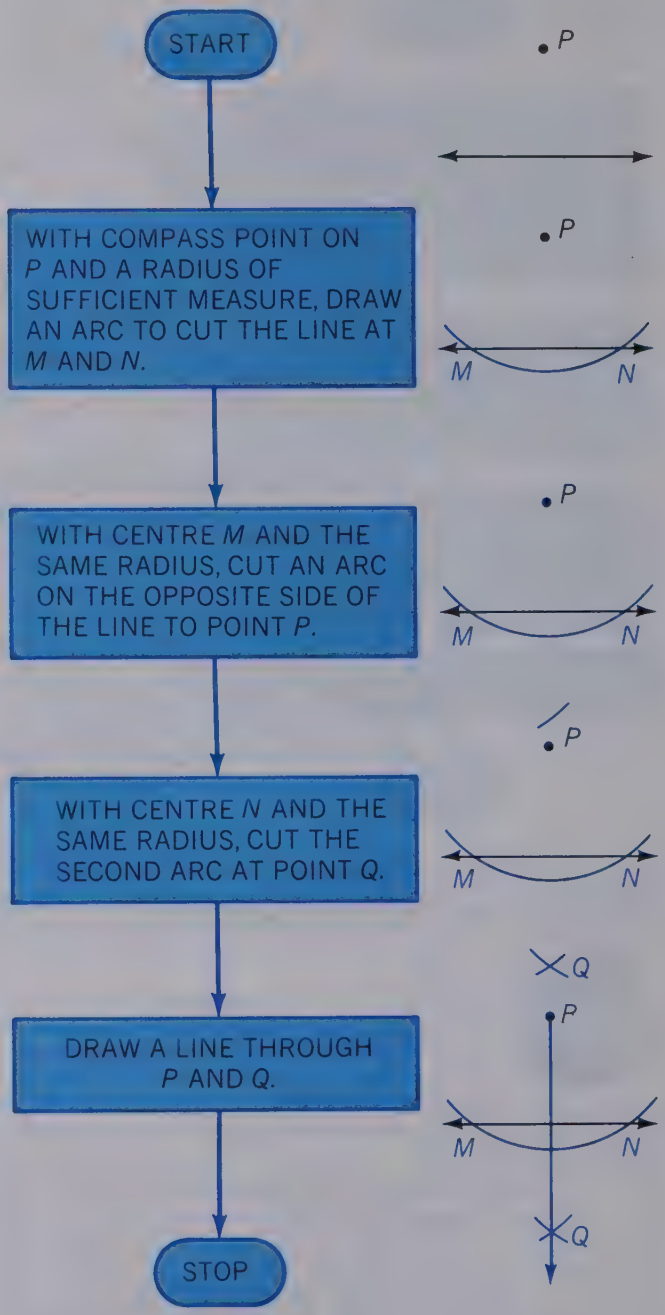
$\angle APC \equiv \angle BPC$ ($\equiv$ is read "congruent".)

$\angle APC$ is a right angle. It has a measure of 90°.

Do Questions 9, 10, and 11 in Exercise 9·3 before proceeding.

Construction E

To construct a perpendicular to a line through a point not on the line:


START
WITH COMPASS POINT ON P AND A RADIUS OF SUFFICIENT MEASURE, DRAW AN ARC TO CUT THE LINE AT M AND N.
WITH CENTRE M AND THE SAME RADIUS, CUT AN ARC ON THE OPPOSITE SIDE OF THE LINE TO POINT P.
WITH CENTRE N AND THE SAME RADIUS, CUT THE SECOND ARC AT POINT Q.
DRAW A LINE THROUGH P AND Q.
STOP
P
M
N
Q

Line PQ is perpendicular to the given line.

Do the balance of the questions in Exercise 9·3.

EXERCISE 9·3

1. With a ruler, draw a straight line segment 10 cm long. Construct a perpendicular bisector to the 10 cm segment by using only a compass and straightedge. Do not use the ruler to make any measurements at this time.
   (a) Measure the angle between the new line and the given segment. Is the line truly perpendicular?
   (b) Now measure each "half" of the 10 cm line segment with a ruler. Are the "halves" congruent; that is, are they equal in measure?

2. Draw a line segment 8 cm long and label it AB. Construct a right bisector to the line AB and label the intersection C. Use the method employed in Question 1.
   (a) Measure the two parts of the line segment, $\overline{AC}$ and $\overline{BC}$. Are they congruent?
   (b) Measure the angles formed by the bisector and the line segment. Is the bisector perpendicular to the line segment?

3. Draw any angle. Construct a bisector of this angle with compass and ruler. Measure the two "half" angles with a protractor. Are they of equal measure?

4. Draw an angle similar to this one. Make each of the rays of the angle 10 cm long.

   Construct a bisector of this angle as you did in Question 3. Measure the two smaller angles that are formed. Are their measures equal?

5. Draw an angle similar to this one.

   Again, bisect it as before. Measure the angles to determine if they are congruent.

6. Use a protractor to construct an angle of 64°. Construct another angle of 64° using only a compass and ruler. Measure the constructed angle with a protractor as a check.

7. Using a protractor and ruler, construct $\angle ABC$ so that it measures 55°. Use the same method as in Question 6 to construct another angle, PQR, that is equal in measure to $\angle ABC$. Measure $\angle PQR$. Is it also 55°?

8. Repeat Question 7, but this time, start with $\angle DEF$ which measures 84°.

9. Draw a line segment 12 cm long. Mark a point R on the line 7 cm from one end. Construct a perpendicular to the line segment at point R using a compass and ruler. Measure one angle to check the perpendicular.

10. Refer to Construction D in Section 9·3 above. What is the sum of the measures of $\angle APC$ and $\angle BPC$?

11. Draw a line segment 9 cm long. Mark a point A on the line 3 cm from one end. Construct a perpendicular to the line segment at point A by using the method illustrated in Construction D (Section 9·3 above). Measure the angles formed by your construction. Did you construct a perpendicular at point A?

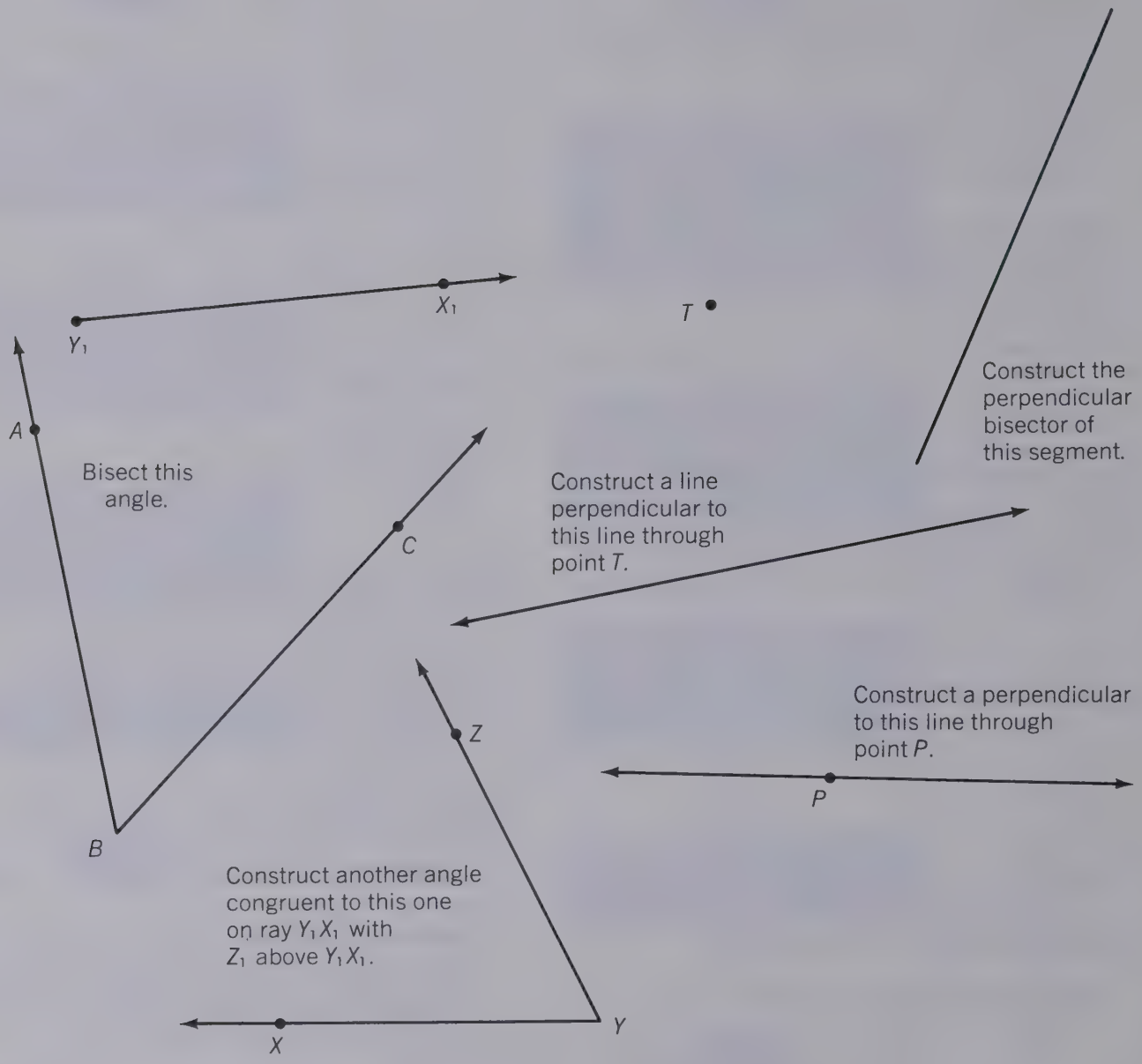
12. Draw a line segment. Mark a point T about 5 cm below the line. With compass and ruler, construct a perpendicular from point T to the line segment.

13. Refer to Construction E in Section 9·3. Tell as many things as you can about the lines PQ and MN. What do you notice about the angles formed by these two intersecting lines?

14. Draw a line and locate a point M about 4 cm above it. Construct a perpendicular to the line segment from point M as in Construction E (Section 9·3). Measure the angles to check if you have constructed a perpendicular.

15.

2	33	19	43	6	1	0	9	18	21	13	24	12	73	49	2	17	23	13	61	50	3	11	91	25	26	30	41	31	65	34	10



Place a thin sheet of paper over this page. Use a straightedge to trace the line and its numbers at the top; then do not move your sheet of paper until you have also traced the other segments and the angles.

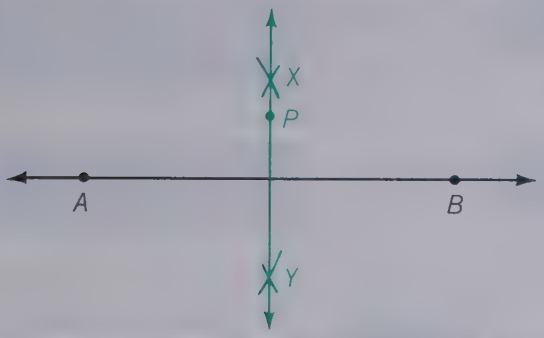
Perform the constructions.

Extend each constructed line to intersect a number at the top of the page.

If your score is 100, you've done a good job!

16. Practise the five constructions in Section 9·3 until you are able to do them without any difficulty. We will use these constructions later in more difficult situations.

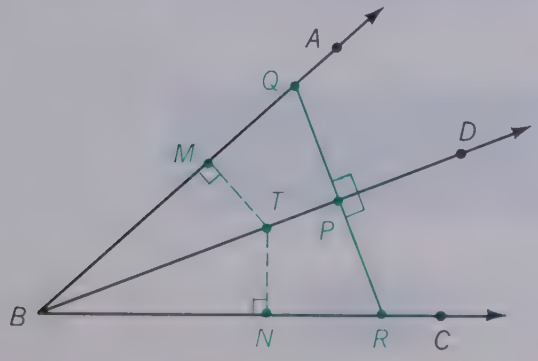
17. Take any point P on the perpendicular bisector of $\overline{AB}$. Tell as much as you can about point P. What do you know about *every* point on $\overleftrightarrow{XY}$?



18. Draw any angle ABC and bisect it. Label the bisector BD.

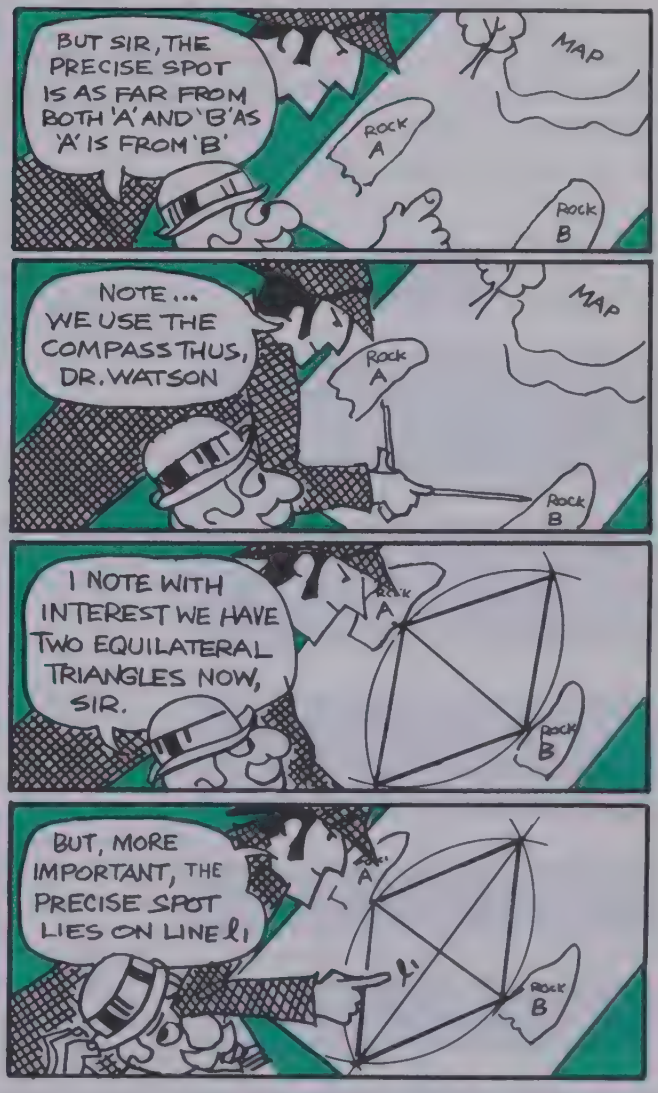
    (a) Construct a perpendicular to ray BD at any point P. Extend the perpendicular in both directions until it intersects the two arms of $\angle ABC$. Label the points of intersection Q and R. What do you notice about the line segments PQ and PR? Repeat for several different points P. What conclusion can you draw about every point on an angle bisector?

    (b) Pick any point on the ray BD and label it T. From T, draw a line perpendicular to the arm AB and label the intersection M. Now draw a perpendicular to arm BC from T and label the intersection N. What do you notice about the segments TM and TN? Repeat for several different points T. What conclusion can you draw about every point on an angle bisector?



9·4 Angle Construction

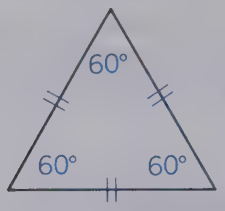
There are various angles that can be constructed quite accurately without the use of a protractor. Have you any idea how to construct an angle of 67.5° using only a compass and ruler? You will be able to do this later.


BUT SIR, THE PRECISE SPOT IS AS FAR FROM BOTH 'A' AND 'B' AS 'A' IS FROM 'B'
MAP
ROCK A
ROCK B
NOTE... WE USE THE COMPASS THUS, DR. WATSON
MAP
ROCK A
ROCK B
I NOTE WITH INTEREST WE HAVE TWO EQUILATERAL TRIANGLES NOW, SIR.
ROCK A
ROCK B
BUT, MORE IMPORTANT, THE PRECISE SPOT LIES ON LINE ℓ_1
ROCK A
ℓ_1
ROCK B

Write a flowchart and show a diagram for the construction of an equilateral triangle (a triangle in which all sides are congruent).

Explain Mr. Homes' last comment: "The precise spot lies on line ℓ_1."

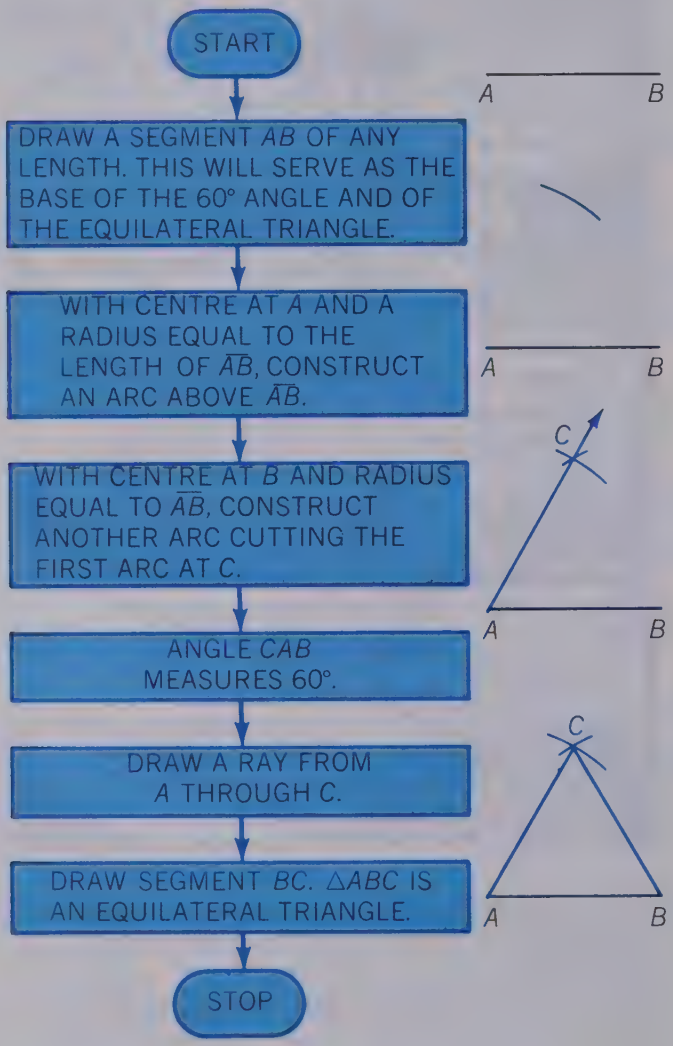
Let us first construct an angle of 60°. This is done through the construction of an equilateral triangle. How many degrees is the sum of the measures of the three angles of a triangle?



$180 \div 3 = 60$

The measure of each angle in an equilateral triangle is 60°. Therefore, an *equilateral* triangle is also an *equiangular* triangle.

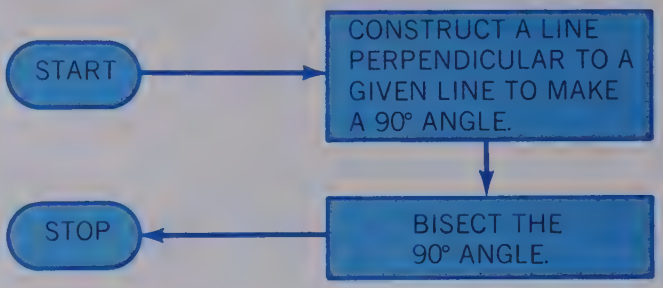
To construct an angle of 60°:

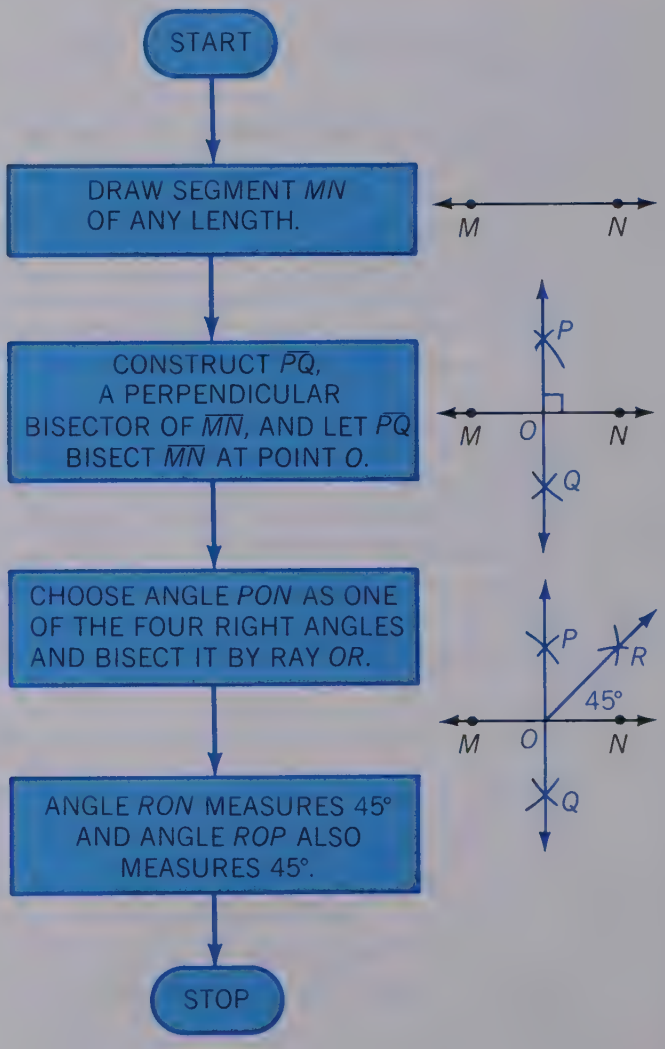

START
DRAW A SEGMENT AB OF ANY LENGTH. THIS WILL SERVE AS THE BASE OF THE 60° ANGLE AND OF THE EQUILATERAL TRIANGLE.
WITH CENTRE AT A AND A RADIUS EQUAL TO THE LENGTH OF AB, CONSTRUCT AN ARC ABOVE AB.
WITH CENTRE AT B AND RADIUS EQUAL TO AB, CONSTRUCT ANOTHER ARC CUTTING THE FIRST ARC AT C.
ANGLE CAB MEASURES 60°.
DRAW A RAY FROM A THROUGH C.
DRAW SEGMENT BC. △ABC IS AN EQUILATERAL TRIANGLE.
STOP
A
B
C

Check your construction by measuring each angle and each side of the triangle. What do you find?

To construct an angle of 45°:

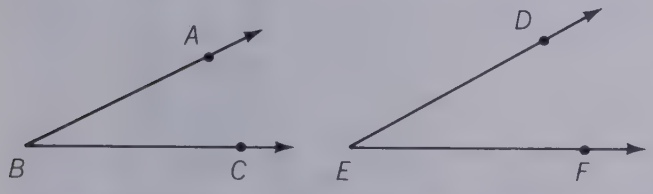
General scheme:

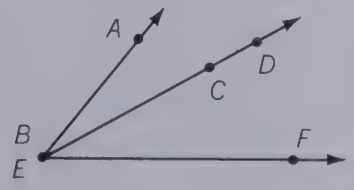



START
DRAW SEGMENT MN OF ANY LENGTH.
CONSTRUCT PQ, A PERPENDICULAR BISECTOR OF MN, AND LET PQ BISECT MN AT POINT O.
CHOOSE ANGLE PON AS ONE OF THE FOUR RIGHT ANGLES AND BISECT IT BY RAY OR.
ANGLE RON MEASURES 45° AND ANGLE ROP ALSO MEASURES 45°.
STOP
M
N
P
O
Q
R
45°

EXERCISE 9·4

Do all constructions with compass and ruler only (no protractors).

1. Draw a ray. Use this ray to construct an angle of 60°.
   (a) Bisect the 60° angle. What is the measure of each angle produced?
   (b) How can a 15° angle be constructed? Construct a 15° angle.
2. Construct another angle measuring 60°. By drawing one more ray, make an angle of 120°. Explain why the angle measures 120°.
3. Draw a ray for one arm of an angle. Use this ray to construct an angle of 45°.
   (a) Bisect the 45° angle. What size are the resulting angles?
   (b) Can you identify an angle of 67.5° on your diagram?
   (c) How can an angle of 135° be constructed? Explain.
4. 

   Construct one angle that is the sum of angles *ABC* and *DEF*. Your diagram will look like this when completed.


5. Using the method of construction in Question 4, show how to construct an angle of 75°.
   Hint! 75° is the sum of 45° and 30°.
6. Show how to construct the following angles:
   (a) 52.5° (b) 150° (*Hint!* 180° − 30° = 150°.)
   (c) 105° (d) 135° (e) 67.5°

9·5 Similar Triangles

Most of us have seen a film projected onto a screen. The film is much smaller than the picture on the screen. The picture is an *enlargement* of the film. We can represent the projector, the film, and the picture on the screen like this:

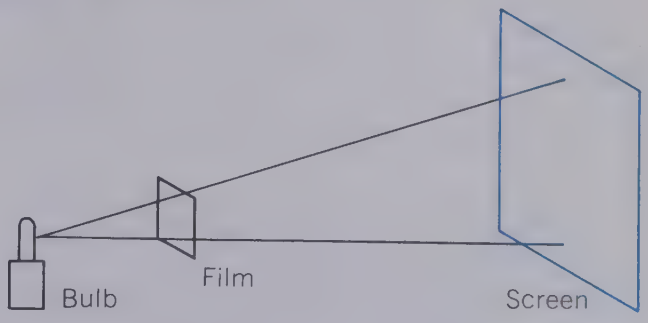


Figure 9-1

Let us investigate simple enlargements. One can use a flashlight and a triangle made from drinking straws and pipe cleaners to illustrate a simple enlargement.

Hold the triangle parallel to the wall.

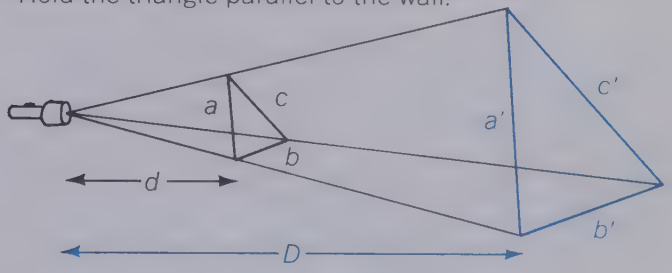
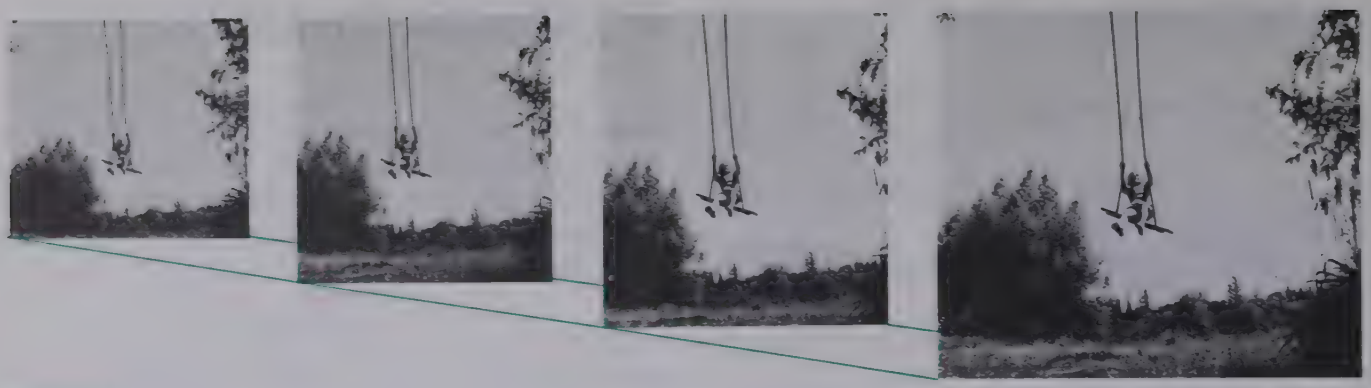


Figure 9-2

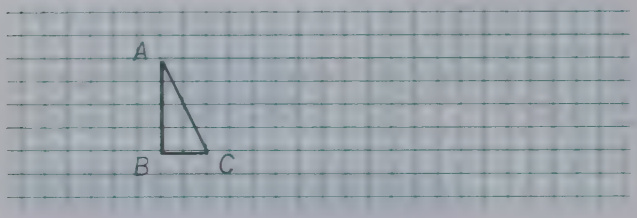
If we set up the apparatus so that D is twice the length of d, then a' is twice the length of a, b' is twice the length of b, and c' is twice the length of c. This enlargement has a *scale factor* of 2.

If D is three times d, what do you think the relation will be between a' and a, b' and b, and c' and c? This enlargement will have a scale factor of 3. Obtain the necessary equipment and test your hypothesis. Another way to illustrate enlargements is shown in Example 1.



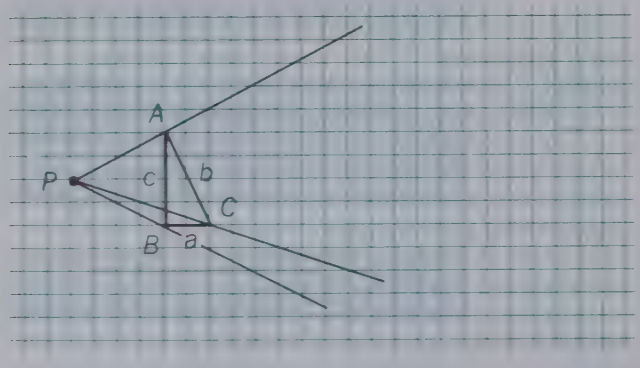
Example 1

Enlarge triangle ABC by a scale factor of 2.



Solution

Draw the triangle on grid paper as shown. Choose some point P to the left of the triangle.

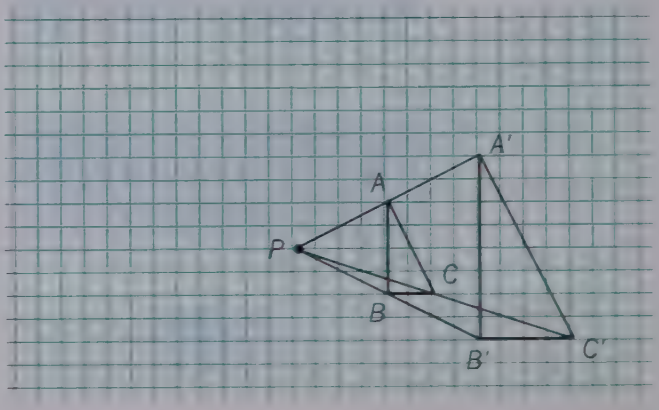


Draw lines through P and each point A, B, and C in turn.

Mark a point A', on line PA, such that $m(\overline{PA'})$ is twice $m(\overline{PA})$. $m(\overline{PA})$ means "the measure of segment PA." (Use your compass to mark off the length.)

Repeat this procedure for $\overline{PB'}$ and $\overline{PC'}$.

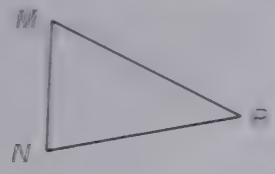
Draw segments $A'B'$, $B'C'$, and $C'A'$.



Do Questions 1 and 2 in Exercise 9·5.

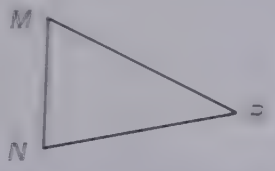
Example 2

Enlarge any triangle *MNR* by a factor of 3.

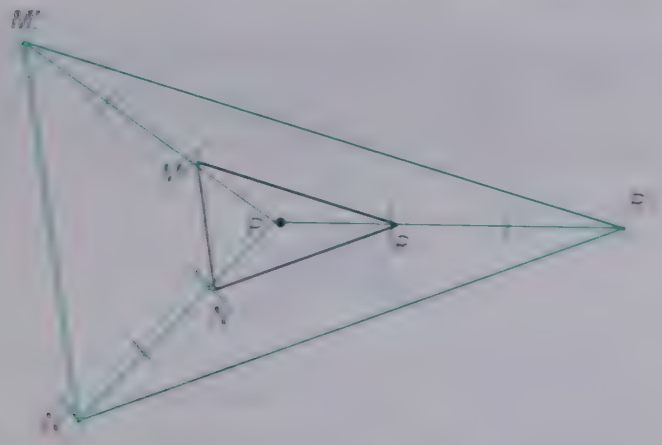


Solution

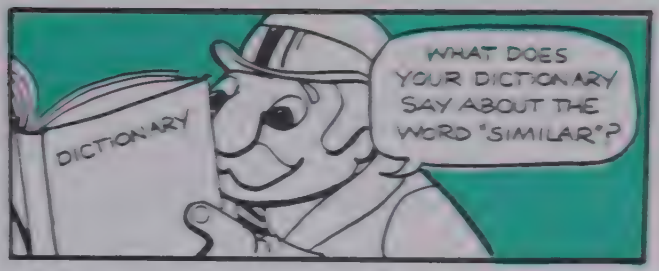
We do not need to work on grid paper. Draw any triangle *MNR* on plain paper. Choose any point *P* near or in, the triangle. (We will choose a point in the interior.)



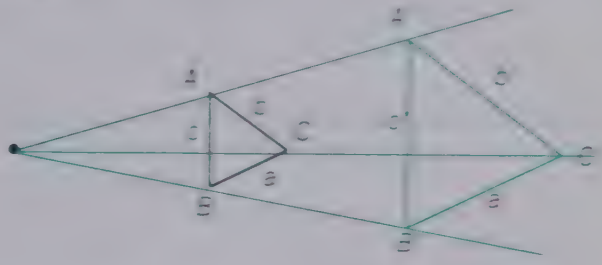
Draw lines through *P* and each vertex. Use a compass to mark off *R'* such that $m(\overline{PR'})$ is three times $m(\overline{PR})$. Repeat this procedure for *M'* and *N'*.



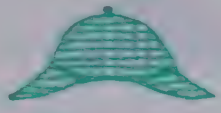
Do Questions 3, 4, and 5 in Exercise 9·5.

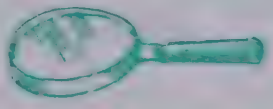


After doing Questions 1 through 5 in Exercise 9·5, you will notice that the triangle formed by an enlargement is related in a special way to the original triangle. In mathematics, we say that the two triangles are *similar*.



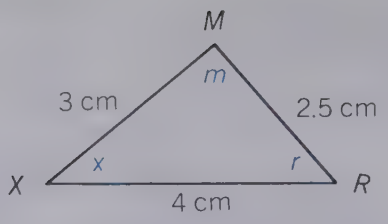
Triangle *ABC* is enlarged to form triangle *A'B'C'*.
The two triangles are *similar triangles*.
Sides *AB* and *A'B'* are corresponding sides.
Sides *BC* and *B'C'* are corresponding sides.
Sides *AC* and *A'C'* are corresponding sides.
Angles *A* and *A'* are corresponding angles.
Angles *B* and *B'* are corresponding angles.
Angles *C* and *C'* are corresponding angles.

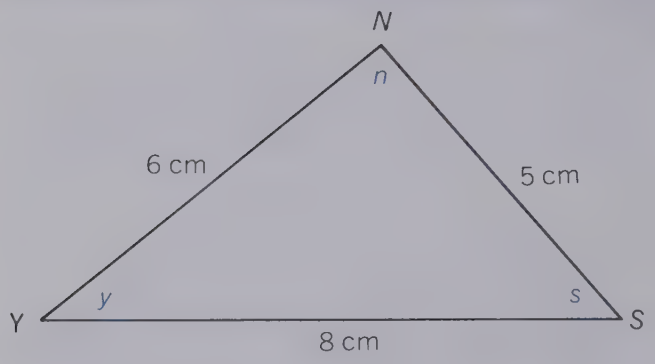




Do Question 6 in Exercise 9·5.

Study $\triangle XMR$ and $\triangle YNS$.





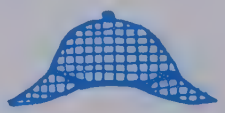
If we compare the ratios of corresponding sides, we find that the ratios are equal.

$$\frac{6}{3}=2 \qquad \frac{5}{2.5}=2 \qquad \frac{8}{4}=2$$

If we measure the angles, we find that the corresponding angles have equal measures.

$$x=y \qquad m=n \qquad r=s$$

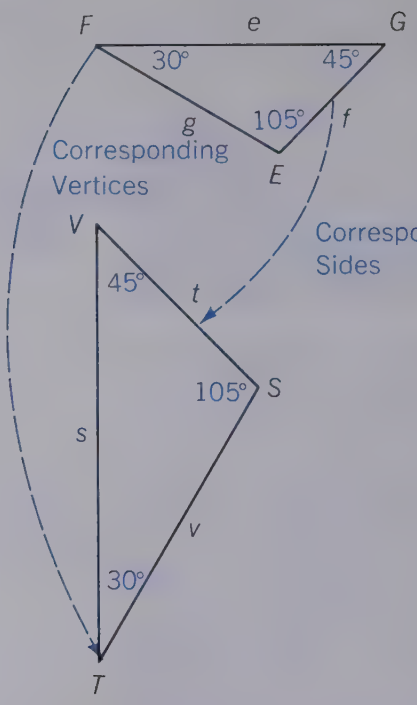
(Note that lower case letters are used as measures of the respective angles.)



If the ratios of the corresponding sides of two triangles are equal, then the corresponding angles are congruent and the triangles are similar.

Do Question 7 in Exercise 9·5.

Study $\triangle EFG$ and $\triangle STV$.

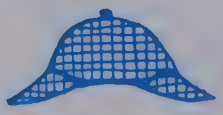


Note! One of the triangles has been turned. Do not let this confuse you. Which angles are corresponding angles?

We see that the measures of corresponding angles are equal.

Now measure the sides and compare the ratios of corresponding sides.

Do you find that $\frac{s}{e}=1.5$, $\frac{v}{g}=1.5$, and $\frac{t}{f}=1.5$?



If the corresponding angles of two triangles are equal, then the ratios of the measures of the corresponding sides are equal and the two triangles are similar.

EXERCISE 9·5

1. Refer to Example 1 in Section 9·5.

   (a) Compare these ratios:

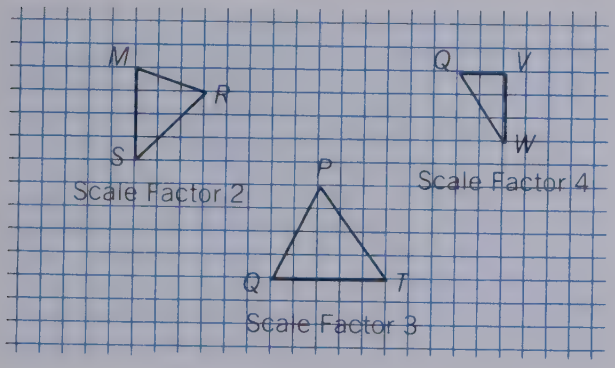
   $$\frac{m(\overline{PA'})}{m(\overline{PA})} \qquad \frac{m(\overline{PB'})}{m(\overline{PB})} \qquad \frac{m(\overline{B'C'})}{m(\overline{BC})}$$

   What do you observe?

   (b) Compare the measures of:
   (i) angles A and A';
   (ii) angles B and B';
   (iii) angles C and C'.

   What do you observe?

2. Enlarge each triangle by the scale factor indicated. Use grid paper.



3. Refer to Example 2 in Section 9·5.

   (a) Compare these ratios:

   $$\frac{m(\overline{M'N'})}{m(\overline{MN})} \qquad \frac{m(\overline{N'R'})}{m(\overline{NR})} \qquad \frac{m(\overline{M'R'})}{m(\overline{MR})}$$

   What do you observe?

   (b) Compare the measures of:
   (i) $\angle N$ and $\angle N'$;
   (ii) $\angle M$ and $\angle M'$;
   (iii) $\angle R$ and $\angle R'$.

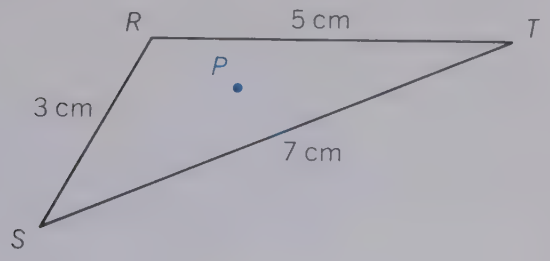
   What do you observe?

4. Draw any triangle on your page but don't make it too large.

   (a) Enlarge the triangle by a scale factor of 2 using a compass and straightedge.

   (b) Repeat (a) for a scale factor of 3.

5. Copy this figure in the centre of a sheet of paper.



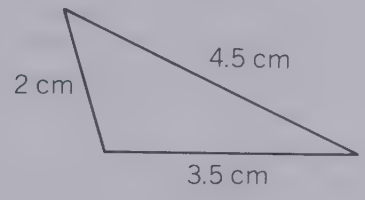
   (a) Enlarge the triangle by a scale factor of 2.

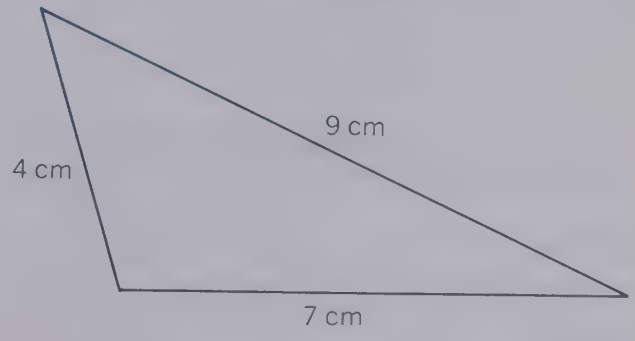
   (b) Enlarge the triangle by a scale factor of 3.

   (c) Enlarge the triangle by a scale factor of 4.

6. Refer to Questions 4 and 5. Compare the measures of corresponding angles in each set of triangles. Compare the ratios of corresponding angles in each set of triangles. What are your observations?

7. Study the following triangles:



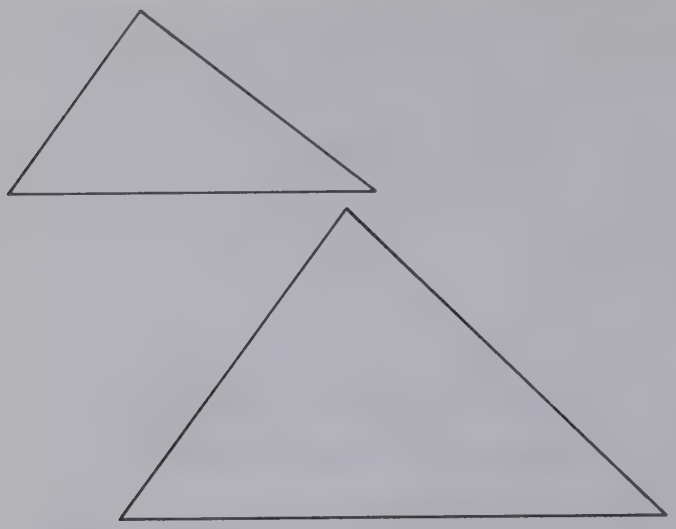


   (a) Are the ratios of the corresponding sides equal?

   (b) Measure the angles.

   (c) Are the measures of corresponding angles equal?

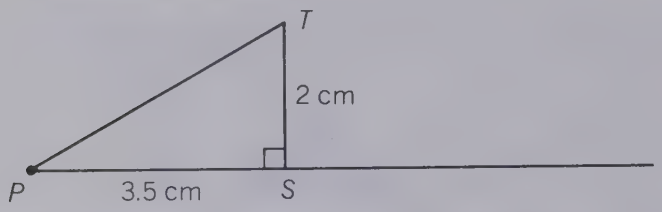
8. Use your protractor and ruler to measure the angles and the sides of these triangles.



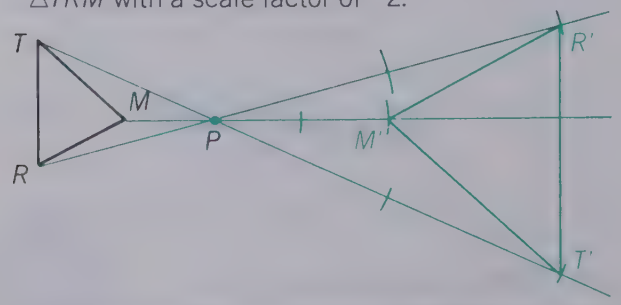
(a) What do you observe about the angles?

(b) What do you observe about the ratios of the measures of the sides?

9. Draw the following diagram in your notebook. Use point P and enlarge $\triangle PST$ by a scale factor of 3. (*Hint!* Point P becomes P' and does not move.) Compare the ratios of the measures of the corresponding sides.



10. Notice how we perform the enlargement of $\triangle TRM$ with a scale factor of -2.



(a) Tell how this enlargement differs from those we did earlier in this section.

(b) Name the corresponding sides and the corresponding angles.

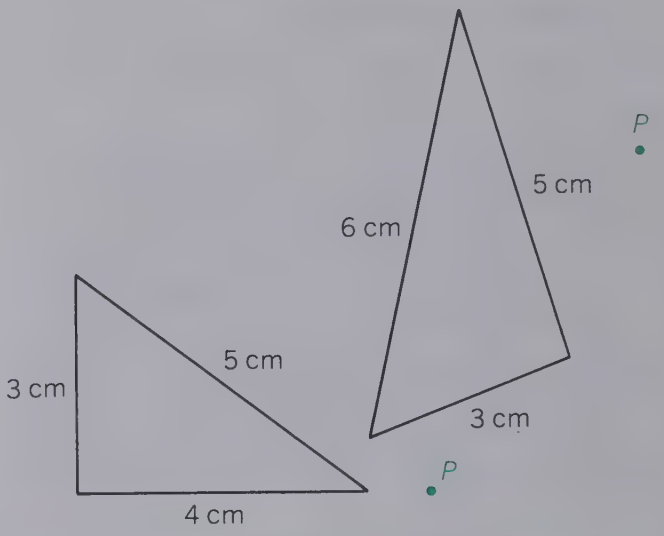
(c) How do the positions of the triangles compare?

(d) How do the ratios of the measures of corresponding sides compare?

(e) How do the measures of the corresponding angles compare?

(f) Why do you think we call this a scale factor of *negative two*?

11. Enlarge each of these triangles by a scale factor of -2 using the point P indicated in each case.



9·6 Using Similar Triangles

We will use two properties from the previous sections:

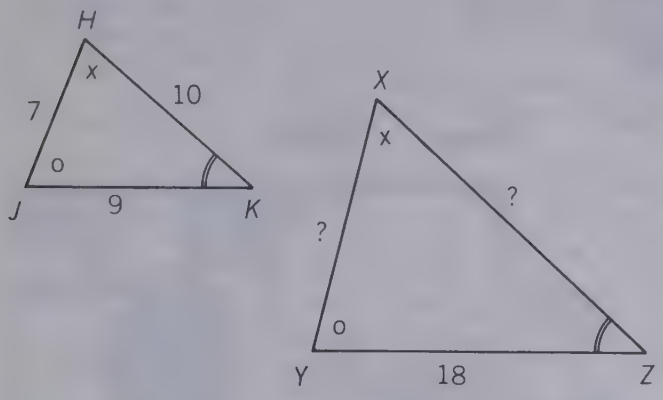
(a) If two triangles have corresponding angles congruent, then the two triangles are similar.

(b) If two triangles are similar, the ratios of the measures of corresponding sides are equal.

$$\frac{d}{a} = \frac{e}{b} = \frac{f}{c}$$

Example 1

Triangles *HJK* and *XYZ* are similar triangles. If the measures of the sides are as on the diagrams, find the measures of sides *XY* and *XZ*.



Solution

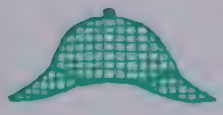
Because the two triangles are similar,

$$\frac{m(\overline{JK})}{m(\overline{YZ})} = \frac{m(\overline{JH})}{m(\overline{YX})} = \frac{m(\overline{HK})}{m(\overline{XZ})}$$

By substitution, in $\frac{m(\overline{JK})}{m(\overline{YZ})} = \frac{m(\overline{JH})}{m(\overline{YX})}$

$$\frac{9}{18} = \frac{7}{z} \quad \text{(Let measure of side } YX = z.\text{)}$$

Remember:



If $\frac{a}{b} = \frac{c}{d}$

then $ad = bc$.

$$9z = 18 \times 7$$

$$\frac{9z}{9} = \frac{126}{9} \quad \text{(Divide each side by 9.)}$$

$$z = 14$$

Therefore, the measure of side *YX* is 14.

Similarly, $\frac{m(\overline{JK})}{m(\overline{YZ})} = \frac{m(\overline{HK})}{m(\overline{XZ})}$

and $\frac{9}{18} = \frac{10}{y}$ (Let $m(\overline{XZ}) = y$.)

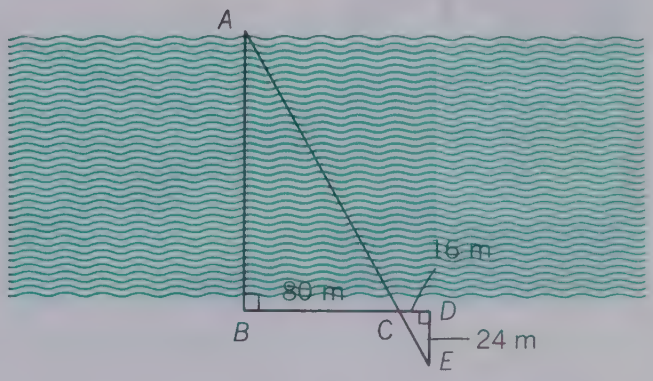
$$9y = 10 \times 18$$

$$\frac{9y}{9} = \frac{180}{9} \quad \text{(Divide each side by 9.)}$$

$$y = 20$$

Therefore, the measure of side *XZ* is 20.

Example 2



To find the distance across a river, Ted and Jim spotted an object on the far bank at *A*. They measured off a line along the river bank so that the distance $\overline{BC}$ was 80 m. They chose points *B* and *C* so that angle *ABC* was a right angle. They extended *C* to *D* making the distance $\overline{CD}$ 16 m. At *D*, they marked off another right angle and located *E* so that (i) $\angle CDE$ was a right angle, and (ii) points *E*, *C*, and *A* were in a straight line. They found the distance $\overline{DE}$ to be 24 m. What was the width of the river?

Solution

First, triangles *ABC* and *CDE* are similar. Can you see why?

Then, $\frac{m(\overline{AB})}{m(\overline{ED})} = \frac{m(\overline{BC})}{m(\overline{DC})}$ (Do you agree?)

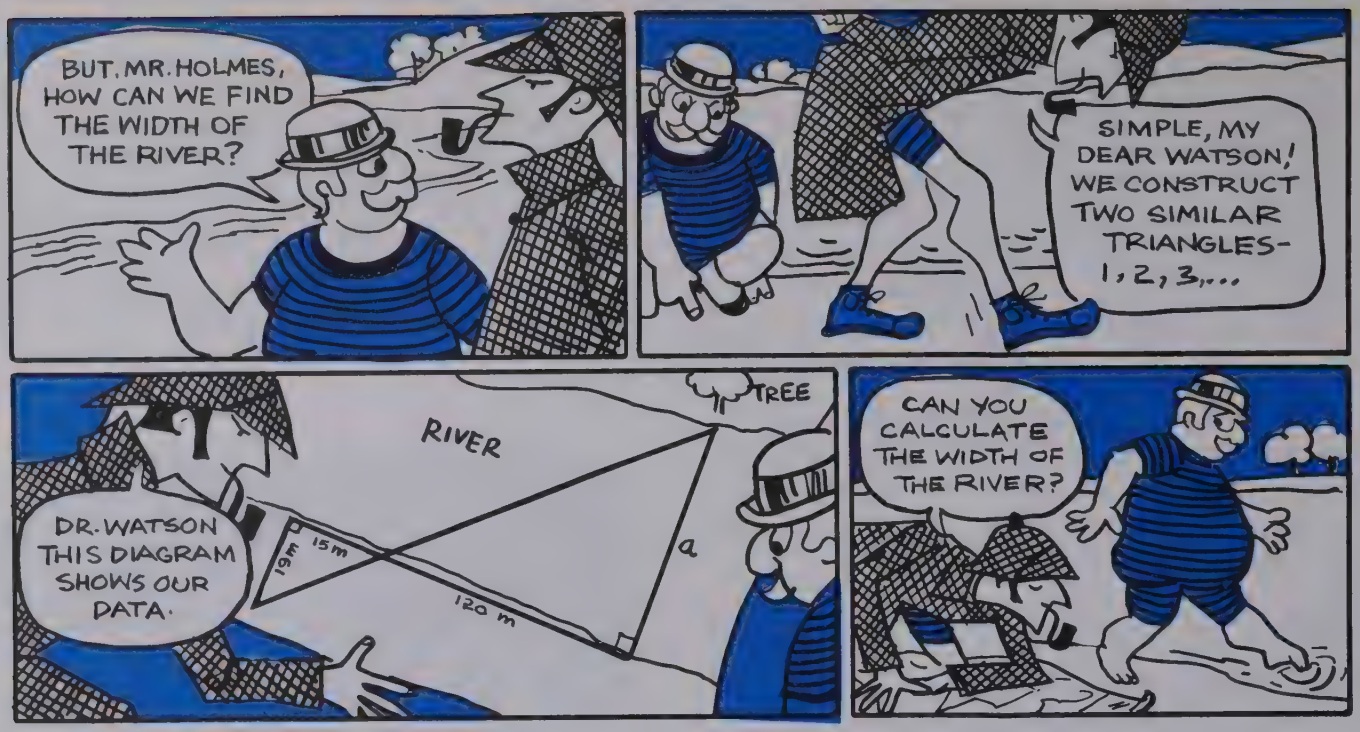
By substitution, $\frac{x}{24} = \frac{80}{16}$ (Let $m(\overline{AB}) = x$.)

$$16x = 80 \times 24$$

$$x = \frac{1920}{16} \quad \text{(Divide each side by 16.)}$$

$$x = 120$$

The river is 120 m wide from point *A* to point *B*.

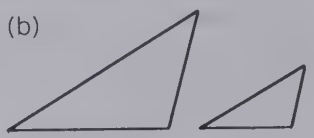

BUT, MR. HOLMES, HOW CAN WE FIND THE WIDTH OF THE RIVER?
SIMPLE, MY DEAR WATSON! WE CONSTRUCT TWO SIMILAR TRIANGLES- 1, 2, 3,...
TREE
RIVER
DR. WATSON THIS DIAGRAM SHOWS OUR DATA.
19m
15m
120 m
a
CAN YOU CALCULATE THE WIDTH OF THE RIVER?

EXERCISE 9·6

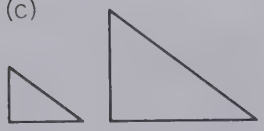
1. By observation, which pairs of triangles appear to be similar and which pairs do not?

(a)

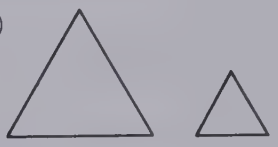
(b)



(c)



(d)



(e)

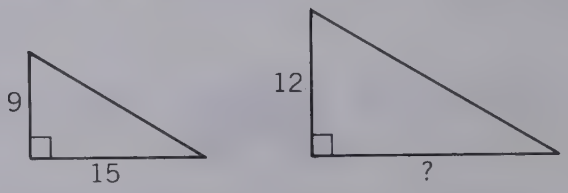
(f)

(g)

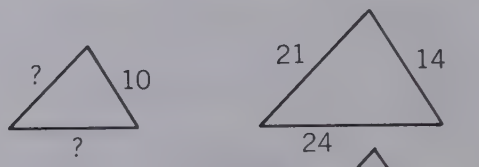
(h)

2. Name the corresponding sides for each of the following pairs of triangles. Find the measures of the sides marked with "?". The triangles in each pair are similar.

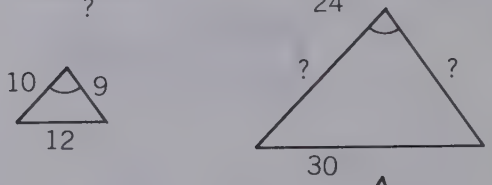
(a)



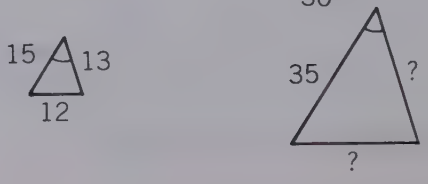
(b)



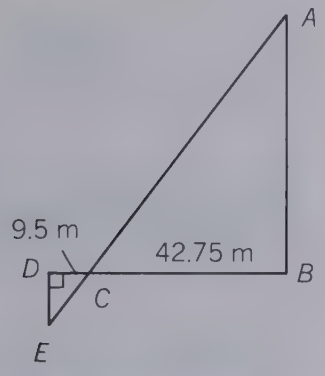
(c)



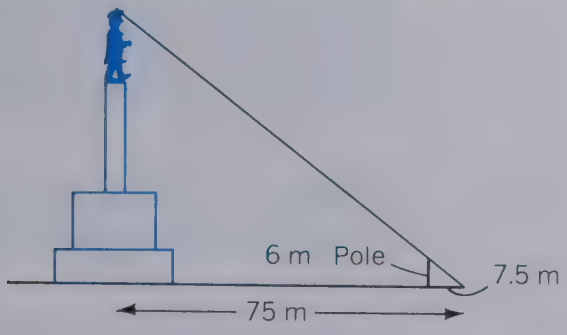
(d)



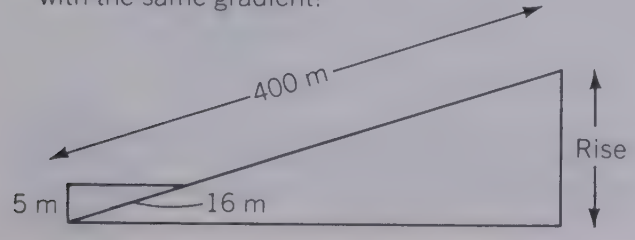
3. To find the width of a river between points A and B, Mark and Len laid out a line BD parallel to the river and perpendicular to the side AB. Then, point E was marked so that the side DE measured 12 m and was perpendicular to line DB. Point C was located on $\overline{BD}$ so that C was collinear (in a straight line) with points E and A. The distance $\overline{DC}$ was found to be 9.5 m and the distance $\overline{BC}$ was found to be 42.75 m. Find the measure of the distance $\overline{AB}$.



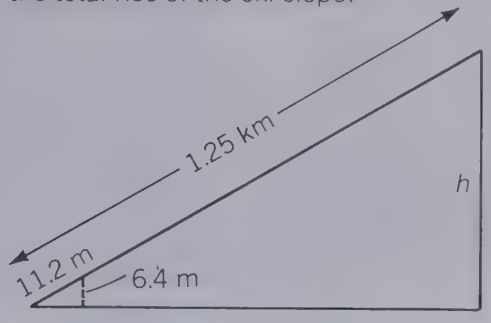
4. Susan and Jane measured the shadow cast by a monument to be 75 m. At the same time, an upright pole, 6 m high, was found to cast a shadow 7.5 m long. How high was the monument? (*Hint!* Identify the two similar triangles.)



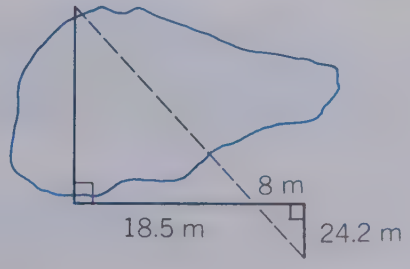
5. With the aid of a transit, Jim found the rise in a road gradient to be 5 m for every 16 m of road travelled. How high is the rise for 400 m of road with the same gradient?



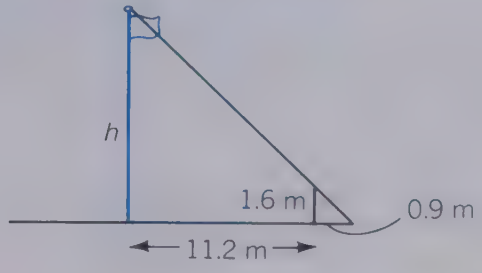
6. Jill and Bill approximated the gradient of a ski slope at 6.4 m rise for each 11.2 m of slope. If the full slope of 1.25 km has the same gradient, what is the total rise of the ski slope?



7. Calculate the width of the lake.



8. Calculate the height of the flagpole.



9. Use the procedure in Question 3 to calculate the width of:
   (a) a street near your playground;
   (b) an imaginary river that you mark out on your playground.

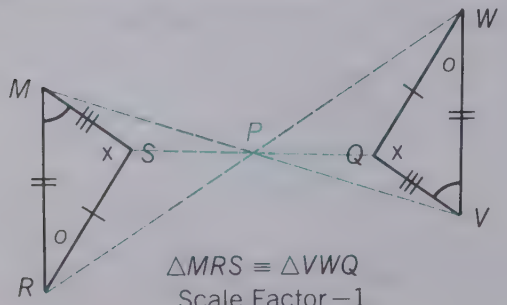
10. On a sunny day, use the method of Question 4 to find the height of:
   (a) a flagpole (b) a streetlight pole
   (c) a telephone pole (d) a tree

What is an electrocardiogram? The record made by an electrocardiogram is a graph—an electrocardiograph. A doctor must interpret the graph to determine the condition of a patient's heart.

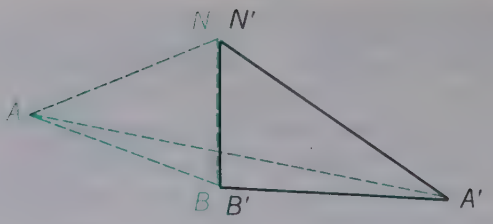
9·7 Congruent Triangles

Similar triangles with an enlargement scale factor of 1 or −1 are congruent triangles.

The symbol for *congruent* is ≡.



$\triangle MRS \equiv \triangle VWQ$
Scale Factor −1

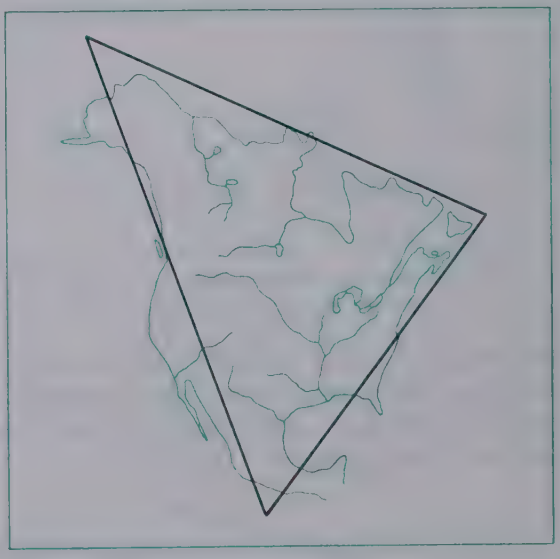


$\triangle NBA \equiv \triangle N'B'A'$
Scale Factor 1

Do Questions 1, 2, and 3 in Exercise 9·7.

If we are required to draw two congruent triangles, what is the minimum amount of information we need to know? The following three constructions and the exercises will help you answer this question.

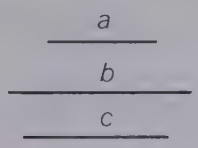
There are three basic triangle constructions, depending in each case on the particular measurements given. Let us learn how to do each construction. Every triangle has six distinct measurable parts: three angles and three sides.

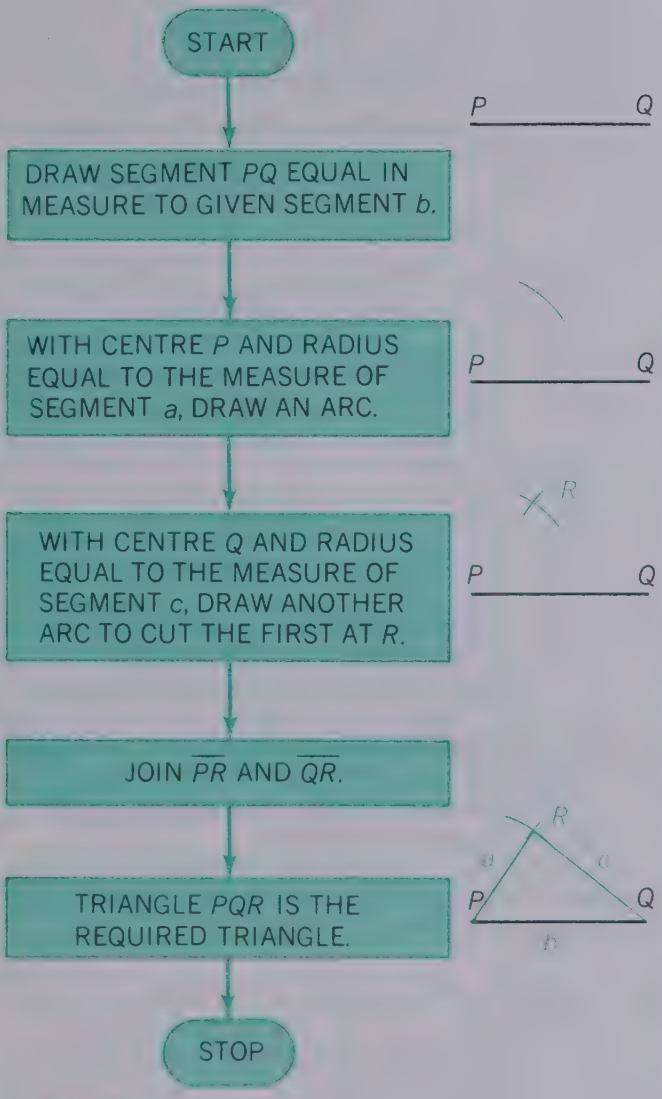


North America can be compared to a triangle. Draw a triangle with sides twice as long as those in the diagram. Sketch a map of North America over your triangle. How does your diagram compare to this one?

To construct a triangle given three sides:

Let a, b, and c represent the measures of the three sides of the required triangle.

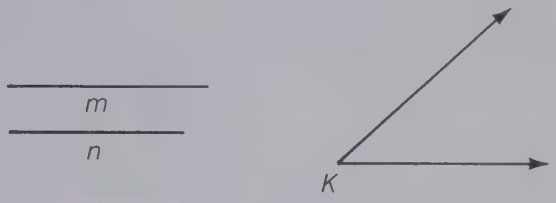


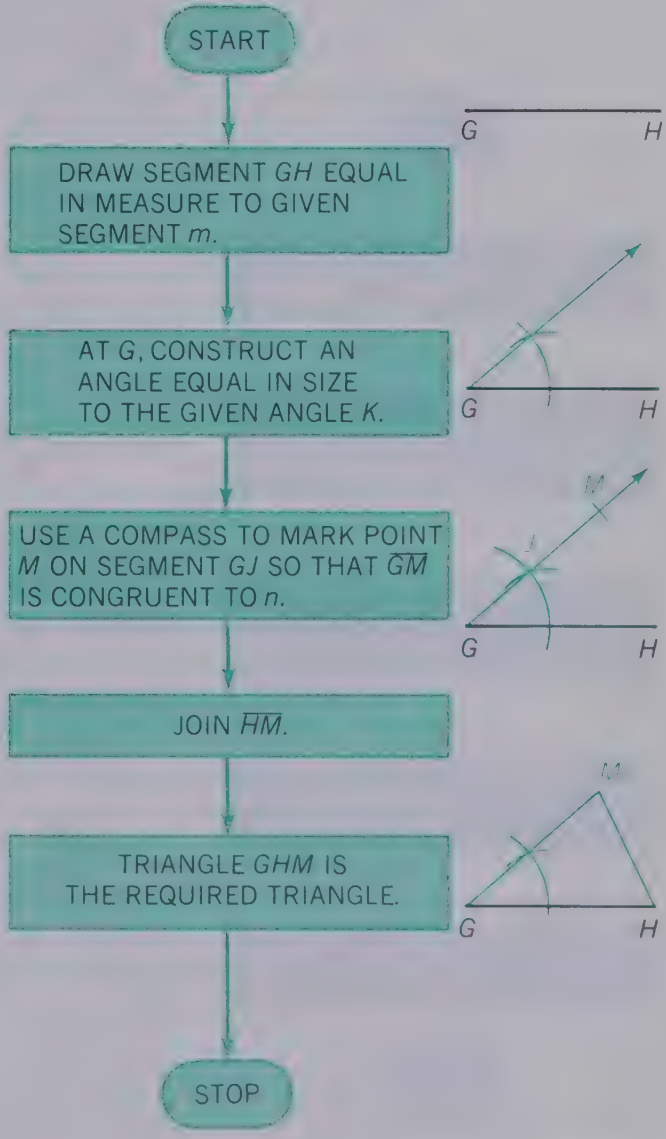


Do Questions 4 and 5 in Exercise 9·7.

To construct a triangle given two sides and the angle contained by them:

Let segments m and n represent the measures of the two sides and let angle K represent the contained angle. (This means that segments m and n must be the arms of angle K.)

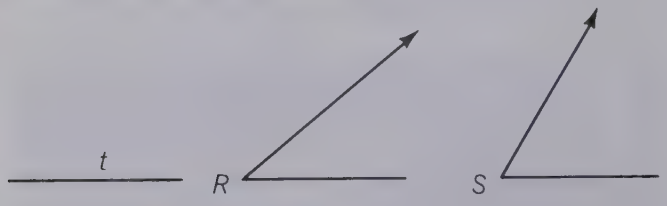


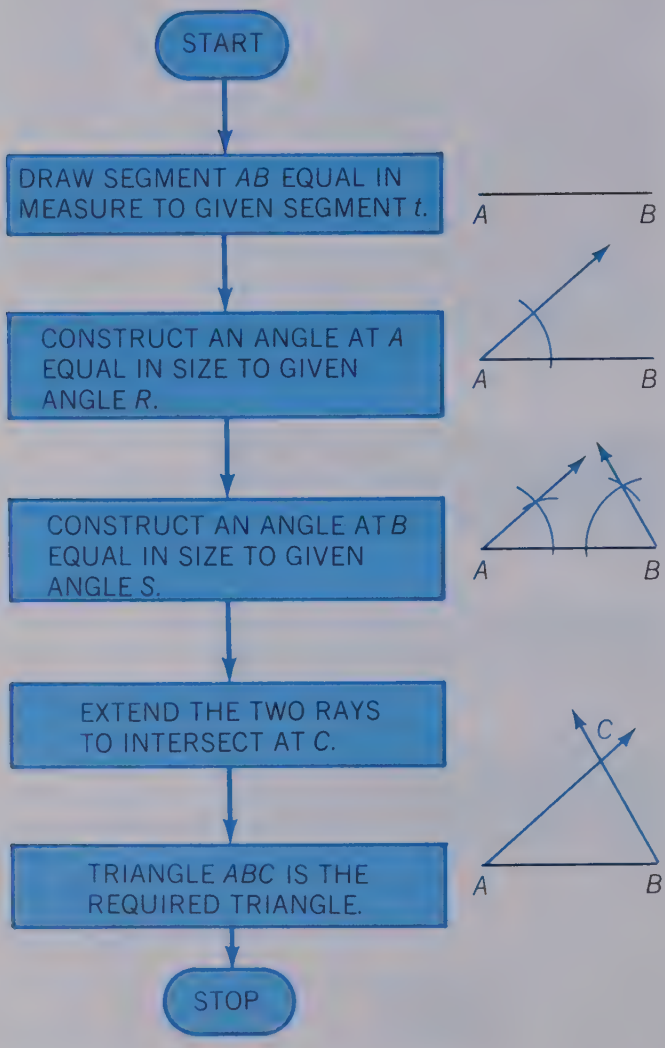


Do Questions 6 and 7 in Exercise 9·7.

To construct a triangle given two angles and the side common to them:

Let segment t and angles R and S represent the given parts of the required triangle.





EXERCISE 9·7

1. $\triangle CRY \equiv \triangle TEA$

From the diagrams, determine:

(a) which angles are congruent;

(b) which sides are congruent.

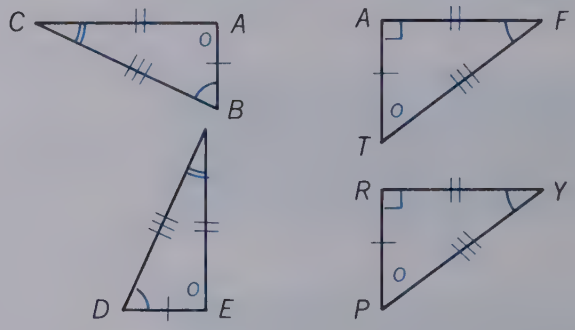
Have you noticed the way in which the statement is written?

$$\triangle CRY \equiv \triangle TEA$$

What do you notice?

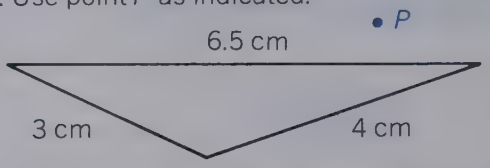
Why do you think it is written this way?

2. Note how we indicate which sides and angles are congruent.



Write a statement in symbols saying that each pair of triangles is congruent.

3. Copy and enlarge this triangle by a scale factor of -1. Use point P as indicated.



(a) Compare the measures of corresponding angles.

(b) Compare the lengths of corresponding sides.

(c) What do you observe?

Construct each of the following triangles. Use care and precision in your constructions (a sharp pencil is needed). In each case, compare your triangle with a classmate. Are your triangles congruent?

4. Triangle ABC with $m(\overline{AB}) = 12$ cm, $m(\overline{BC}) = 10.5$ cm, and $m(\overline{AC}) = 9$ cm.

5. Triangle MNP with $m(\overline{MN}) = 8.2$ cm, $m(\overline{NP}) = 9.7$ cm, and $m(\overline{MP}) = 11.3$ cm.

6. Triangle GHK with $m(\overline{GH}) = 9.4$ cm, $m(\overline{HK}) = 7.8$ cm, and $m\angle H = 85°$.

7. Triangle STV with $m(\overline{TV}) = 8.6$ cm, $m(\overline{SV}) = 12$ cm, and $m\angle V = 45°$.

8. Triangle PQR with $m\angle Q = 72°$, $m\angle R = 57°$, and $m(\overline{QR}) = 10.4$ cm.

9. Triangle DEF with $m\angle D = 48°$, $m\angle F = 63°$, and $m(\overline{FD}) = 12$ cm.

Accurate measurements will be necessary to answer the following questions.

10. In the triangle of Question 4, what are the measures of angles A, B, and C? What is the sum of the measures of all three angles?

11. In the triangle of Question 6, what are the measures of side GK and angles G and K? What is the sum of the measures of all three angles?

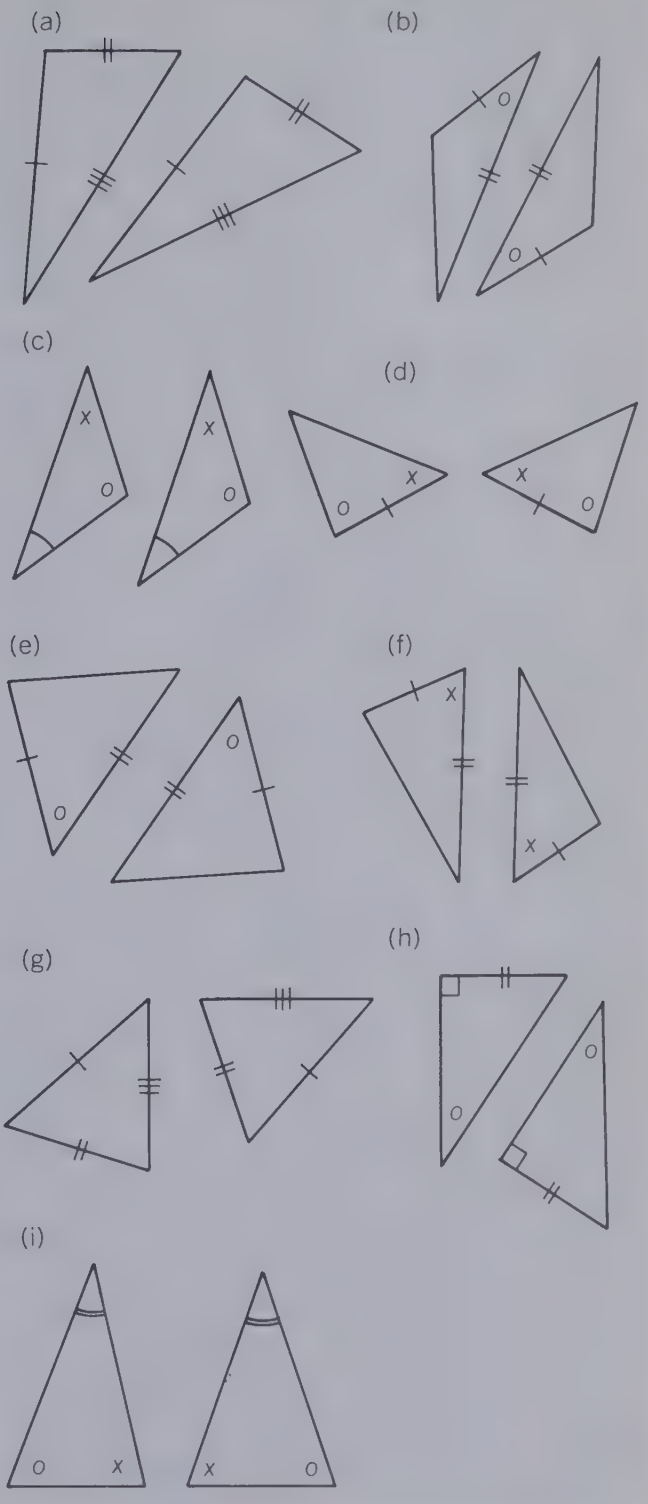
12. In the triangle of Question 7, what are the measures of angles S and T and of side ST? What is the sum of the measures of all three angles?

13. In the triangle of Question 8, what are the measures of angle P and of sides PQ and PR? What is the sum of the measures of all three angles?

14. In the triangle of Question 9, what are the measures of sides DE and EF? What is the sum of the measures of all three angles?

15. There are three different sets of information from which we can construct congruent triangles. Refer to the three constructions in Section 9·7. In your own words, state what information is required in each case.

16. Which of the following pairs of triangles are congruent? Give a reason from Question 15 for each pair of congruent triangles.



Projects

Project 2

Purpose

To study the circle and its inscribed quadrilateral(s).

Method

Use a very sharp pencil for all drawings and make all measurements with care.

Note to the Teacher: If the class is divided into five groups, each group need make only one circle. The table can be drawn on the chalkboard and each group can fill in the appropriate information. Each student then copies the table into his book.

Draw five circles with radii greater than 5 cm. Label the circles by numbers 1, 2, 3, 4, and 5.

1. Inscribe a different quadrilateral in each of the five circles. (*To inscribe is to construct on the inside so that each vertex is touching the circle.*)

2. Label the vertices of the quadrilateral in circle 1 in this order: A_1, B_1, C_1, D_1.

   Label the vertices of the quadrilateral in circle 2 as: A_2, B_2, C_2, D_2.

   Label the rest in a similar fashion.

3. Very carefully measure each angle of the quadrilaterals to the nearest degree. Copy this table in your exercise book and enter the angle measures.

Complete the table. The data collected is approximate. Keep this in mind when answering the following questions.

Questions

1. What did you observe about the sums of the measures of angles A and C?

2. What did you observe about the sums of the measures of angles B and D?

3. What relation is the measure of angle C to the measure of angle A in each quadrilateral?

4. What relation is the measure of angle D to the measure of angle B in each quadrilateral?

5. What generalization can you make about the angles of a quadrilateral inscribed in a circle? Do you believe it will hold for all inscribed quadrilaterals in any size of circle?

Quadrilateral	Measures of Angles A	Angles C	Sum of Measures of A and C	Measures of Angles B	Angles D	Sum of Measures of B and D
$A_1B_1C_1D_1$						
$A_2B_2C_2D_2$						
$A_3B_3C_3D_3$						
$A_4B_4C_4D_4$						
$A_5B_5C_5D_5$						

STOP

Do Questions 1 and 2 in Exercise 9·8.

Project 3

Purpose

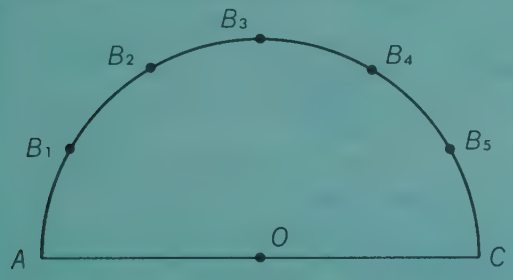
To study the angle inscribed in a semicircle.

Method

Again, use very sharp pencils for all constructions and measure very carefully.

See the Note to the Teacher in Project 2.

1. Construct five semicircles and label them 1, 2, 3, 4, and 5.
2. On each semicircle, draw a diameter AC through the centre O as shown in the diagram.



3. On semicircle 1, locate point B_1 in about the position shown in the diagram.
4. Join AB_1 and B_1C.
5. On semicircle 2, locate B_2 in about the position shown in the diagram.
6. Join AB_2 and B_2C.
7. Continue the same pattern for semicircles 3, 4, and 5.
8. Carefully measure each angle ABC for the five triangles and record the measurement on a copy of the following table.

Triangle	AB_1C	AB_2C	AB_3C	AB_4C	AB_5C
Measure of	$\angle AB_1C$	$\angle AB_2C$	$\angle AB_3C$	$\angle AB_4C$	$\angle AB_5C$

Questions

1. What relation do you observe among the measures of the five angles?
2. What kind of angle does each angle ABC form?
3. Can you generalize your observation? Do you believe this holds true for all sizes of semicircles?
4. Are all your measurements exact? What do you know about approximate numbers?

STOP

Do Question 3 in Exercise 9·8.

Project 4

Purpose

To study one property of the perpendicular bisector of a chord of a circle.

Method

See the Note to the Teacher in Project 2.

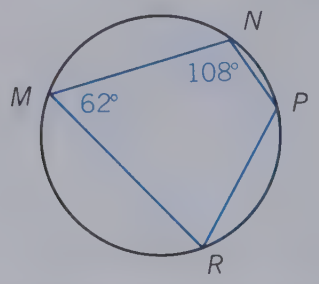
1. Draw four circles with radii between 5 and 10 cm.
2. Choose any two points on one circle and join them. Now you have a chord.
3. Construct a perpendicular bisector of the chord.
4. Draw chords for each of the other three circles.
5. Construct perpendicular bisectors of each of these chords.

Questions

1. What do you observe about the relation between the perpendicular bisector of each chord and the centre of the circle?
2. Would every perpendicular bisector of every chord of a circle pass through the centre?
3. Is a diameter of a circle a chord? Where is the centre of the circle in relation to the diameter?
4. Does the perpendicular bisector of a diameter pass through the centre?

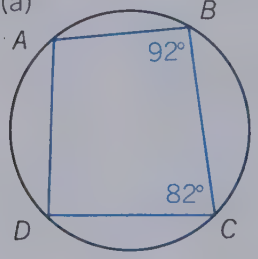
EXERCISE 9·8

1. If quadrilateral $MNPR$ is inscribed in a circle so that angle M measures $62°$ and angle N measures $108°$, find the measures of angles P and R.

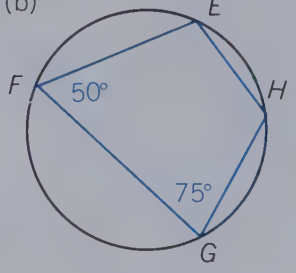


2. Calculate the measures of each of the unknown angles in these quadrilaterals.

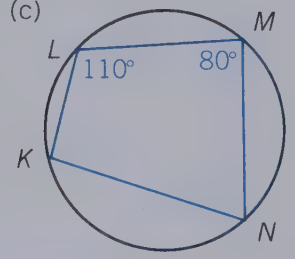
(a)



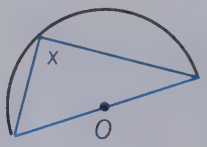
(b)

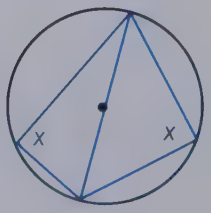


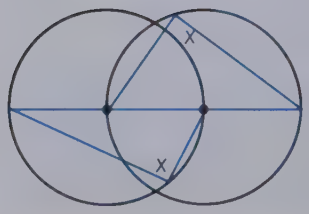
(c)



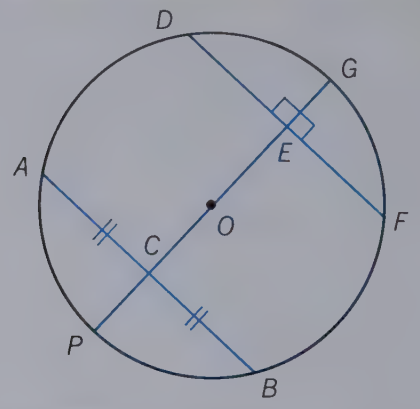
3. Calculate the measure of each angle indicated by x.





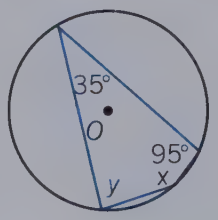


4. What can you tell about:
   (a) the measures of segments DE and EF?
   (b) the measures of angles ACP and BCP?

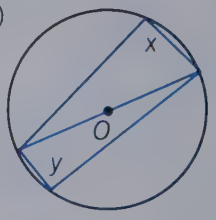


5. Calculate those angles indicated by the variables x and y in each diagram. The centre of each circle is indicated by O.

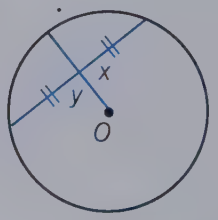
(a)



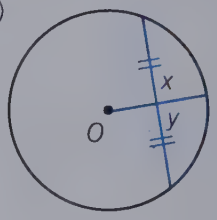
(b)



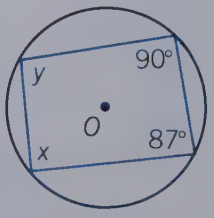
(c)



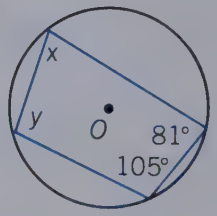
(d)



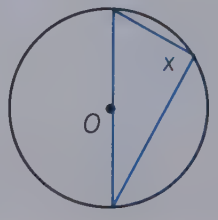
(e)



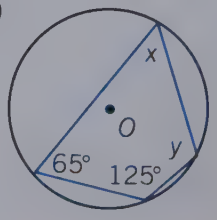
(f)



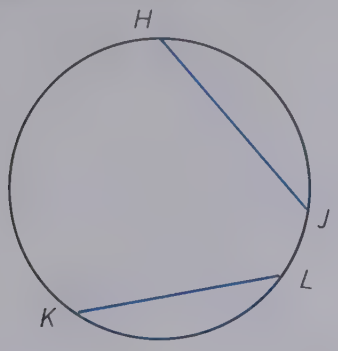
(g)



(h)



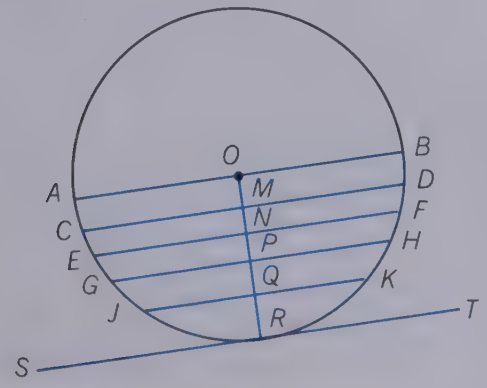
6. Two chords HJ and KL are given for the accompanying circle. Locate the centre of this circle. Recall Project 4 of this chapter.



7. Draw a circle with a radius of 5 cm. On the circle, draw a chord 8 cm long. Find the centre of the chord by measurement. Join the centre of the chord to the centre of the circle. Measure the sizes of the angles formed at the centre of the chord. Will the sizes of the angles always be the same if you repeat this construction for other chords of the circle?

   What conclusion do you make about joining the centre of the circle to the midpoint of the chord?

8. For the circle shown in the diagram, the radius OR is perpendicular to the diameter AB. Chords CD, EF, GH, and JK are all parallel to diameter AB. Line segment ST is also parallel to diameter AB and passes through point R on the circumference of the circle.



   (a) What angles are formed at points M, N, P, Q, and R?

   (b) Does that make radius OR perpendicular to each chord and to segment ST?

   (c) If OR is perpendicular to chord CD, what does OR do to CD at M?

   (d) What kind of points are N, P, and Q of their respective chords?

   (e) ST is *tangent* to the circle at point R. A tangent intersects a circle at one point only. Can $\overline{SRT}$ be tangent to the circle when $\overline{OR}$ is not perpendicular to $\overline{SRT}$? Discuss.

9. Construct a right triangle given that the hypotenuse is equal to 12 cm and that one side of the triangle is equal to 7 cm. *Hint!* Construct a semicircle with a diameter equal to 12 cm. Apply your study of Project 3 to complete the construction.

10. Construct a right triangle with a hypotenuse of 10 cm and one side of 6 cm. Measure the third side of the triangle to the nearest centimetre. Use the Pythagorean Relation, $a^2 + b^2 = c^2$, to check on the measurements of the triangle.

Use only a compass and straightedge for questions 11 to 14.

11. Draw any quadrilateral and bisect each of the four sides. Now join successive midpoints and describe your new quadrilateral. Do you think you always get the special new figure? Repeat this several times with quadrilaterals of different shapes to check your conclusion.

12. Draw any quadrilateral and bisect the angles. Do the bisectors *always* meet at one common point? Are there special quadrilaterals in which the bisectors always meet at a common point?

13. Draw any three triangles.

   (a) In the first, bisect the angles. What do you notice about the bisectors?

   (b) Bisect a side in the second triangle and join the midpoint of the side and the vertex opposite. This line is called a median. Now construct the medians of the other two sides. What do you notice about the three medians?

   (c) Construct perpendicular bisectors of each side of the third triangle. What do you notice about the three perpendicular bisectors?

14. In the figure you drew for Question 13(a), place a compass point on the intersection of the three bisectors. Can you draw a circle inside the triangle so that it just touches all three sides?

    In the figure you drew for Question 13(b), place a compass point on the intersection of the three medians. Can you draw a circle outside the triangle so that it just touches all three vertices of the triangle?

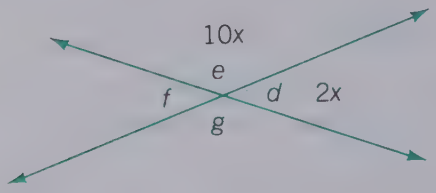
    In the figure you drew for Question 13(c), place a compass point on the intersection of the three perpendicular bisectors. Can you draw a circle that "fits" the triangle in any way? Describe any such circle.

9·9 Geometry and Algebra Combined

We can use algebra to solve problems related to the geometry we have been doing.

Example 1

Solve for the measure of each angle d, e, f, and g.



Solution

We know that angles d and e are supplementary; that is, we know the sum of the measures is 180°.

$$\begin{aligned} m\angle d + m\angle e &= 180° \\ 2x + 10x &= 180° \\ 12x &= 180° \\ x &= 15° \end{aligned}$$

Therefore, $2x = 30°$

and $10x = 150°$

Therefore, $m\angle d = 30°$

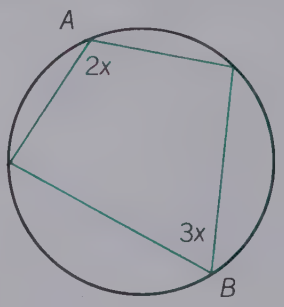
$m\angle e = 150°$

$m\angle f = 30°$ (Because $\angle f$ is vertically opposite to $\angle d$.)

$m\angle g = 150°$ (Because $\angle g$ is vertically opposite to $\angle e$.)

Example 2

Solve for the measures of $\angle A$ and $\angle B$.

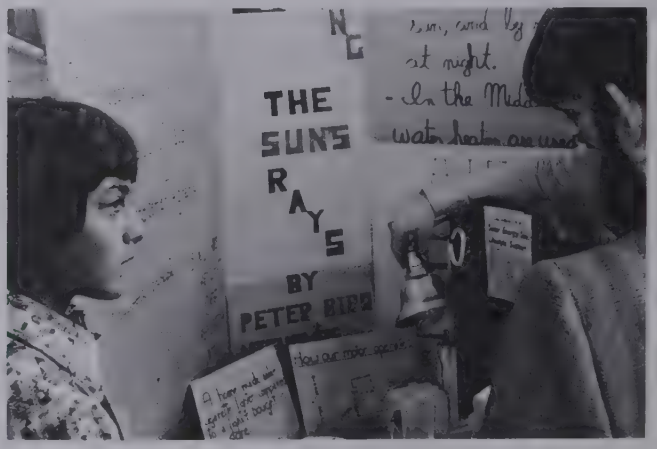


Solution

We know the sum of the opposite angles of an inscribed quadrilateral is 180°.

$$\begin{aligned} 2x + 3x &= 180° \\ 5x &= 180° \\ x &= 36° \\ 2x &= 72° \\ 3x &= 108° \end{aligned}$$

Therefore, the measure of $\angle A = 72°$ and $\angle B = 108°$.

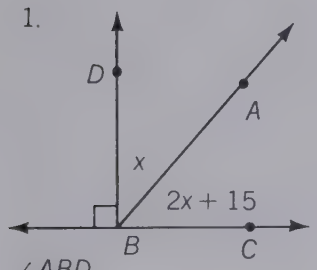


By using mathematical and scientific concepts, these students have illustrated how the sun's energy can be harnessed. This use of solar energy is especially appropriate because of the energy crisis in recent years.

EXERCISE 9·9

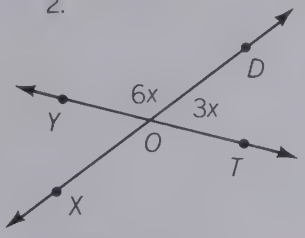
Calculate the measure of each indicated angle.

1.



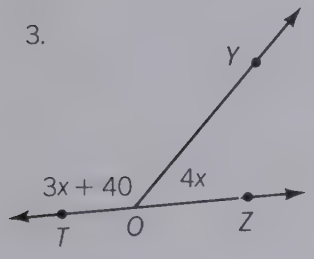
∠ABD
∠ABC

2.



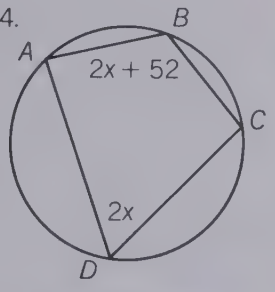
∠DOT ∠DOY
∠YOX ∠TOX

3.



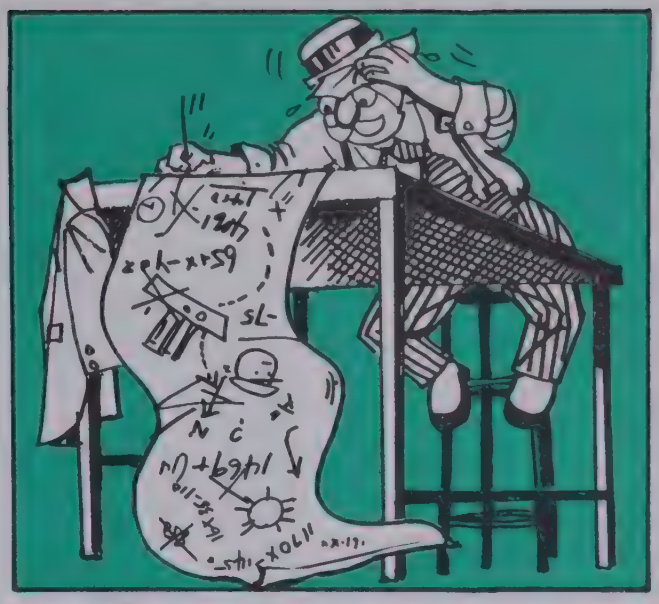
∠TOY ∠YOZ

4.



∠ADC ∠ABC

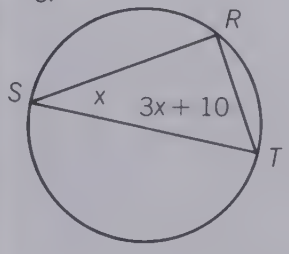
A large combine can harvest 40 ha a day. Each hectare may yield 45 t of wheat. Calculate the number of hectares and the number of tonnes of wheat that can be harvested by 6 combines in 20 days.



Since you are now studying geometry and trigonometry, I will give you a problem. A ship sails the ocean. It left Boston with a cargo of wool. It grosses 20 t. It is bound for Le Havre. . . . There are 12 passengers aboard. The wind is blowing East-North-East. The clock points to 1515. It is the month of May. How old is the captain?

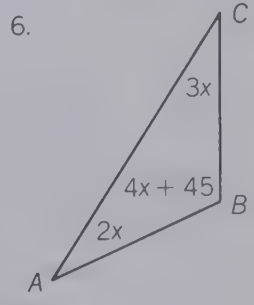
Gustave Flaubert

5.



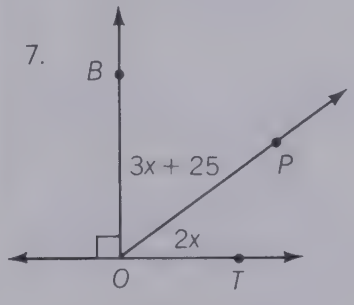
∠SRT ∠RST ∠RTS

6.



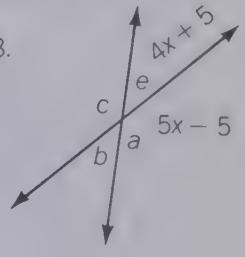
∠A ∠B ∠C

7.



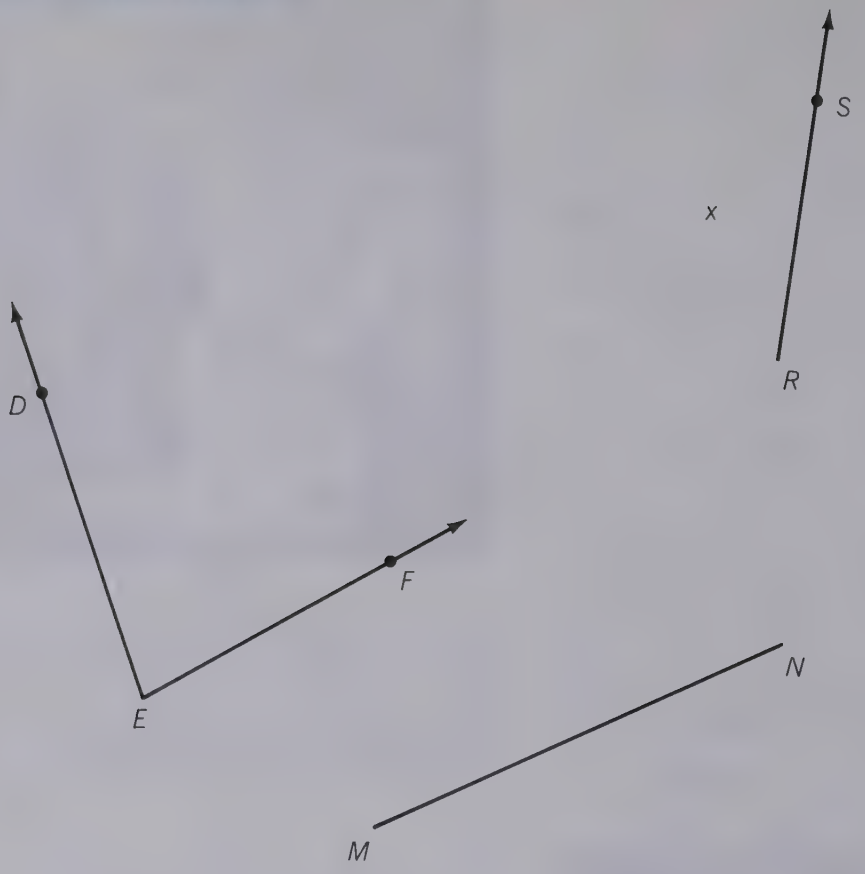
∠BOP ∠POT

8.



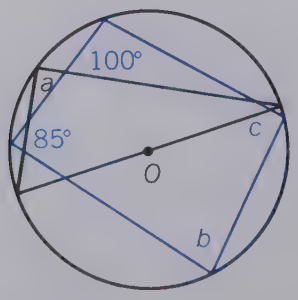
∠a ∠b
∠c ∠e

REVIEW EXERCISE 9



1. Place a sheet of paper over this page and trace the three figures. Construct a perpendicular bisector of segment MN. Construct a bisector of $\angle DEF$. Copy $\angle DEF$ on ray RS to form $\angle DRS$ with ray RD on the side of ray RS marked x. *Self-check*: If all the constructed lines or rays meet at one point, your work has been done very well.

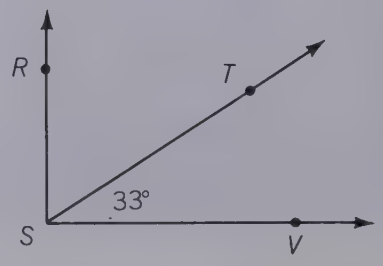
2. Calculate the measure of each angle a, b, and c.



3. Construct an equilateral triangle.

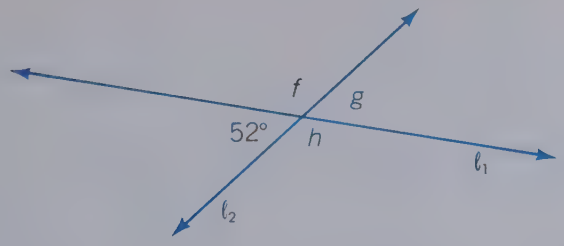
4. $\angle ABC$ and $\angle KLM$ are supplementary angles. The measure of $\angle ABC$ is 43°. What is the measure of $\angle KLM$?

5. The measure of $\angle RSV$ is 90°. What is the measure of $\angle RST$?

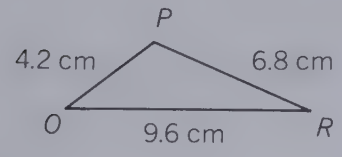


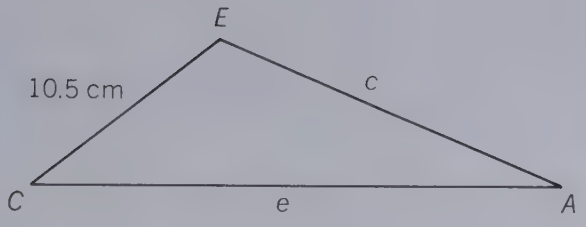
6. Construct an angle of 45° using a compass and ruler.

7. Calculate the measures of angles f, g, and h when it is given that ℓ_1 and ℓ_2 are two straight lines.

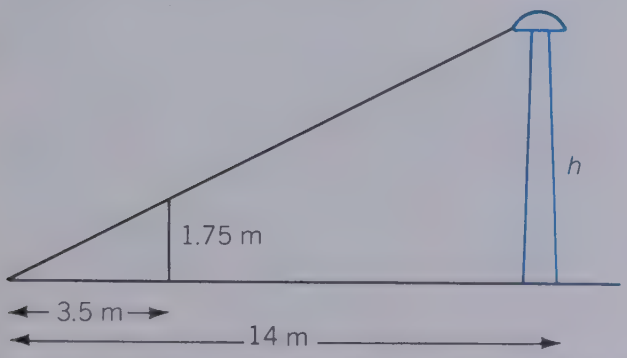


8. $\triangle ROP$ and $\triangle ACE$ are similar triangles. Calculate the measures of sides c and e.





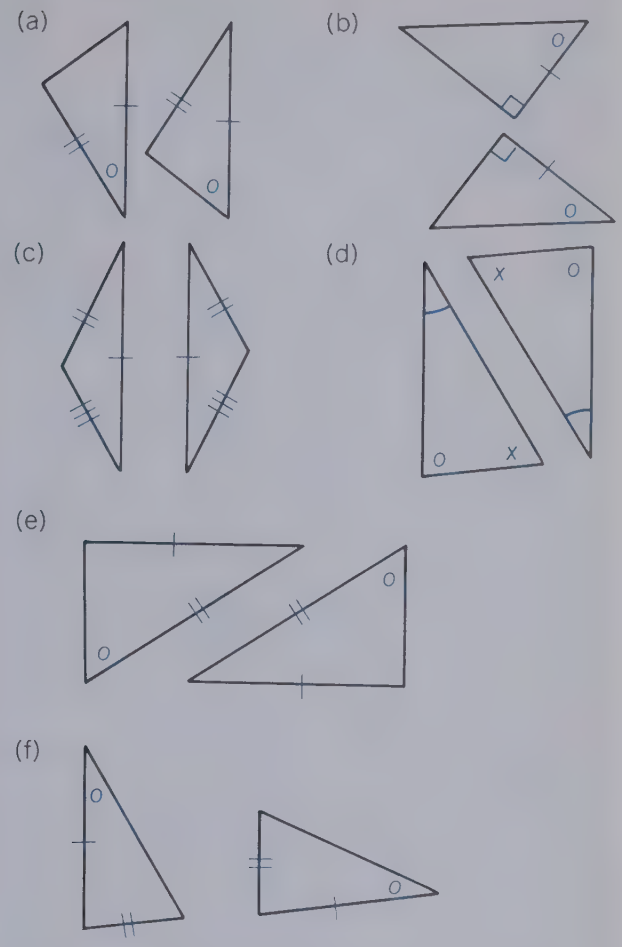
9. Calculate the height of the light pole.



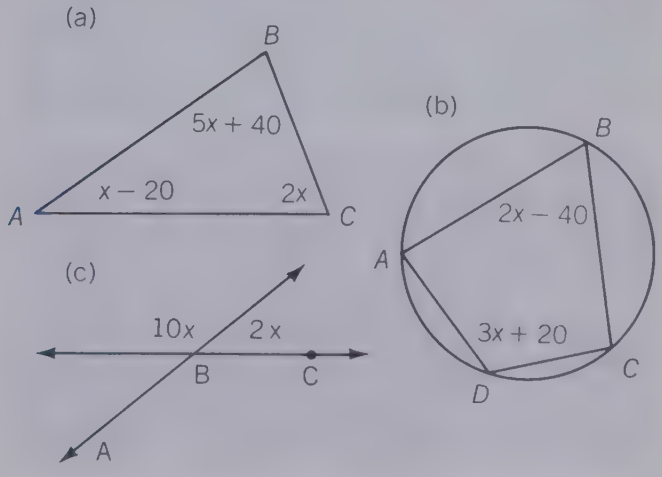
10. Draw any triangle on your paper. Enlarge it by a scale factor of 2.

11. Draw a triangle on your paper. Enlarge it by a scale factor of -3.

12. Which of the following pairs of triangles are congruent?



13. Calculate the measure of $\angle ABC$ in each case.



Using the Library

Do You Remember that . . .

. . . information may be shown by circle graphs?

1. Illustrate the information in this article by means of a circle graph.

Which color is safest?

For years, traffic safety experts have pondered causes and possible solutions to traffic accidents on streets and highways. Invariably, the main factor is the driver operating the vehicle. A study of the colors of cars involved in accidents conducted in Britain resulted in some interesting information.

The following colors were involved in auto accidents in varying degrees:

Blue	25%	Black	4%
Green	20%	Beige, brown .	3%
Grey	17%	Yellow, gold . .	2%
White, cream .	12%	Other	9%
Red, maroon .	8%		

White cars are generally considered to stand out as exceptionally safe against darker backgrounds, whereas blue cars recorded the highest percentage of accidents. Blue, of course, is a much more popular color than those such as black or beige, and obviously the number of vehicles on the road would affect the outcome of this study.

The safest color is bright orange, but few people want to drive an orange car.

Hint! Follow these steps to find the degree measure of each section.

Step 1. Change percents to fractions.

Blue $25\% = \frac{25}{100}$

Step 2. Calculate the percent of the circle measure: $\frac{25}{100} \times 360° = 90°$.

Step 3. Measure a section of 90°.

Step 4. Repeat this method for other sections.

Step 5. Title your graph.

2. Survey your class to find how many students:
   (a) have a driver's licence;
   (b) have a learner's permit;
   (c) have neither.

   Show this information in a circle graph.

3. Collect the necessary information and draw two other circle graphs.

Something to Think About

Project

Project 5

Purpose

To diagram the Pythagorean Relation that $c^2 = a^2 + b^2$ in the Figure 9-3.

Method

On a large sheet of paper, very accurately construct a right triangle with $b = 8$ cm, $a = 6$ cm, and $c = 10$ cm. First, draw $b = 8$ cm; label the segment AC. On your compass, measure off a span of 6 cm. With the point of the compass on C, draw an arc directly above C (any point on the arc will be 6 cm from C). Now, measure off a span of 10 cm on your compass. With the point of the compass on A, draw an arc to cut the first arc above C. Label the intersection of these arcs B. Join B to A and B to C. Label the three sides of the triangle with small letters a, b, and c as in Figure 9-3. Measure angle C. Angle C should be a right angle in your triangle.

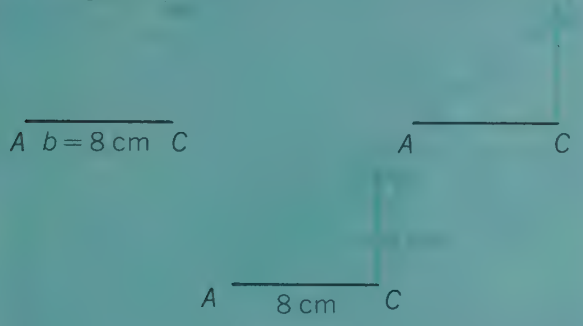


Figure 9-3

Construct a square on each side of your triangle as shown in Figure 9-4. Let the sides of your triangle be the measures of the sides of the squares.

Figure 9-4

How many square centimetres are in the squares of sides a, b, and c?

Compare the area on side c with the sum of the areas on sides a and b.

Explain: $c^2 = a^2 + b^2$

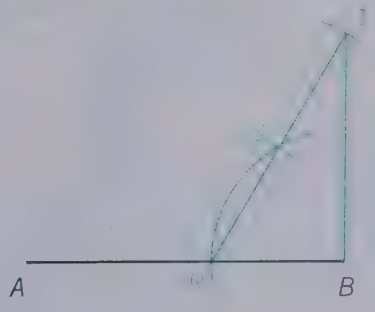
Discuss.

Note! Try another triangle where $b = 12$ cm, $a = 5$ cm, and $c = 13$ cm.

What conclusion do you arrive at with this triangle?

Have You Discovered that...

. . . you can construct a perpendicular at the end of a line segment without extending the line segment and without using a protractor or a right-angle tool?

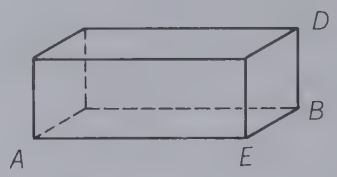


Let $\overline{AB}$ be the segment on which we wish to draw a right angle at B.

1. With B as centre and radius less than $\overline{AB}$, construct an arc cutting $\overline{AB}$ at R. Extend the arc well above $\overline{AB}$.
2. With R as centre and radius $\overline{BR}$, cut the first arc at S. Draw a line segment from R through S and extend it well above B.
3. With centre S and radius $\overline{BR}$, cut the extended line segment at T.
4. Join $\overline{TB}$.
5. $\overline{TB}$ is perpendicular to $\overline{AB}$ at point B. Does this method always work? Repeat it two or three times.

Have You Discovered that...

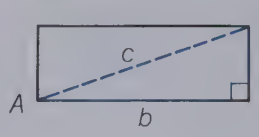
. . . you can calculate the distance from one upper corner of the room to the opposite lower corner of the room?



Let the diagram represent a room. The problem is how to find the distance from A to D given the length, width, and height of the room. The problem has to be handled in two parts.

Part One

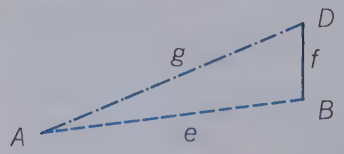
First find the length of the diagonal of the floor ($\overline{AB}$). Use the Pythagorean Relation $c^2 = a^2 + b^2$.



Floor Plan

Part Two

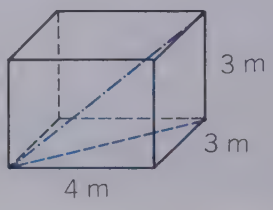
Use the diagonal AB and side BD to form a right triangle ABD. Solve for $m(\overline{AD})$.



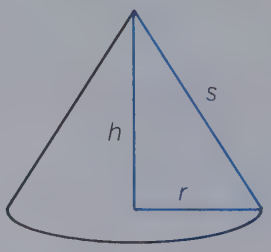
Use: $e^2 + f^2 = g^2$

Now try this.

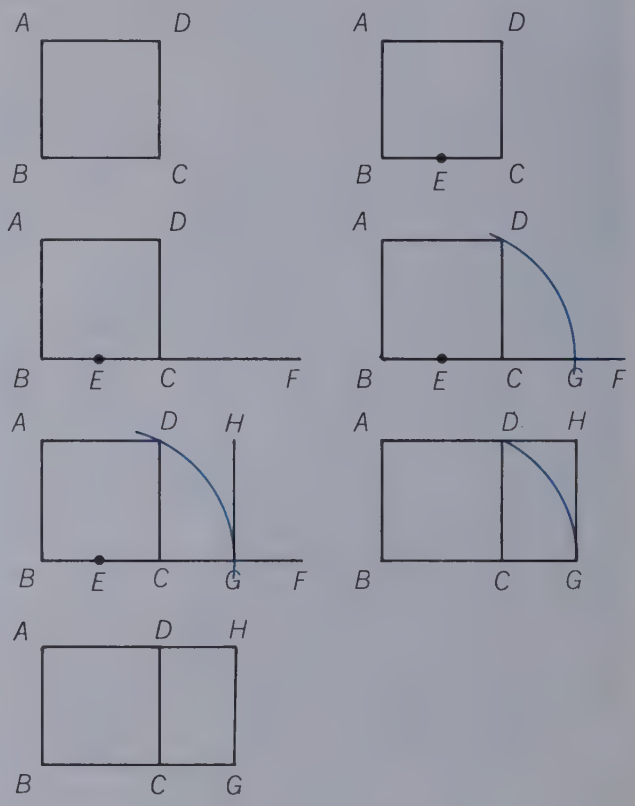
Let the dimensions of the room be as follows: length 4 m, width 3 m, and height 3 m. Find the distance from one corner at the ceiling to the opposite corner at the floor. (Use square root tables to assist you.)



. . . it is possible to calculate the measure of the side of a circular cone? Find the slant height of the circular cone shown in the diagram. The radius (r) is 3.0 cm and the vertical height (h) is 7.2 cm.

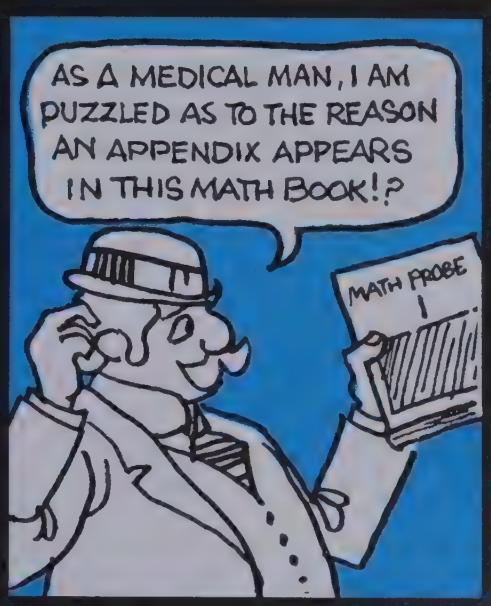


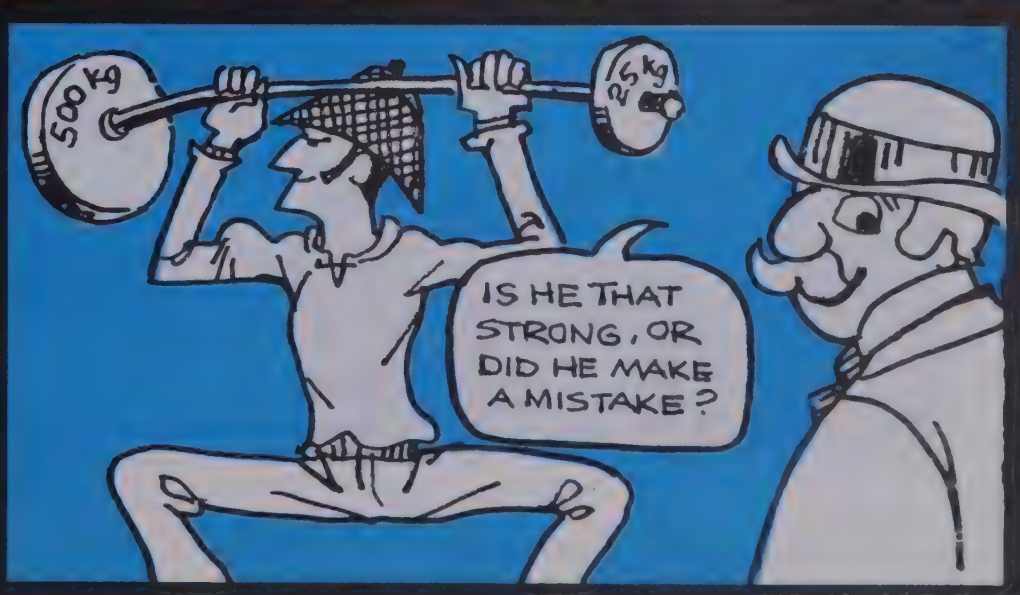
. . . there is an easy method of constructing a Golden Rectangle? Follow the steps from 1 to 7, then write the steps in flowchart form.



How much do you know about the Golden Rectangle? What civilization made extensive use of it in many of its architectural designs? Visit your library and check for all the information you can find on the Golden Rectangle.

Chapter Ten
APPENDIX
Measurement





10-1 Length

The material in this chapter is entirely optional. It has been included for those who are unfamiliar with metric measurement or for those who may need some review. However, the more advanced work on the force of gravity and on pressure (Sections 10·6 and 10·7) will be of interest to students already well-acquainted with basic metric measurement.

Projects

Project 1

Purpose

To gain experience in estimating and measuring lengths in metres.

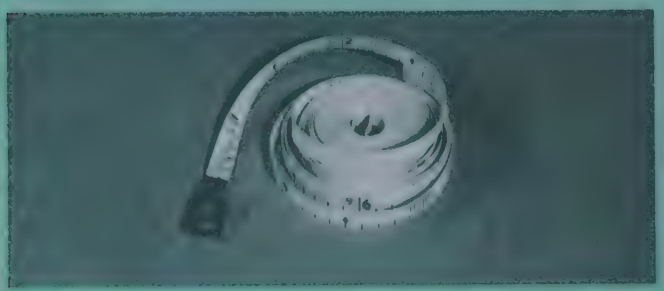
Materials

One metrestick (preferably unmarked) per person or group.

Method

Answer these questions to the nearest whole number of metres. Begin by making a guess as to the correct answer for each question and then measure to obtain the actual length to the nearest whole number of metres. Make a chart and record your results as shown.

Item	Estimated Length	Correct Length	Computed Error	Percentage Error
Your pace				
A door				



$$\text{percentage error} = \frac{\text{computed error}}{\text{correct length}} \times 100$$

Notice that m is a symbol and not an abbreviation. A symbol is not followed by a period and does not change when it is used in the plural. We write:

1 m for one metre;
3 m for three metres;
10 m for ten metres;
15 m for fifteen metres;

and so on.

Work in pairs or small groups to guess, and then measure, the following. Remember to round your answers to the nearest whole number of metres and enter them in the chart.

1. Is your pace 1, 2, or 3 m long?
2. Is a door 1, 2, 3, or 4 m high? How wide is a door?
3. How many metres long is your classroom? How wide and how high is it?
4. How wide is a street?

5. How many handspans make up a metre?
6. Find the length and width of your school ground in metres.
7. How long and how wide is a swimming pool?
8. What are the lengths of the sprint races in the Olympic Games?
9. What is the distance of the marathon race in the Olympics?
10. Find ten common objects a metre or more in length, height, or width. First guess their size and then measure them.
11. Use a geography book to find the height in metres of five famous mountains.

Project 2

Purpose

To gain experience in estimating and measuring lengths in decimetres.

Materials

One metrestick and one decimetre ruler per group. Make your own decimetre rulers if they are not available.

Method

Work in small groups.

Part A

This ruler is actually one decimetre (1 dm) in length.

1. How many decimetres are there in a metre?
2. What does the prefix *deci* mean?

Part B

10 dm = 1 m

1. A hall is 3 m wide. How many decimetres wide is the hall?
2. A man's pace is 9 dm. What is the man's pace in metres? Give your answer correct to one decimal place.
3. A man's arm span is 18 dm. How many metres is this?
4. A room is 11 m by 14 m. What are the length and width in decimetres?
5. Write each measure in decimetres.
   (a) 50 m (b) 4.5 m (c) 3.2 m
   (d) 8 m (e) 0.6 m (f) 0.7 m
6. Write each measure in metres.
   (a) 4.0 dm (b) 39 dm (c) 114 dm
   (d) 5 dm (e) 8 dm (f) 3141 dm

Project 3

Purpose

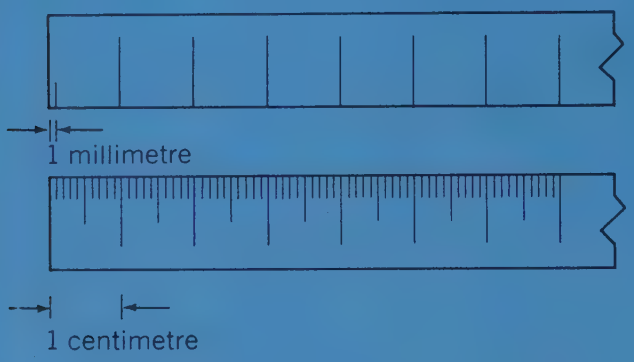
To gain experience in estimating and measuring in centimetres (cm) and millimetres (mm).

Materials

A ruler marked in centimetres and millimetres for each group.

Method

Work in small groups.



Part A

The rulers show actual length in centimetres and millimetres.

1. How many centimetres are there in a decimetre?
2. How many centimetres are there in a metre?
3. What does the prefix *centi* mean?

Part B

Answer these questions by giving the measures to the nearest whole number of centimetres. Use the chart to find the symbols for the metric units.

10 cm = 1 dm	1 cm = 0.1 dm
10 dm = 1 m	1 dm = 0.1 m
100 cm = 1 m	1 cm = 0.01 m

1. What is the width of your thumb?
2. What is the distance from the tip of your thumb to the first knuckle?
3. What is the thickness of this book?
4. What is the diameter of a nickel?
5. What are the length and width of this page?
6. What is the length of your second finger?
7. What is the length of your handspan?
8. What is the length of your foot?
9. What are the length and width of a dollar bill?
10. How long is a new pencil?
11. How long and how wide is your desk?
12. How long is a new piece of chalk?
13. What is the length of your arm span?
14. Locate ten other objects suitable for measuring in centimetres and give their length.

Part C

1. Write each measure in centimetres.

	m	cm
One pace	0.85 m	
Width of a door	0.75 m	
Height of a desk	0.70 m	
Circumference of a student's head	0.53 m	
Circumference of a student's waist	0.69 m	
Height of a chair's back	0.44 m	
Height of a chair's seat	0.80 m	
Length of a key	0.05 m	
Length of a used pencil	0.09 m	
Width of an envelope	0.08 m	

2. Write a rule for changing metres to centimetres.

3. Express each of the following measures in metres (correct to two decimal places).

	cm	m
Length of a baseball bat	78 cm	
Length of a tennis racket	56 cm	
Height of a high jump	192 cm	
Height of a basketball player	211 cm	
Width of a television screen (portable)	18 cm	
Width of a television screen (cabinet)	51 cm	
Length of a coffee table	96 cm	
Width of a coffee table	46 cm	
Length of a ski	190 cm	
Wheelbase of a compact car	244 cm	
Width of a compact car	183 cm	
Length of a compact car	320 cm	
Height of a compact car	126 cm	

4. Write a rule for changing centimetres to metres.

Part D

The smallest unit of measure on the common ruler is the millimetre (mm).

10 mm = 1 cm
100 mm = 1 dm
1000 mm = 1 m

How are shirt sizes designated in metric units?

1. Copy and complete these charts.
   0.1 cm = ■ mm 10 mm = ■
   0.01 dm = ■ mm 10 cm = ■
   0.001 m = ■ mm 10 dm = ■

2. What does the prefix *milli* mean?

3. How many millimetres are there in a centimetre?

4. Estimate the length of the objects in Part B of this project and then measure them to the nearest millimetre. Record your answers in a chart and determine the computed error and the percentage error.

5. Express each of the measures in Part C, Questions 1 and 3, in millimetres.

6. Copy and complete this chart.

	m	dm	cm	mm
Length of a chesterfield	2.13			
Height of a refrigerator	1.64			
Height of a kitchen range		9.2		
Width of a kitchen range	0.75			
Length of a standard car		48.7		
Height of a garage door	2.44			
Width of a garage door	3.05			

7. Copy and complete this chart.

	mm	cm	dm	m
Length of a postage stamp	25	2.5	0.25	0.025
Width of a postage stamp	20			
Length of a paper clip	35			
Length of a credit card	95			
Width of a credit card		6		
Height of a pop bottle	26			
Length of a playing card		8.8		
Width of a playing card	57			

8. How many items can you find with a measurement of approximately 1 mm? List them.

9. How many items can you find with a measurement very close to 1 cm? List them.

10. How many items can you find with a measurement very close to 1 m? List them.

11. Skis are sold in metric sizes in lengths that are multiples of 5 cm; that is, 150 cm, 155 cm, 160 cm, 165 cm, and so on.

	Height of Skier (in cm)	Guide for Downhill Skis (in cm)	Guide for Cross-country Skis (in cm)
Short	150-155 155-170	160-165 165-180	175-185 185-200
Medium	175-180 180-185	170-185 185-190	205-210 210-215
Tall	185-190 190-195	195-200 200-215	215-220 220-225

Check your own height and determine what length of ski you should use for each type of skiing.

12. Here is another way to calculate roughly the length of ski required. Use this method to calculate the length of each type of ski. Compare your results with the lengths given in the guide. (What do h_D and h_C mean?)

Project 4

Purpose

To gain experience involving distances of one kilometre (1 km) or more.

Materials

A tape measure 100 m long and a wristwatch with a second hand.

Method

Work in teams of four or more.

Part A

Just as we have names for units smaller than a metre, we also have names for units larger than a metre. Notice that the ratio of one unit to the next smaller unit is 10 to 1.

ten metres = one decametre
10 m = 1 dam

ten decametres = one hectometre
10 dam = 1 hm

ten hectometres = one kilometre
10 hm = 1 km

Decametres and hectometres are rarely used.

1. Which is larger:
   (a) 1 m or 1 dam? (b) 1 dam or 1 km?
   (c) 1 hm or 1 km? (d) 1 m or 1 km?

2. Copy and complete this table.

3000 m = ■ dam = ■ hm = ■ km
1875 m = ■ dam = ■ hm = ■ km
4350 m = ■ dam = ■ hm = ■ km
875 m = = ■ km
743 m = = ■ km
1035 m = = ■ km
375 m = = ■ km

What is the rule for writing metres as kilometres?

3. Give the meaning of the prefixes: *deca*, *hecto*, and *kilo*.

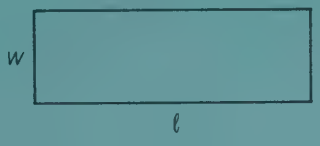
4. How many metres are there in a kilometre?

5. Copy and complete this table.

1 dam = ■ m	1 m = ■ dam
1 hm = ■ m	1 dam = ■ hm
1 km = ■ m	1 hm = ■ km

Part B: Review

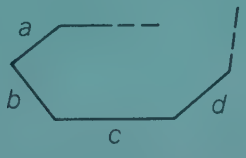
Perimeter of a rectangle:
$P = 2\ell + 2w$.



Perimeter of a square:
$P = 4s$.

Perimeter of an equilateral triangle:
$P = 3s$.

Perimeter of any polygon:
$P = a + b + c + \ldots$



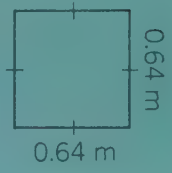
Circumference of a circle:
$C = \pi d$ ($\pi = 3.14$)
$C = 2\pi r$.

What are the length and width of a football field in metres? What is its perimeter?

Calculate the perimeter of each polygon.

(a)

(b)

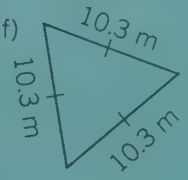


(c)

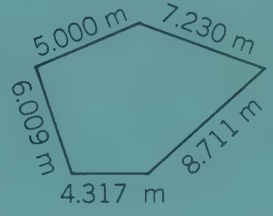
(d)

(e)

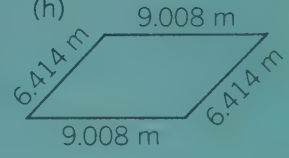
(f)



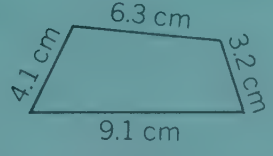
(g)



(h)

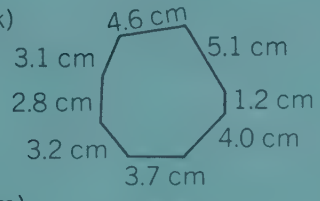


(i)



(j)

(k)



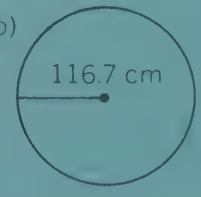
(l)

(m)

(n)

(o)

(p)



Part C

1. Measure a distance of 1000 m in a straight line. This is a kilometre. (Do this in a park or large playground.)

   OR

   Measure and stake out a course in your schoolyard which is 1000 m (1 km) in length.

2. Use a watch to measure the time it takes each member of your group to *walk* 1 km. Use this information to calculate the walking speed of each in kilometres per hour (km/h).

   If the time for the walk was 12 min, then

   $$\frac{1}{12} = \frac{x}{60}$$
   $$12x = 60 \times 1$$
   $$\frac{12x}{12} = \frac{60 \times 1}{12} \quad \text{(Divide each side by 12.)}$$
   $$x = 5$$

   The speed is 5 km/h.

3. Repeat Question 2 for "running as fast as you can".

4. Measure the distance around the block your school is on. How long does it take each member of your group to cover this distance by (a) walking, (b) jogging, (c) running as fast as possible, and (d) riding a bicycle? Calculate the speed in kilometres per hour for each.

5. What is the likely speed limit in kilometres per hour in:
   (a) a school zone?
   (b) a playground area?
   (c) a residential area?
   (d) a paved two-lane highway?
   (e) a freeway?

6. Calculate the time for an airplane flying at 790 km/h to fly each route.

(a)	Calgary to Edmonton	320 km
(b)	Vancouver to St. John's	6840 km
(c)	San Francisco to New York	4500 km
(d)	Toronto to Montreal	450 km
(e)	Regina to Winnipeg	560 km
(f)	Los Angeles to Edmonton	2200 km
(g)	Dawson to Winnipeg	2800 km

7. Calculate the time for an airplane flying at 850 km/h to fly each route.

(a)	London to Cape of Good Hope	10 000 km
(b)	Montreal to London	5 400 km
(c)	London to Calcutta	8 500 km
(d)	Calgary to Honolulu	5 100 km
(e)	Toronto to Port au Prince	2 950 km
(f)	Toronto to Lisbon	5 900 km
(g)	Winnipeg to Moscow (nonpolar)	9 300 km
(h)	Winnipeg to Moscow (polar)	7 850 km
(i)	Vancouver to Melbourne	14 000 km
(j)	North Pole to Equator	10 000 km
(k)	Tokyo to Auckland	9 350 km

10·2 Mass

Projects

Project 5

Purpose

To gain experience in estimating and measuring masses in kilograms. (This experience will form the basis for comparison purposes.)

Materials

A personal scale marked in kilograms.

Method

The symbol kg is used to represent a kilogram or kilograms.

1. Measure the mass of each of these items. Make a chart and record the masses to the nearest kilogram.
   (a) Your mass
   (b) A chair
   (c) This book
   (d) Large fruit-juice can full of water
   (e) A pail of water
   (f) Ten volumes of an encyclopedia
2. Estimate the mass of the following and then measure each. Record your measurements in a chart like the one in Project 1.
   (a) A thick novel
   (b) Your shoe
   (c) Chalk box full of sand
   (d) Large dictionary
   (e) Various plastic containers filled with water.
3. Find five other items suitable for measuring in kilograms. Estimate, then measure, the mass of each as you did in Question 2.

Project 6

Purpose

To gain experience in estimating and measuring masses in grams.

The symbol g is used to represent a gram or grams.

Materials

A scale or balance graduated in grams.
A set of the following masses: 1 g, 50 g, 100 g, 500 g, and 1000 g.

Method

1. Pick up each mass to get used to the way it feels.
2. Obtain ten objects suitable for measuring in grams on the scale. Estimate and then measure each in turn. Record your results in a chart similar to the one in Project 1. Your percentage error should become less as you practise.

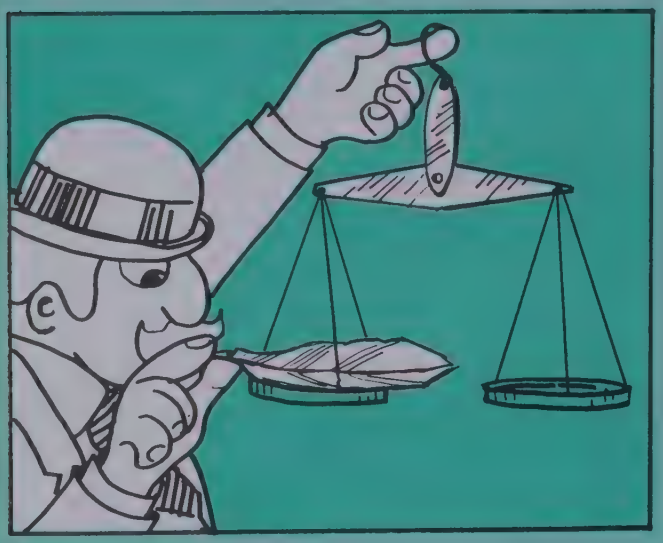
   1000 g = 1 kg

3. Convert each of the following measures into grams.
   (a) Newborn baby: 3.1 kg.
   (b) 5¢ coin: 0.005 kg.
   (c) One dozen eggs: 0.68 kg.
   (d) Glass of water: 0.35 kg.
   (e) Large can of peaches: 0.58 kg.
   (f) Tube of toothpaste: 0.248 kg.
4. Convert each of the following measures into kilograms.
   (a) Small tin of soup: 320 g.
   (b) Can of pop: 250 g.
   (c) Small tin of salmon: 115 g.
   (d) 25¢ coin: 15 g.
   (e) Metal ball-point pen: 25 g.
   (f) Dinner plate: 1250 g.

To find the mass of very light objects, we may use a unit smaller than a gram. We can use a unit 1000 times smaller: the milligram (mg)!

1000 mg = 1 g

What sorts of things do you think might be measured in milligrams?



5. Convert each of the following measurements into grams.
   (a) Medium-size paper clip: 500 mg.
   (b) Large paper clip: 1250 mg.
   (c) Plastic ball-point pen: 10 500 mg.
   (d) Small wooden spool of thread: 8500 mg.
   (e) Pocket comb: 4500 mg.
   (f) Car key: 6750 mg.

6. Convert each of the following measures into milligrams.
   (a) 5¢ coin: 5 g.
   (b) Sugar cube: 2 g.
   (c) 25¢ coin: 0.015 kg.
   (d) Art gum eraser: 13 g.
   (e) Book of paper matches: 3.5 g.
   (f) Metal shoehorn: 18 g.

7. Find the mass of a sheet of paper which is:
   (a) 74 mm by 105 mm (b) 210 mm by 297 mm
   What is the most accurate way to do this?

Very heavy objects are often measured in a unit larger than a kilogram. This large unit is 1000 times larger than a kilogram: the tonne (t).

1000 kg = 1 t

How many grams are there in 1 t?

What is another name for *megagram*?

This plane has an approximate mass of 320 t *(tonnes) and cruises at 1000* km/h *at an altitude of 10 000* m. *A tonne* (t) *is 1000* kg.

8. Calculate the mass of your class. Is the total mass more or less than a tonne? Use this total to find the mass of an average student. Use the average to calculate the mass, in tonnes, of all the students in your school.

9. Copy and complete this table.
   (a) ■ kg = 1 t (b) 1 kg = ■ t
   (c) ■ g = 1 kg (d) 1 g = ■ kg
   (e) ■ mg = 1 g (f) 1 mg = ■ g

10. State the meaning of each of these prefixes.
    (a) kilo (b) deci
    (c) hecto (d) milli
    (e) deca (f) mega

11. This chart provides the ideal masses of men and women of medium frames. Check your own metric dimensions against the chart.

Height	Mass	
	Men	Women
cm	kg	kg
158	58	49
160	59	50
163	60	51
165	62	53
168	64	54
170	66	56
173	68	58
175	70	60
178	71	62
180	74	64
183	76	65
185	78	67
188	80	70
191	82	
193	84	

The members of a class of 35 students are all of ideal masses. This chart shows the numbers of students at various heights.

4 boys are 160 cm tall.
7 boys are 165 cm tall.
6 boys are 168 cm tall.
2 boys are 173 cm tall.
3 girls are 163 cm tall.
4 girls are 165 cm tall.
7 girls are 168 cm tall.
2 girls are 175 cm tall.

What is the total mass of the class in kilograms? in tonnes? What is the mass of an average student in the class?

12. Look at various items in your local supermarket. Which items are measured in grams? kilograms? litres? millilitres? How is toothpaste measured? Why is it measured in this way?

13. Why will you rarely need milligrams in your measurements?

Projects

Project 7

Purpose

To gain experience in estimating and measuring capacity in litres and millilitres.

The symbols ℓ and ml represent litres and millilitres respectively.

Materials

A set of metric containers with measures of 1 ℓ, 500 ml, and 100 ml.

A graduated cylinder of at least 500 ml (0.5 ℓ).

A selection of various containers as indicated below.

Method

1. Study the set of containers to get an idea of a litre.

2. Use the metric containers to pour water into the graduated cylinder to check the capacity of each container.

   Study the diagrams to see the correct way to read the level of liquid in a graduated cylinder.

3. Use the 100 ml container to fill the litre container. How many times must one empty the 100 ml container in order to fill the litre container? Write this as a number sentence. Is 100 ml equivalent to a decilitre? Explain.

4. How many 500 ml containers does it take to fill the litre container? Write this as a number sentence.

5. How many millilitres are there in a litre? Write this as a number sentence.

6. Complete this table.

   ■ ml = 1 ℓ
   ■ ml = 0.5 ℓ
   ■ ml = 0.1 ℓ
   ■ ml = 0.01 ℓ
   ■ ml = 0.0001 ℓ

7. Use a graduated cylinder to determine the number of litres or millilitres for each container in the list. (Use other containers if some of these are not available.)

Container	Estimate of Capacity	Actual Capacity	Computed Error	Percentage Error
Small soft drink can				
Large soft drink can				
Large soft drink bottle				
Small soft drink bottle				
Small vegetable tin				
Large vegetable tin				
Small salmon tin				
Water glass				
Cream cartons (2 sizes)				
Milk cartons (2 sizes)				
Spoons (4)				
Cups (several sizes)				
Coffee can				
Pail of water				
Five other containers				

Some of these containers may have the capacity of their contents marked on them. Why do you think it is possible to put more in these containers than is marked on them?

1000 ml = 1ℓ

8. In appropriate units, estimate the capacity of each of the following.

   (a) Shoe box (b) Marmalade jar
   (c) Peanut butter jar (d) Soup bowl
   (e) King-size pop bottle (f) Root beer jug
   (g) Coffee mug (h) Teacup
   (i) Gas tank (compact car) (j) Ketchup bottle

9. Pour what you think is 100 ml of water into a juice can. Check your estimation by pouring the water into a graduated cylinder or into a 100 ml container. Was your estimation too much or too little? Repeat for:

   (a) 500 ml (b) 1 ℓ (c) 1.5 ℓ (d) 250 ml

10. Repeat Question 9 using a water pail for:

    (a) 1 ℓ (b) 2 ℓ (c) 3 ℓ (d) 4 ℓ

Project 8

Purpose

To learn the relationship between a mass of water and its volume.

Materials

A 100 ml container, a scale or balance, and a square milk carton.

Method

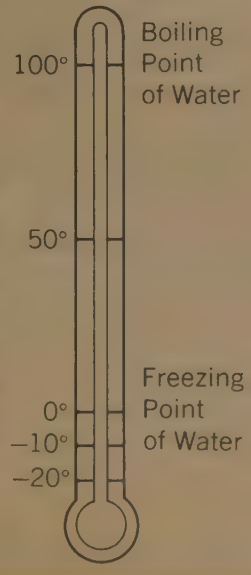
1. Find the mass of the 100 ml container.
2. Pour exactly 100 ml of water into the container.
3. Find the mass of the container and the water together.
4. Calculate the mass of 100 ml of water.
5. Calculate the mass of 1 ml of water to the nearest whole gram.
6. Use your answer to Step 5 to calculate the mass of:
   (a) 500 ml of water (b) 1 ℓ of water

When pure water at 4°C is used, the following is true.
1 ml has a mass of 1 g;
1000 ml has a mass of 1000 g;
and 1 ℓ has a mass of 1 kg.

Note. In this book, we will assume that these relationships hold at all temperatures.

7. Using the information you have gathered in this project, calculate the mass of each of the following amounts of water.
   (a) 4.5 ℓ (b) 155 ℓ (c) 3 ℓ
   (d) 0.5 ℓ (e) 0.25 ℓ (f) 0.75 ℓ
   (g) 400 ml (h) 1300 ml (i) 10 ml
   (j) 2500 ml (k) 1 ml (l) 850 ml

The Celsius thermometer was named after a Swedish scientist, Anders Celsius (1701-1744). He presented the idea of separating the point at which water freezes and the point at which water boils by 100 equal units or degrees. Water freezes at 0°C and boils at 100°C. Do your findings agree with this? If not, can you give reasons for the differences?



10·4 Temperature

Project

Project 9

Purpose

To gain experience in estimating and measuring temperatures in degrees Celsius.

Materials

A thermometer graduated in Celsius (range −5°C to 100°C).
A clinical thermometer, beaker, ice cubes, burner, and stand.

Method

1. Record the normal room temperature.
2. Record the temperature outdoors in the shade and in the sun. Describe the day.
3. Record your own temperature by placing the clinical thermometer in your mouth for 2 min. (Disinfect the thermometer after each use by placing it in some suitable disinfectant.)
4. Place ice cubes in cold water and leave them there for 15 min. Record the temperature of the ice water.
5. Record the temperature of warm water, hot water, and boiling water.
6. Record the temperature in a refrigerator.
7. If an oven thermometer in Celsius is available, record the temperature suitable for baking a cake.
8. As a long term exercise, prepare graphs to record for a month:
   (a) the high and low temperatures each day;
   (b) the amount of rain daily in millimetres;
   (c) the amount of snow daily in centimetres;
   (d) the velocity of the wind at some given time each day in kilometres per hour.

10·5 Area and Volume Measurement

Five metric units which can be used for area are:

Square millimetre	mm^2
Square centimetre	cm^2
Square metre	m^2
Square kilometre	km^2
Hectare	ha

The actual sizes of the following squares are:

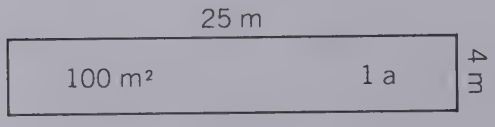
1 mm^2

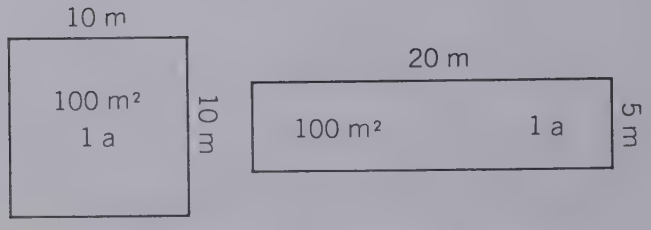
1 cm^2

Do Questions 1 to 8 in Exercise 10·5.

Hectare

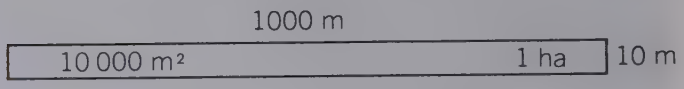
A hectare is a unit of land measure. A hectare is 10 000 m^2. The word hectare comes from *hecto* and *are*. An are is a smaller unit of land measure that will not be used extensively in Canada except for its relation to the hectare. An are is 100 m^2. The symbol a designates an are.

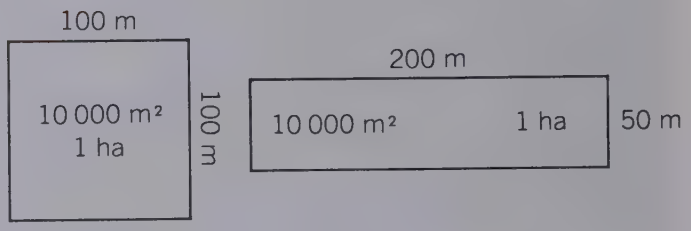




The prefix *hecto* indicates there are 100 a in one hectare. Therefore:

1 ha = 10 000 m^2





Do Question 9 in Exercise 10·5.

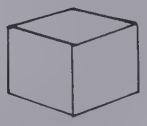
Volume

Some units which can be used for measurement of volumes are:

Cubic millimetre	mm^3
Cubic centimetre	cm^3
Cubic metre	m^3
Cubic kilometre	km^3

The actual sizes of the following cubes are:

1 mm^3



1 cm^3

1000 cm^3 is equivalent to 1 ℓ.

Project

Project 10

Find the mass of an empty square milk carton in grams and then put 500 ml of water into the carton and find the total mass. Before emptying the carton, place it on a flat surface and mark the height of the water level in the carton.

After emptying the carton, measure its inside dimensions and the height of the water level as marked. Calculate the volume of 500 ml of water in cubic centimetres.

Calculate the mass of 1 cm^3 of water to the nearest whole gram.

Calculate the volume of 1 ml of water.

1 cm^3 of water has a mass of 1 g.

1 ml of water occupies 1 cm^3.

EXERCISE 10·5

1. How many millimetres long is each side of a square centimetre? How many square millimetres are there in a square centimetre?

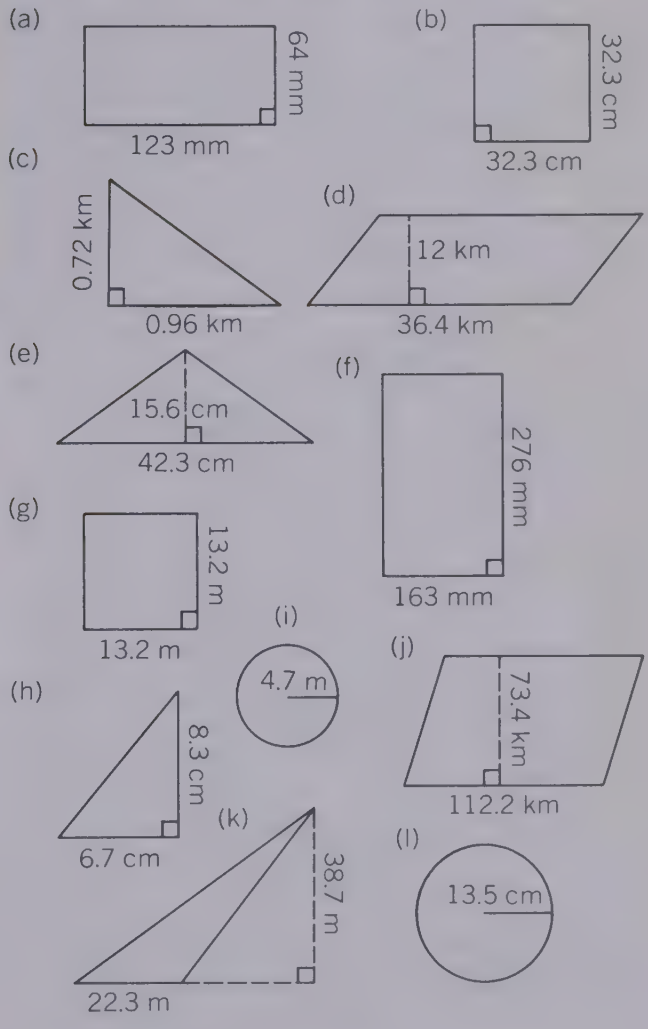
2. How many centimetres long is each side of a square metre? How many square centimetres are there in a square metre?

An average refrigerator has a capacity of about 400 ℓ. How many cubic centimetres is this?

3. How many millimetres long is each side of a square metre? How many square millimetres are there in a square metre? What kind of drawing could you make to illustrate the ratio 1 000 000 to 1?

4. How many metres long is each side of a square kilometre? How many square metres are there in a square kilometre?

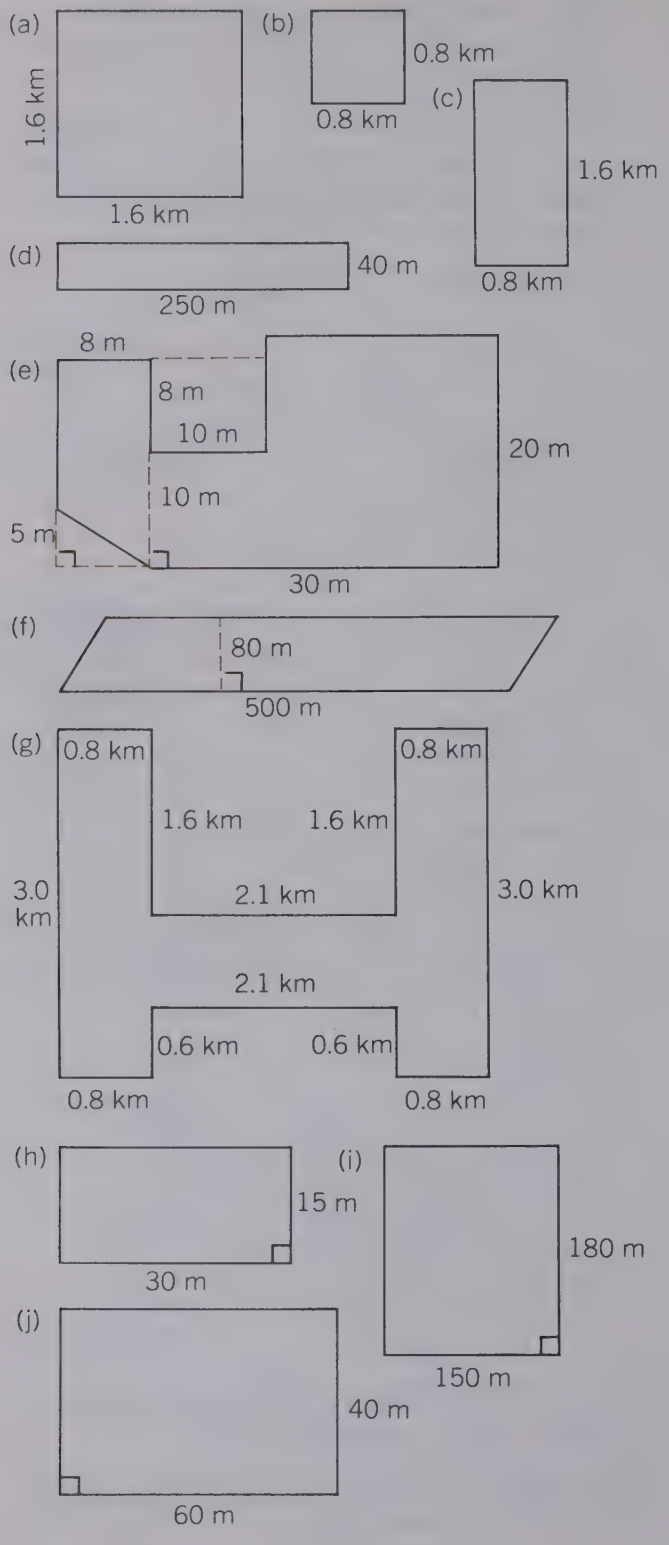
5. Calculate the area of each region.



6. Calculate the area of the figures in 5(a), (f), (g) and (k) in square centimetres.

7. Calculate the area of the figures in 5(b), (c), (d), (f) and (h) in square metres.

8. Calculate the number of hectares in each field.



9. Calculate the volume of:
   (a) A cornflakes box (b) A cracker box
   (c) A shoe box and five other available boxes

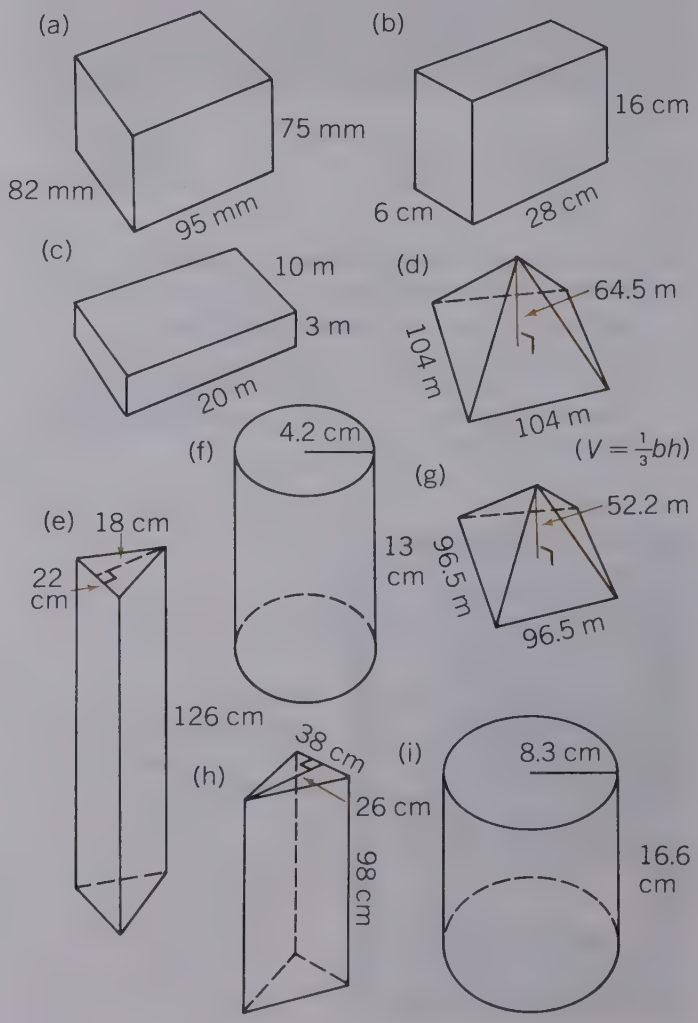
10. How many cubic millimetres are there in a cubic centimetre?

11. How many cubic centimetres are there in a cubic metre?

12. Name three things you would measure in:
    (a) cubic centimetres (b) cubic metres

13. Can you name anything that would be measured in cubic millimetres? in cubic kilometres?

14. Calculate the volume of each:



15. The Great Pyramid of Cheops is a gigantic tomb. The length of a side of the square base is about 226.5 m, while its height is 144.4 m. What is its volume in the following units of measurement? (Volume of a pyramid $= \frac{1}{3}$ area of base $\times$ height.)
    (a) cubic metres (b) cubic kilometres

16. Fill a large fruit-juice can with water. Measure the quantity of water in any way you like. Describe the quantity of water in three different ways.

17. The engine capacities of most motor vehicles are measured in cubic centimetres. Find the cubic centimetre ratings of different kinds of motor vehicles.

Project

Project 11

Purpose

To gain experience in estimating and measuring large areas.

Materials

A tape measure marked in metres.

Method

Part A

1. Calculate the area of your schoolyard in:
   (a) square metres (b) square kilometres
   (c) hectares

2. Calculate the area of a city block in each of the three units in Question 1.

3. Calculate the area of a city residential lot in:
   (a) square metres (b) hectares

4. Calculate the area of a football field and a baseball field in:
   (a) square metres (b) hectares

5. Calculate the area of three other large areas in your community.

Part B

1. If you live in a rural area, you may wish to omit Questions 2 and 3 in Part A. Instead, calculate the number of hectares in different fields near your school.

2. In the metric system, grain production may be given in tonnes per hectare (t/ha). This table will help you understand this system.

Yield in tonnes per hectare

Grain	Low	Average	Bumper Crop
Wheat	1.0	3.0	6.1
Oats	1.5	3.2	6.8
Barley	1.2	3.5	6.4

Calculate the amount of wheat, oats, and barley produced in a bumper crop for each field in Question 1, Part B. Repeat your calculations for average and low yield crops.

3. In whole numbers, give the dimensions of five rectangles having an area of 1 ha each. Make up examples which are different from those already given in this section.

Problem

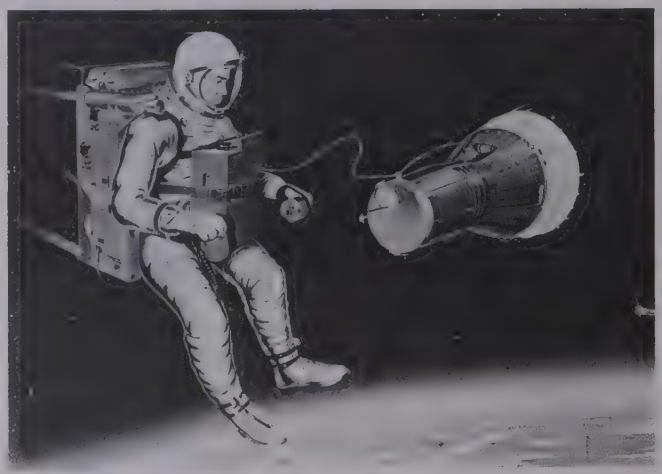
The New Turf Company laid turf on a boulevard 3 km *long and* 5 m *wide. What was the area in hectares of the boulevard?*

Lawn sod costs $7.50/m^2$. *What is the cost per hectare? What is the total cost for the boulevard?*

10·6 Force of Gravity and Newtons

Have you wondered why an astronaut can walk in space without falling back to earth, but you cannot walk off into space from the roof of your house without falling back to earth?

The earth attracts or pulls objects towards it. This pull is called the *force of gravity*. The force of gravity is often symbolized by *G*. The force of gravity is greater on a large mass than on a small mass. The unit of force is a newton (N), and it is named after Sir Isaac Newton. Sir Isaac Newton published a very famous scientific law in 1687. This law stated that the force of gravity on an object varied directly with the mass of the object.



The earth's force of gravity on an object decreases as the distance of the object from the earth increases. Therefore, the earth's force of gravity is very slight on a small object a great distance from the earth. As the object in space nears the moon, the moon exerts a greater and greater force of gravity on it. The moon's force of gravity on an object on its surface is less than the earth's force of gravity on the same object (when it is on the surface of the earth).

IT HAS PUZZLED ME FOR YEARS THAT I AM SO FIRMLY TIED ON THIS EARTH BY GRAVITY...
...AND YET THE MOON RIDES ON HIGH UNAFFECTED.
WHAT INEFFABLE TWADDLE! FOR CERTAIN THE MOON IS HELD BY THE EARTH!
BOG

Project

Project 12

Purpose

To gain experience in measuring forces.

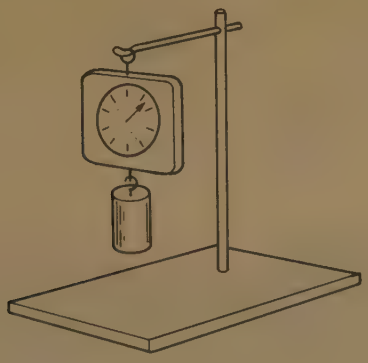
To notice the relationship between the force of gravity on an object and the mass of the object.

Materials

A balance in kilograms.
A spring balance marked in newtons (N).
Several masses suitable for use with the spring balance.

Method

1. Hook the lightest object to the spring balance and record the reading in newtons.



2. Repeat Step 1 for each of the other objects.
3. Measure and record the mass of each object.
4. Copy and complete this chart.

Object	Force of Gravity (G) in newtons	Mass (M) in kilograms	Ratio of $\frac{G}{M}$ Rounded Off
1			
2			
3			
.			
.			
.			

Do you see a relation between the mass and the force of gravity on the object?

5. You may have noticed that the force (in newtons) is about ten times the mass in kilograms. We can state this by the proportion:

$$\frac{G}{M} = \frac{10}{1}.$$

The statement in Step 5 of Project 12 says that the force of gravity on an object (in newtons) is ten times the mass (in kilograms) of that object.

Example 1

If an object has a mass of 45 kg, what is the force of gravity on the object?

Solution 1

We know the force (in newtons) is ten times the mass (in kilograms). Therefore,

$$G = 10 \times 45 \text{ N}$$
$$G = 450 \text{ N}.$$

Solution 2

$$\frac{G}{\text{mass in kg}} = \frac{10}{1} \text{ N/kg}$$
$$\frac{G}{45} = \frac{10}{1} \text{ N/kg}$$
$$45 \times \frac{G}{45} = \frac{10}{1} \times 45 \text{ N}$$
$$G = 450 \text{ N}$$

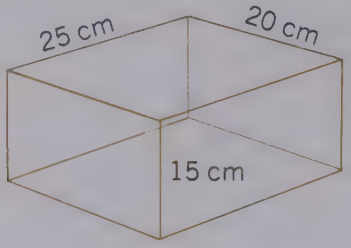
The force of gravity on the object is 450 N.

Do Questions 1 to 3 in Exercise 10·6.

The force of gravity is what keeps us from falling off the earth into space. Gravity causes objects to press towards the centre of the earth. For instance, the force of water on the bottom of an aquarium is due to the force of gravity pulling downward on the water. The bottom of the tank must withstand this force.

Example 2

Calculate the force exerted on the bottom of the full aquarium shown.



Solution

Step 1. Calculate the volume of water.

$$V = \ell wh$$
$$= 20 \times 25 \times 15$$
$$= 7500$$

The volume is 7500 cm³.

Step 2. Calculate the mass of the volume of water.

1 cm³ of water has a mass of 1 g.
7500 cm³ of water has a mass of x g.
7500 cm³ of water has a mass of 7500 g.

Step 3. Convert mass in grams to kilograms.

$$7500 \text{ g} = 7.5 \text{ kg}$$

Step 4. Use one of the methods from Example 1 to calculate the force of gravity on 7.5 kg of water.

For example,

$$G = 10 \times 7.5$$
$$= 75.$$

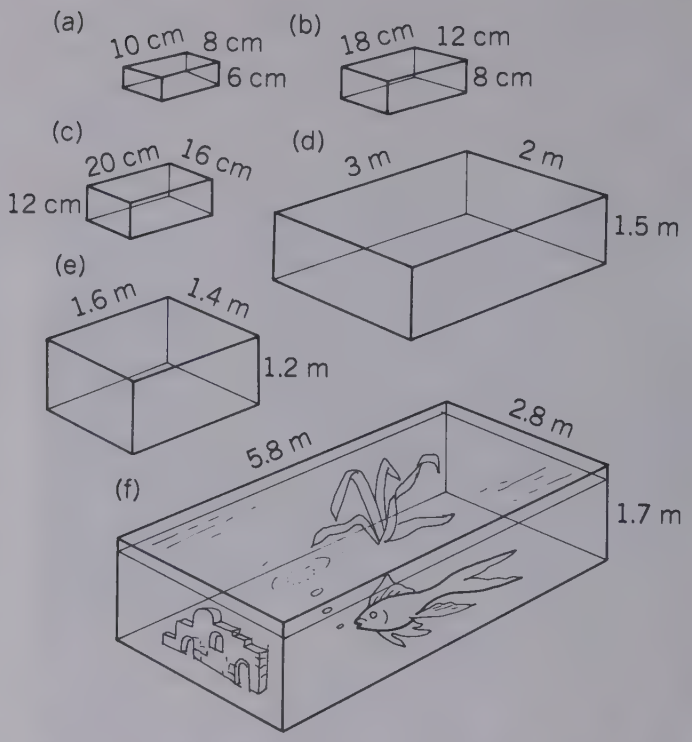
The force on the bottom of the aquarium is 75 N.

EXERCISE 10·6

1. Calculate the approximate force of gravity on each mass when the mass is on the earth. Give your answers in newtons.

   (a) 25 kg (b) 32 kg (c) 52 kg (d) 61 kg
   (e) 81.3 kg (f) 12.7 kg (g) 0.45 kg (h) 0.72 kg
   (i) 1500 g (j) 1750 g (k) 500 g (l) 750 g
   (m) 200 g (n) 150 g (o) 1000 g (p) 1.5 kg

2. Calculate the volume of water in each aquarium. ($V = \ell wh$.) ℓ is used here to represent length, not as the symbol for *litre*.

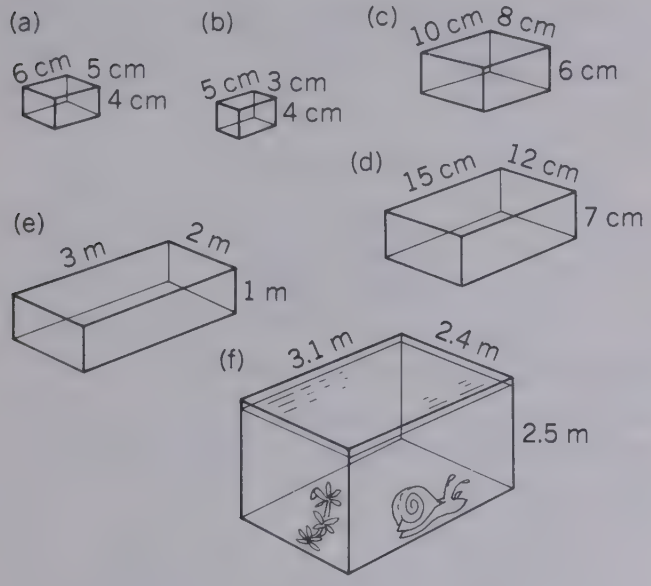


These students are calculating the relationship between the force on a certain mass and the resulting acceleration. A formula showing this relation is $F = ma$, where F is the force in newtons, m is the mass in kilograms, and a is the acceleration in metres per second.

3. What is the mass of water in each aquarium in Question 2?

4. Calculate the force in newtons exerted on the bottom of each aquarium in Question 2.

5. Calculate the force of gravity exerted on the bottom of each aquarium.



6. A student with a mass of 46 kg stands on a plank across a stream. What force is exerted on the plank?

7. Another student carrying two pails of water walks across the plank in Question 6.

Mass of student	42 kg
Mass of one pail	2 kg
Capacity of one pail	5 ℓ

What is the total force on the plank now?

8. The force of gravity in newtons on an object on the surface of the moon is about 1.6 times the mass in kilograms. The proportion is:

$$\frac{G \text{ (in newtons)}}{M \text{ (in kilograms)}} = \frac{1.6}{1}.$$

(a) An astronaut has a mass of 79 kg. What force of gravity does the astronaut experience?

(b) The survival pack carried by the astronaut has a mass of 52 kg. Calculate the force of gravity on the pack on the earth and on the moon.

(c) What is the heaviest mass you think you can lift on earth? on the moon?

9. On the surface of the planet Jupiter, the force of gravity on an object is expressed by the proportion:

$$\frac{G \text{ (in newtons)}}{M \text{ (mass in kg)}} = \frac{26.4}{1} \text{ N/kg}.$$

Answer parts (a), (b), and (c) of Question 8 for Jupiter.

10·7 Pressure and Pascals

Have you ever gone snowshoeing? Why doesn't a person sink into deep snow when one wears snowshoes? Why do you sink if you step from them into the snow?

When we stand on the snow, we are pulled to the earth by the force of gravity. This force is opposed by the snow. If the snow is firm enough, it may hold and not allow us to sink. What determines whether or not we will sink?

If the mass of the snowshoer is 40 kg, then the force of gravity is 400 N. If a person is wearing snowshoes, the area of the snowshoes may be as much as 0.4 m². The average force in newtons per square metre is $\frac{400}{0.4}$ or 1000. That is, the pressure is 1000 N/m² (1000 newtons per square metre).

If a person has no snowshoes (only normal shoes), the area of the shoes on the snow is about 0.02 m². In this case, the average force in newtons per square metre is $\frac{400}{0.02}$ or 20 000. The pressure is 20 000 N/m².

With snowshoes	1000 N/m²
Without snowshoes	20 000 N/m²

The pressure on the snow without snowshoes is 20 times what it is with snowshoes. No wonder we sink without them!

> Pressure is the force exerted on a unit area. The unit for pressure in the metric system is a pascal.

> One pascal (1 Pa) is equivalent to a force of 1 N on 1 m².

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

Blaise Pascal (1623-1662), a French philosopher, mathematician, and scientist stated that the level of a mercury column in a barometer is determined by atmospheric pressure rather than a vacuum as previously believed. In addition to this, Pascal worked on projective geometry (at the age of 16). He also invented a mechanical adding machine and contributed to the theory of probability.

Example 1

Calculate the pressure exerted on a floor by each hoof of a horse. The mass of the horse is 800 kg and this mass is distributed evenly over the 4 hoofs. The area of a hoof is 0.05 m².

Solution

The force is 200 × 10 = 2000 N and the hoof area is 0.05 m². The pressure is 2000 N on 0.05 m². Therefore, the pressure is x N on 1 m².

Proportion:

$$\frac{2000}{x} = \frac{0.05}{1}$$
$$0.05x = 2000 \times 1$$
$$\frac{0.05x}{0.05} = \frac{2000}{0.05}$$
$$x = 40\,000$$

The pressure is 40 000 Pa (or 40 kPa).

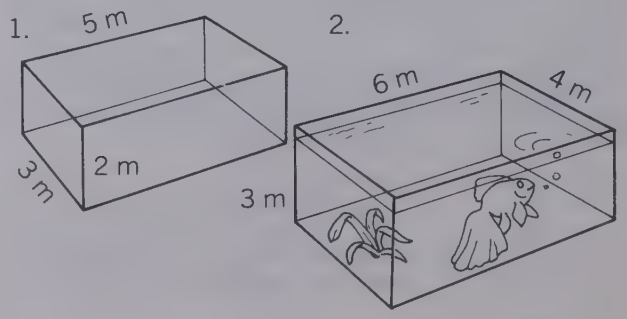
We measure air pressure, tire pressure, and other pressures such as water and gas pressure in pascals.

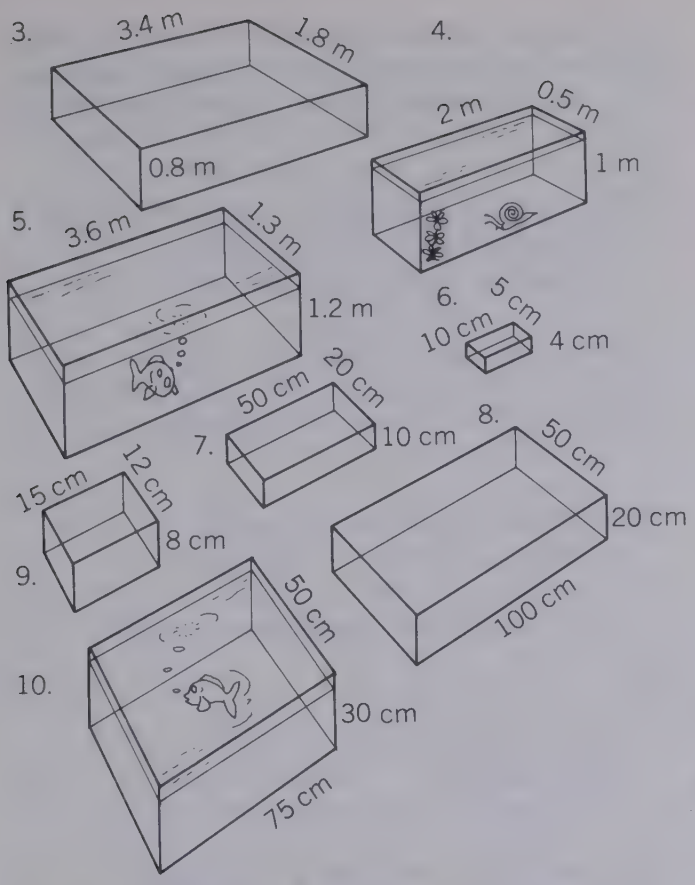
Typical Pressures

Bicycle tire	400 kPa
Car tire (low-pressure tire)	180 kPa
Truck tire	600 kPa
Cabin pressure inside a jet liner	75 kPa
Air pressure 10 km above sea level	25 kPa
Air pressure at sea level	100 kPa

EXERCISE 10·7

Calculate the pressure in pascals on the bottom of each aquarium.





10·8 Using Metric Units

Gasoline is sold by the litre. The distance a car or motor vehicle travels is recorded in kilometres. We often want to know the relation between how much gasoline a car uses and how far it goes. Two methods are used to determine this.

First Method

One method is to state how far the car goes on a litre of gasoline.

Motorbike	17 km/ℓ
Compact car	10 km/ℓ
Standard-size car	6 km/ℓ
Big Truck	2 km/ℓ

In this method, the larger the number of kilometres per litre, the more economical the vehicle is to operate.

Example 1

A car used 56 ℓ of gasoline in travelling 616 km. What was the rate of gasoline consumption in kilometres per litre?

Solution

Set up the proportions:

$$\frac{\text{kilometres travelled}}{\text{litres used}} = \frac{\text{kilometres}}{1\ \ell}$$

Known: kilometres travelled $= 616$
litres used $= 56$
kilometres $= x$

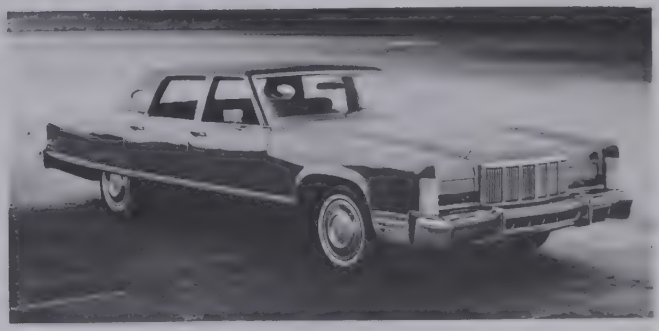
Replacements: $\frac{616}{56} = \frac{x}{1}$

$56x = 616 \times 1$ (Do you remember this?)

$\frac{56x}{56} = \frac{616}{56}$ (Divide each side by 56.)

$x = 11$

The rate of gasoline consumption is 11 km/ℓ.



There are various reasons why a car might have a high gas consumption rate. Can you think of several?

Second Method

The European method of calculating the relation between gasoline consumption and distance travelled expresses gasoline consumption in litres per 100 km.

These comparisons of approximate consumption will be of help.

Compact car	10 ℓ/100 km
Standard-size car	15 ℓ/100 km

You will notice that, the lower the number of litres, the more economical the car is to operate.

Example 2

A car went 250 km on 42.5 ℓ. What was the gasoline consumption per 100 km? What kind of car do you think this is?

Solution

First we set up the proportion:

$$\frac{\text{litres used}}{\text{km travelled}} = \frac{\text{litres}}{100 \text{ km}}$$

Known: litres used $= 42.5$
kilometres travelled $= 250$
litres $= x$

$$\frac{42.5}{250} = \frac{x}{100} \qquad \text{(Making replacements.)}$$

$$250x = 42.5 \times 100$$

$$\frac{250x}{250} = \frac{42.5 \times 100}{250}$$

$$x = 17$$

The consumption rate is 17 ℓ/100 km. It is likely that this was either a large car with a high gas consumption rate or a small car running very inefficiently or heavily overloaded.

EXERCISE 10·8

1. A tourist filled the gas tank on his subcompact car when the odometer showed 4234.6 km. Later when he stopped at a service station, he needed 45 ℓ to fill the tank. The odometer showed 4609.6 km. What was the gasoline consumption per 100 km? What was the gasoline consumption in kilometres per litre?

2. The odometer reading before a trip was 6721.5 km. The odometer read 7066.8 km after the trip. The amount of gasoline used was 32.4 ℓ. Calculate the rate of gasoline consumption in two ways.

3. A motorcycle odometer read 1723.6 km before a trip and 2098.5 km afterwards. The amount of gasoline used was 15.6 ℓ. Calculate the rate of gasoline consumption in two ways.

4. A tourist plans a trip of 853 km. His rate of gasoline consumption is 11.4 ℓ/100 km. Gasoline costs 18¢/ℓ. Calculate the cost of the gasoline for this trip.

5. Which of the two methods of calculating gasoline consumption is more useful? Why? Which method would you suggest we adopt in Canada?

6. The length of a planned trip is 900 km and the rate of gasoline consumption is 9.8 ℓ/100 km. If gasoline costs 19.5¢/ℓ, calculate the cost of the gasoline for this trip.

7. Length of planned trip – 1238 km.
   Kilometres per litre – 8.6.
   Cost of gasoline – 15.2¢/ℓ.
   Calculate the cost of gasoline for the trip.

8. Length of trip – 709.8 km.
   Kilometres per litre – 15.6.
   Cost of gasoline – 14.2¢/ℓ.
   Calculate the cost of gasoline for the trip.

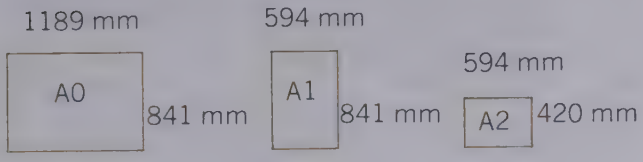
9. Shoe sizes can be based on the length of the foot in millimetres and multiples of 5 can be used. Hence, sizes such as 255, 260, 265, and 270 are four consecutive sizes. Measure and record the shoe sizes for 10 students in your class. What will be the shoe size for a foot 261 mm long?

10. An aquarium is 54 cm by 27 cm by 26 cm.
    (a) What is its capacity in cubic centimetres? in litres?
    (b) What is the force on the bottom of the tank?
    (c) What is the pressure on the bottom in pascals?

11. If neck sizes and sleeve lengths of shirts are in millimetres, record the neck size and sleeve length of five students in your class.

12. Why might it be useful to have two methods for calculating gasoline consumption?

13. The most common size of paper is A4 which measures 297 mm by 210 mm. A smaller size is A5 which measures 210 mm by 148 mm. The advantage of this method of sizing is that each size has approximately one half the area of the next larger size. The trimming of paper to size is done in the following manner. We begin with the largest size paper, A0.

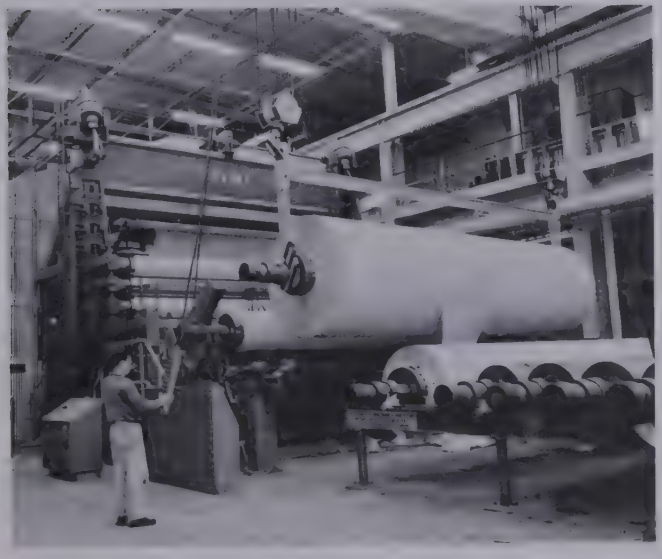


(a) What are the approximate dimensions of the following sheets of paper?
(i) A3 (ii) A6 (iii) A7

(b) What is the area of each of these sheets of paper?
(i) A4 (ii) A5 (iii) A6 (iv) A7 (v) A3

(c) What is the area of A0 paper?

(d) Envelopes for paper are similarly named. A C5 envelope will take an A5 sheet; or an A4 sheet folded once; or an A3 sheet folded twice. Show how to fold an A3 sheet to fit a C5 envelope.

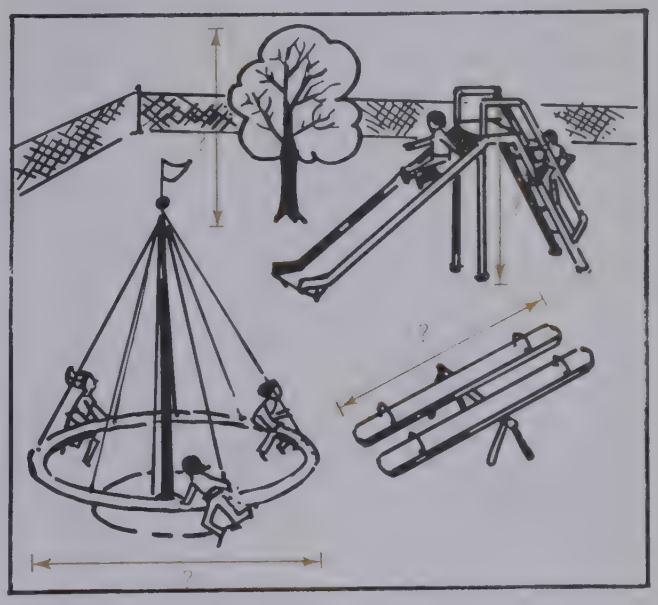


14. Does the newspaper in your community have any of the dimensions mentioned in Question 13?

REVIEW EXERCISE 10

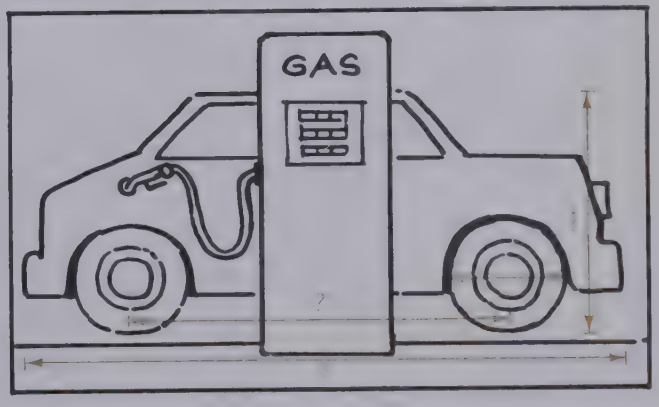
Look at the pictures and estimate the dimensions, quantities, or measures indicated.

1.



2. What is the normal speed limit on a city street?

3.

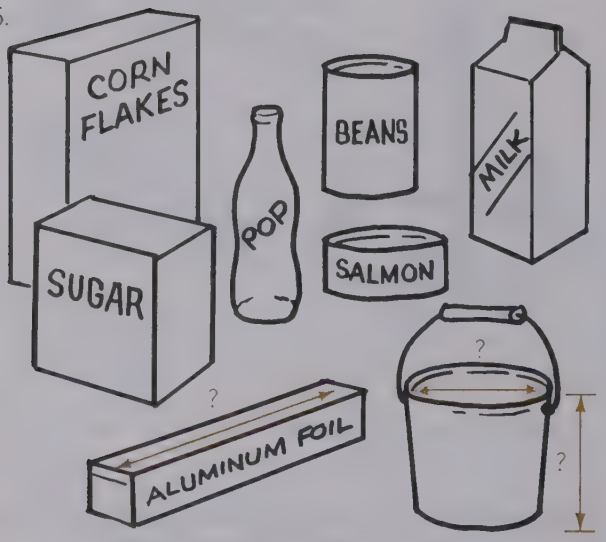


What is the tank capacity and rate of gas consumption per 100 km? What is the average price per litre of gasoline? What is a small car's rating in cubic centimetres?

4. What is the usual air temperature at the beach on a hot day?

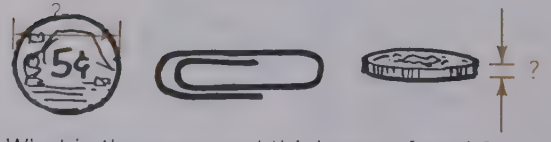
5.

6.



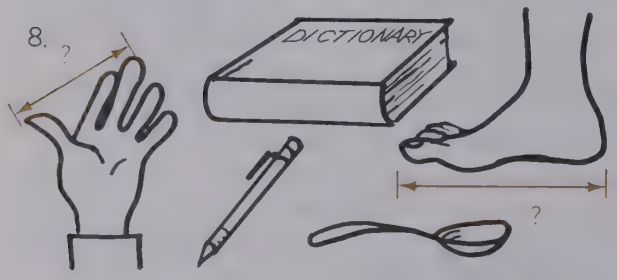
What is the capacity of each?

7.



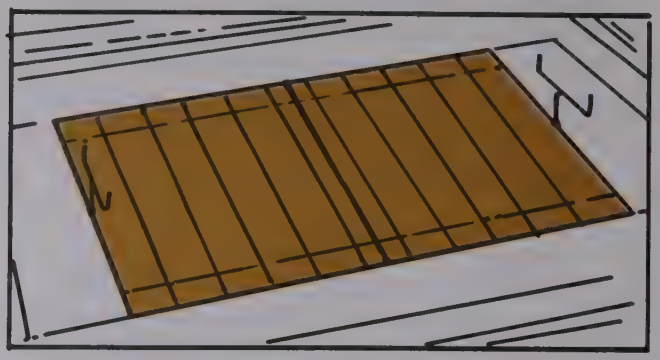
What is the mass and thickness of each?

8.



What is the mass of a book? pen? water in a spoon?

9. What are the dimensions and area? height of goal post?

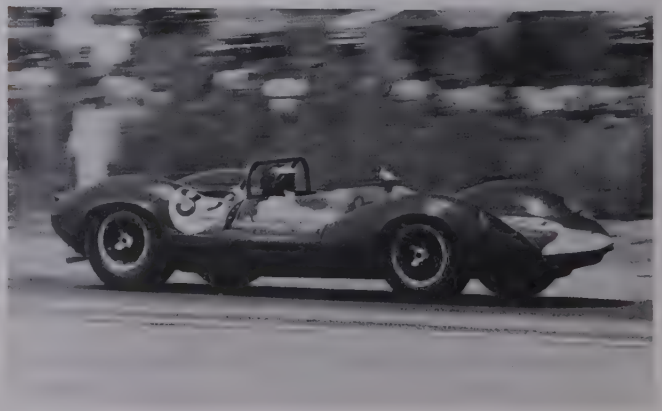


10. If the fever is gone, then what is the person's temperature?

11. What is the meaning of each prefix?

(a) deci (b) hecto
(c) deca (d) milli
(e) kilo (f) centi

12. What is the speed of the car?



Using the Library

1. Use the library to write reports on the following topics.
   (a) Report on the origin of the metre. What was the original definition of a metre?
   (b) When did Canada decide to adopt the metric system for business and our everyday lives?
   (c) When did other countries of the world adopt the metric system?
   (d) What has been the effect on Canada of full metric conversion?
   (e) What is the SI (Systeme International)?
   (f) Why has Canada converted from the Imperial System (the foot pound system) to the SI or metric system?
   (g) Write a brief history of measurement. Each student or group should take one aspect such as distance, time, temperature, mass, etc. Prepare a picture report.

2. *Prepare Now for a Metric Future*. Donovan, F. Clarke, Irwin and Company Ltd. (1970).
   (a) Read pages 80-109. Briefly report on why some people opposed metric conversion on a religious basis.
   (b) Read pages 56-57 and 81-83. If clocks and calendars were decimalized, describe what you think a clock would look like and what the weeks and months would be like. How many months would there be in a year?

3. What is the World Calendar? What are its advantages and disadvantages?

4. *Reading in Mathematics*, Book 1. Adler, I. Ginn and Company (1972).
   (a) Read page 149. Calculate which of these is more economical per unit of mass or volume. Find the prices at your local supermarket.
      (i) A small package or a giant economy size of soap powder.
      (ii) A small jar or a giant jar of peanut butter.
      (iii) A small bottle or a large bottle of ketchup.
      (iv) Three other products that come in different sizes of containers.
   (b) Read pages 150-154. What aspect of Archimedes' life most intrigues you? Write a short story based on this aspect. You may want to get more details from other books in your library.

5. Make a time line to show the growth of modern-day communication. Include the telegraph, telephone, radio, and television.

6. Write a report on the history of units used to measure mass.

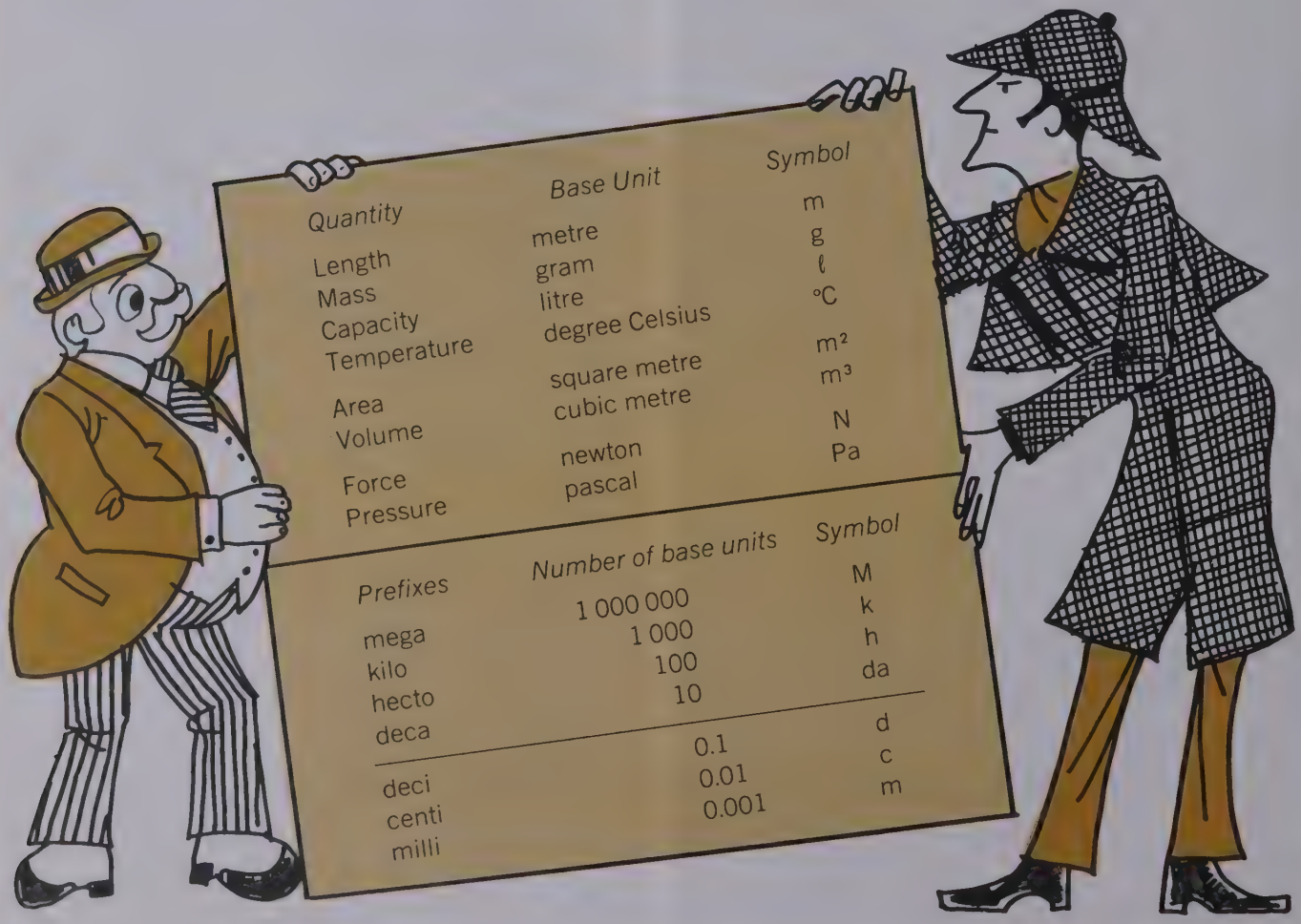
7. *String, Straight Edge And Shadow, The Story of Geometry*. Diggins, J. E. Viking Press (Macmillan Company of Canada, Ltd.) (1965).
   (a) Read pages 59-78. Who were the Ionian Greeks? What was the most interesting part of this section?
   (b) How do you construct a Golden Mean? What is a Golden Section?
   (c) Read any other chapter in this book. Write a short report on what you found interesting.

Something to Think About . . .

Do *one* of each set of questions and then exchange questions with a classmate. Remember to keep the answers to your own questions.

1. (a) Prepare three questions which relate to problems in home economics. Use SI units in your questions.
   (b) Prepare three questions which relate to problems in automotive mechanics. Use SI units in your questions.
2. (a) Prepare three questions on consumer education topics (retail stores, purchasing, selling, banking, etc.) which utilize SI units.
   (b) Prepare three woodworking questions which utilize SI units.
3. (a) Prepare three science questions which utilize SI units.
   (b) Prepare three geography questions which utilize SI units.

Summary: The Metric System



Quantity	*Base Unit*	*Symbol*
Length	metre	m
Mass	gram	g
Capacity	litre	ℓ
Temperature	degree Celsius	°C
Area	square metre	m^2
Volume	cubic metre	m^3
Force	newton	N
Pressure	pascal	Pa

Prefixes	*Number of base units*	*Symbol*
mega	1 000 000	M
kilo	1 000	k
hecto	100	h
deca	10	da
deci	0.1	d
centi	0.01	c
milli	0.001	m

SQUARES AND SQUARE ROOTS

n	n^2	$\sqrt{n}$
1	1	1.000
2	4	1.414
3	9	1.732
4	16	2.000
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162
11	121	3.317
12	144	3.464
13	169	3.606
14	196	3.742
15	225	3.873
16	256	4.000
17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472
21	441	4.583
22	484	4.690
23	529	4.796
24	576	4.899
25	625	5.000
26	676	5.099
27	729	5.196
28	784	5.292
29	841	5.385
30	900	5.477
31	961	5.568
32	1 024	5.657
33	1 089	5.745
34	1 156	5.831
35	1 225	5.916
36	1 296	6.000
37	1 369	6.083
38	1 444	6.164
39	1 521	6.245
40	1 600	6.325
41	1 681	6.403
42	1 764	6.481
43	1 849	6.557
44	1 936	6.633
45	2 025	6.708
46	2 116	6.782
47	2 209	6.856
48	2 304	6.928
49	2 401	7.000
50	2 500	7.071

n	n^2	$\sqrt{n}$
51	2 601	7.141
52	2 704	7.211
53	2 809	7.280
54	2 916	7.348
55	3 025	7.416
56	3 136	7.483
57	3 249	7.550
58	3 364	7.616
59	3 481	7.681
60	3 600	7.746
61	3 721	7.810
62	3 844	7.874
63	3 969	7.937
64	4 096	8.000
65	4 225	8.062
66	4 356	8.124
67	4 489	8.185
68	4 624	8.246
69	4 761	8.307
70	4 900	8.367
71	5 041	8.426
72	5 184	8.485
73	5 329	8.544
74	5 476	8.602
75	5 625	8.660
76	5 776	8.718
77	5 929	8.775
78	6 084	8.832
79	6 241	8.888
80	6 400	8.944
81	6 561	9.000
82	6 724	9.055
83	6 889	9.110
84	7 056	9.165
85	7 225	9.220
86	7 396	9.274
87	7 569	9.327
88	7 744	9.381
89	7 921	9.434
90	8 100	9.487
91	8 281	9.539
92	8 464	9.592
93	8 649	9.644
94	8 836	9.695
95	9 025	9.747
96	9 216	9.798
97	9 409	9.849
98	9 604	9.899
99	9 801	9.950
100	10 000	10.000

n	n^2	$\sqrt{n}$
101	10 201	10.050
102	10 404	10.100
103	10 609	10.149
104	10 816	10.198
105	11 025	10.247
106	11 236	10.296
107	11 449	10.344
108	11 664	10.392
109	11 881	10.440
110	12 100	10.488
111	12 321	10.536
112	12 544	10.583
113	12 769	10.630
114	12 996	10.677
115	13 225	10.724
116	13 456	10.770
117	13 689	10.817
118	13 924	10.863
119	14 161	10.909
120	14 400	10.955
121	14 641	11.000
122	14 884	11.045
123	15 129	11.091
124	15 376	11.136
125	15 625	11.180
126	15 876	11.225
127	16 129	11.269
128	16 384	11.314
129	16 641	11.358
130	16 900	11.402
131	17 161	11.446
132	17 424	11.489
133	17 689	11.533
134	17 956	11.576
135	18 225	11.619
136	18 496	11.662
137	18 769	11.705
138	19 044	11.747
139	19 321	11.790
140	19 600	11.832
141	19 881	11.874
142	20 164	11.916
143	20 449	11.958
144	20 736	12.000
145	21 025	12.042
146	21 316	12.083
147	21 609	12.124
148	21 904	12.166
149	22 201	12.207
150	22 500	12.247

n	n^2	$\sqrt{n}$
151	22 801	12.288
152	23 104	12.329
153	23 409	12.369
154	23 716	12.410
155	24 025	12.450
156	24 336	12.490
157	24 649	12.530
158	24 964	12.570
159	25 281	12.610
160	25 600	12.649
161	25 921	12.689
162	26 244	12.728
163	26 569	12.767
164	26 896	12.806
165	27 225	12.845
166	27 556	12.884
167	27 889	12.923
168	28 224	12.962
169	28 561	13.000
170	28 900	13.038
171	29 241	13.077
172	29 584	13.115
173	29 929	13.153
174	30 276	13.191
175	30 625	13.229
176	30 976	13.267
177	31 329	13.304
178	31 684	13.342
179	32 041	13.379
180	32 400	13.416
181	32 761	13.454
182	33 124	13.491
183	33 489	13.528
184	33 856	13.565
185	34 225	13.602
186	34 596	13.638
187	34 969	13.675
188	35 344	13.711
189	35 721	13.748
190	36 100	13.784
191	36 481	13.820
192	36 864	13.856
193	37 249	13.892
194	37 636	13.928
195	38 025	13.964
196	38 416	14.000
197	38 809	14.036
198	39 204	14.071
199	39 601	14.107
200	40 000	14.142

Answers

CHAPTER 1

EXERCISE 1·1, Page 3

5. (a) 300 (b) 9010 (c) 15 000 420 **6.** (a) 4799 (b) 307 999 (c) 1 063 899
7. 2 000 700, 801 000, 38 462, 15 007, 9712, 7090 **8.** (a) 121 (b) 38 (c) 672 (d) 6 (e) 753
(f) 428 (g) 32 754 (h) 15 (i) 4110 (j) 1736 (k) 88 596 (l) 53 (m) 13 651 (n) 2575

EXERCISE 1·2, Pages 5-6

6. (b) 350 (c) 3500 (d) 280 (e) 2800 (f) 225 (g) 1000 (h) 600 (i) 6000

EXERCISE 1·3, Pages 8-9

1. Subtraction is not closed on the set of whole numbers.
2. Division is not closed on the set of whole number .
3. The commutative property does not hold for either subtraction or division.
4. H, I, M, O, T, V, X, and Y **5.** The associative property does not hold for either subtraction or division.
7. (i) p, q (ii) k, r (iii) c, h (iv) f, o (v) b, i (vi) d, s (vii) a, l (viii) n, q (ix) a, j, m (x) e, t
8. 1586, 1408, 1124, 1725, 1682, 5035, 1929, 1227, 4738, 52, 9660, 38 913, 25, 200, 70 902, 40 689, 20.4, 26.83, 172.84, 51.31

EXERCISE 1·4, Pages 10-11

2. (a) 20 (b) 20 (c) 60 **3.** (a) 7 (b) 39 (c) 21 (d) 22 (e) 6 (f) 84 (g) 28 (h) 63
4. (a) 3 (b) 1 (c) 1 (d) 3 (e) 1 (f) 3 (g) 4 (h) 47 **5.** (a) 13 (b) 6 (c) 60 (d) 1 (e) 24 (f) 11
6. (a) 62 (b) 16 (c) $2\frac{16}{25}$ or 2.64 (d) $1\frac{1}{10}$ or 1.1 (e) $2\frac{1}{2}$ or 2.5

EXERCISE 1·5, Page 12

1. (a) 7 (b) 7 (c) 8 (d) 3 (e) 72 (f) 6 (g) 21 (h) 51 (i) 22 (j) 21 (k) 41 (l) 56 (m) 12
(n) 8 (o) 67 (p) 42 (q) 55 (r) 23 (s) 40 (t) 75 (u) 65

EXERCISE 1·6, Page 14

1. $\approx$ means "approximately equal to". **2.** (a) 30 (b) 30 (c) 70 (d) 200 (e) 300 (f) 300
(g) 20 (h) 50 (i) 100 (j) 400 (k) 600 (l) 800 **3.** (a) 4800 (b) 200 (c) 800 (d) 4800
(e) 400 (f) 1800 (g) 4000 (h) 4900 (i) 630 000 (j) 180 000 **5.** 125 659 rounded to nearest unit
6. 126 861 **7.** (a) 731 (b) 448 (c) 3015 (d) 2117 (e) 192 832 (f) 148 016
8. (a) 4592 (b) 299 (c) 585 (d) 4290 (e) 490 (f) 1392 (g) 4356 (h) 4810 (i) 641 144
(j) 167 811

EXERCISE 1·7, Pages 15-16

1. (c) **2.** (a) **3.** (a) **4.** (a) **5.** (b) **6.** (a) **7.** (c) **8.** (c) **9.** 1800 **10.** 30 000 **11.** 3600
12. 2400 **13.** 270 000 **14.** 6300 **15.** 60 **16.** 160 **17.** 250 **18.** 1952, 29 070, 3738, 2574, 309 400, 6336, 58.98, 137.75, 228.29 **19.** 470 kg **20.** 11 kg 976 g

EXERCISE 1·8, Page 17

1. (a) 11^3 (b) 5^7 (c) 9^4 (d) 10^{12} (e) x^5 (f) $(3x)^6$ (g) $(16+x)^3$ (h) $3(2x+5)^4$
2. (a) $3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3\cdot3$ (b) $12\cdot12\cdot12\cdot12\cdot12$ (c) $3\cdot3\cdot3\cdot3\cdot a\cdot a\cdot a\cdot a$ (d) $5\cdot x\cdot x\cdot x$
(e) $x\cdot x\cdot x\cdot x\cdot x\cdot z\cdot z$ **3.** (a) 81 (b) 23 328 (c) 1600 (d) −92 (e) 147 (f) 2450 (g) 81
(h) 1092 **4.** (a) x^5 (b) t^6 (c) y^8 (d) x^2 (e) m^2 (f) z^4 **5.** No **6.** No **7.** $\$2\times10^6$

EXERCISE 1·9, Page 19

1. (a) 5 (b) 21 (c) 94 (d) 3554 (e) 19 (f) 46 (g) 164 (h) 35 (i) 14 (j) 18 (k) 7 (l) 10
2. (a) $x \in \{8, 9, 10, \ldots\}$ (b) $x \in \{3, 4, 5, \ldots\}$ (c) $x \in \{0, 1, 2, 3\}$ (d) $x \in \{0, 1, 2, \ldots, 8\}$
(e) $x \in \{6, 7, 8, \ldots\}$ (f) $x \in \{4, 5, 6, \ldots\}$ (g) $x \in \{0, 1, 2, 3, 4, 5, 6\}$ (h) $x \in \{0, 1, 2, \ldots, 9\}$
(i) $x \in \{6, 7, 8, \ldots\}$ (j) $x \in \{8, 9, 10, \ldots\}$ (k) $x \in \{0, 1, 2, 3, 4\}$ (l) $x \in \{0, 1, 2\}$
(m) $x \in \{6, 7, 8, \ldots\}$ (n) $x \in \{0, 1, 2, \ldots, 27\}$
3. (a) $k \in \{6, 7, 8, 9\}$ (b) $k \in \{4, 5\}$ (c) $k \in \{4, 5\}$ (d) $k \in \{4\}$
4. (a) $y \in \{8, 10, 12\}$ (b) $y \in \{2, 4\}$ (c) $y \in \{4, 6, 8, 10, 12\}$ (d) $y \in \{2, 4, 6, 8\}$
(e) $y \in \{2, 4, 6, 8\}$ (f) $y \in \{2\}$ (g) $y \in \{2, 4\}$ (h) $y \in \{2, 4, 6\}$
5. These do not have solutions in W. The solution set is empty. $x \in \{\ \}$.

REVIEW EXERCISE 1, Page 20

1. (d), (f), (a), (c), (b), (e) **2.** (a) $4(a-3)=4a-12$ (b) $4+0=4$ (c) $3\times(4\times5)=(3\times4)\times5$ (d) $12+5=5+12$ **3.** (a) 64 (b) 1 (c) 5 **4.** (a) 128 (b) 35 (c) 510
5. (a) $n=4$ (b) $n=7$ (c) $n\in\{5, 6, 7, \ldots\}$ (d) $n\in\{7, 8, 9, \ldots\}$
(e) $n\in\{0, 1, 2, \ldots, 14\}$ (f) $n\in\{13, 14, 15, \ldots\}$

CHAPTER 2

EXERCISE 2·1, Page 26

1. (a) 11 (b) 11 (c) 15 (d) 9 (e) 15 (f) 16 **2.** (a) 6 (b) 7 (c) 9 (d) 6 (e) 13 (f) 6
4. (a) −5 (b) −9 (c) −13 (d) −13 (e) −15 (f) −9 **5.** (a) −2 (b) +7 (c) −15 (d) +12
(e) −5 (f) +7
7. (a) −10 (b) −13 (c) −23 (d) −17 (e) −17 (f) −21
8. (a) +4 (b) +15 (c) −8 (d) −5 (e) −11 (f) +12 (g) −3 (h) +29
9. (a) +12 (b) −75 (c) +136 (d) −11 (e) +15 (f) −22
10. (b) 446 (c) +4 (d) +13 (e) +9 (f) −88 (g) +4 (h) +18 (i) +79 (j) −17

EXERCISE 2·2, Page 28

1. The dots mean that the sequence goes on to infinity.
2. Zero is a member of the set of whole numbers and it is not considered to be a positive number.
3. (a) 2 (b) 5 (c) 1 (d) 0 (e) −1 (f) −12 (g) −376 (h) −15
4. (a) {−9, −4, 0, 7, 12} (b) {−18, −13, −12, −7, −5} (c) {−200, −140, −70, 20, 150}

EXERCISE 2·3, Pages 32-33

1. +11, +7, −7, 0, +7, −10, −5, −11 **2.** +3, +17, +13, +12, −11, 0, −21, +3
3. +28, −60, −30, −36, −18, +25, −104, +28 **4.** (a) +6 (b) −5 (c) +4 (d) −3 (e) +3
5. (a) −13 (b) −48 (c) +7 (d) −7 (e) +9 (f) +13 (g) +45 (h) +23 (i) −8 (j) +36 (k) −6
(l) +6 (m) −6 (n) +3 (o) −35 (p) −21 (q) −1 (r) +42 (s) +8 (t) +2 (u) −12 (v) −5
6. (a) −5 (b) −28 (c) −304 (d) +3112 (e) +12 (f) +108 (g) +325 (h) +4512 (i) −1 (j) −12
(k) +317 (l) −1276 (m) −8 (n) −142
7. (a) +16 (b) +114 (c) +653 (d) +7807 (e) +1 (f) +15 (g) −103 (h) −5925 (i) −10
(j) −100 (k) −982 (l) −9330 (m) +4 (n) −4 (o) +561 (p) −245
8. (a) −4 (b) −6 (c) and (d) do not have integer solutions.

EXERCISE 2·4, Pages 35-36

1. (a) Commutative (b) Distributive (c) Closure (d) Associative (e) Additive Identity
(f) Distributive (g) Associative (h) Multiplicative Identity (i) Commutative (j) Closure
(k) Additive Identity (l) Associative (m) Distributive (n) Multiplicative Identity (o) Commutative
2. (a) −15 (b) −126 (c) −1512 (d) −14 896 (e) +63 (f) +581 (g) +3420 (h) +34 317 (i) −24
(j) −608 **3.** (a) −6 (b) −14 (c) −23 (d) −45 (e) −9 (f) −42 (g) −56 (h) −64 (i) +8
(j) +28 (k) +28 (l) +67 **4.** (a) True (b) True (c) False $r(s-t)=rs-rt$ (d) True
(e) False $k(m-n)=km-kn$ **5.** Subtraction and division **6.** Addition and multiplication
7. (a) 0 (b) +1 **8.** +1 and −1, +2 and −2, +3 and −3, ... **9.** +1 and +1, −1 and −1
10. (b) $-16x+8y$ (c) $-6x+9y$ (d) $48x+48y$ (e) $-21y+28x+56z$ (f) $-144y+108z$
11. (a) 132 (b) 396 (c) 162 (d) −512 (e) 62

EXERCISE 2·5, Page 38

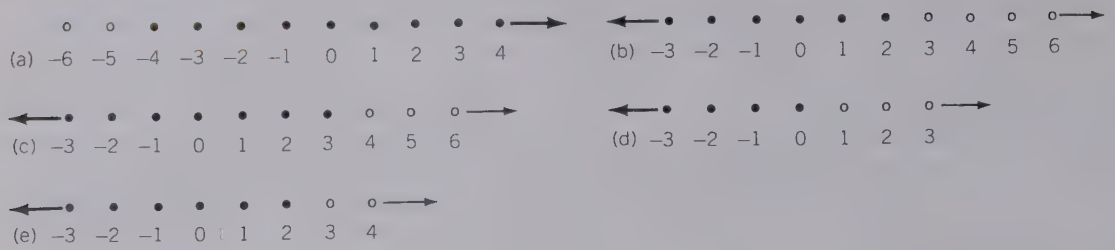
1. (a) 21 (b) 9 (c) 54 (d) 18 (e) −39 (f) 51 (g) −3 (h) 33 (i) −39 (j) 45
2. (a) −45 (b) 75 (c) 31 (d) 160 (e) −90 (f) 110 (g) 600 (h) −125 (i) −140 (j) 21
(k) −146 (l) −165 **3.** (a) −3 (b) −26 (c) 15 (d) 13 (e) −33 (f) 33 (g) −2 (h) 33 (i) 4
(j) 65 (k) 11 (l) 36 (m) 29 (n) 95 (o) 4 **4.** (a) −64 (b) 16 (c) 36 (d) 176 (e) 24
(f) −76 (g) 75 (h) −72 (i) 116 **5.** (a) 3 (b) 1 (c) 14 **6.** $156.25 **7.** (a) 18 m (b) 40 m
(c) 44 m (d) 74 m (e) 120 m **8.** (a) 63 m² (b) 286 m² (c) 156 m² (d) 625 m²
(e) 2500 m² **9.** (a) 270 km (b) 600 km (c) 1035 km (d) 935 km (e) 975 km

EXERCISE 2·6, Pages 39-40

1. (b) $3-5$ (c) $3+4$ (d) $-4+6$ (e) $2n+3$ (f) $-2n+3$ (g) $-5n+8$ (h) $3n+9$
2. (a) −3 (b) −6 (c) 14 (d) 6 (e) 3 (f) 6 (g) −3 (h) −1

3. (a) 11 (b) 5 (c) −7 (d) 5 (e) 20 (f) −9 (g) −3 (h) −5 (i) 0 (j) 0
4. (a) $y \in \{0, 1, 2, \ldots\}$ (b) $y \in \{\ldots, -2, -1, 0\}$ (c) $y \in \{\ldots, -1, 0, 1\}$
(d) $y \in \{\ldots, 0, 1, 2\}$ (e) $y \in \{-2, -1, 0, \ldots\}$ (f) $y \in \{-1, 0, 1, \ldots\}$
(g) $y \in \{\ldots, 1, 2, 3\}$ (h) $y \in \{4, 5, 6, \ldots\}$ (i) $y \in \{5, 6, 7, \ldots\}$
(j) $y \in \{\ldots, -1, 0, 1\}$ (k) $y \in \{\ldots, 1, 2, 3\}$ (l) $y \in \{\ldots, 0, 1, 2\}$
(m) $y \in \{8, 9, 10, \ldots\}$ **5.** (a) $x \in \{-1, 0, 1, 2, 3\}$ (b) $x \in \{-3, -2, -1, 0, 1, 2, 3\}$
(c) $x \in \{-3, -2, -1, 0\}$ (d) $x \in \{3\}$ (e) $x \in \{\ \}$ (f) $x \in \{-3, -2, -1, 0, 1, 2\}$
6. (a) 3 402 000 (b) 226 800 **7.** 25 720

EXERCISE 2·7, Page 41



EXERCISE 2·8, Pages 43-44

1. (a) $n + 4 = 7$ (b) $3n = 18$ (c) $n - 5 = 14$ (d) $4n > 10$ (e) $n + 3 < 10$ (f) $n - 5 = 6$
(g) $n - 3 \leq 8$ (h) $n + 10 = 17$ (i) $n - 4 = 2$ (j) $3n = 15$ (k) $2n + 2(n + 2) = P$, $n(n + 2) = A$
(l) $2n + 4 = 10$ **2.** $n + 8 = 15$, $n = 7$ **3.** $n = 17 + 9$, $n = 26$ **4.** $n - 12 = 7$, $n = 19$
5. $n = 17 - 10$, $n = 7$ **6.** $n - 5 = 9$, $n = 14$ **7.** $13x = 39$, $x = 3$, 9 cm, 12 cm, 18 cm
8. $n + (n + 3) = 25$. Mary is 11 and Billy is 14.
9. $n + 6n + 9n = 160$; flax 10 ha, rye 60 ha, wheat 90 ha.
10. $n + (n + 4) = 22$. 9 and 13. **11.** $n + 7n = 16$. 2 m and 14 m. **12.** $3n + 7 = 22$, $n = 5$.

REVIEW EXERCISE 2, Page 44

1. (a) $\{-7, -3, -1, 0, 2, 5, 7\}$ (b) $\{-17, -14, -12, -9, -6, -3, 2\}$
2. (a) −2 (b) −9 (c) −3 (d) 4 (e) −3 (f) −1 (g) −17 (h) 11 (i) 4 (j) −5 (k) 9 (l) 0
3. $m = 8$ **4.** (a) 3 (b) 28 (c) 2 **5.** (a) 11 (b) −5 (c) 4 (d) 2
6. (a) $y \leq 3$ ← • • • • • • → −2 −1 0 1 2 3 4 5 (b) $y > -4$ ← • • • • • • → −5 −4 −3 −2 −1 0 1 2
7. $n + (n - 7) = 17$; May has 5 books and Jean has 12. (b) $n + 28 + 9$, $n = 37$
8. (a) −5 (b) −23 (c) 1 (d) 12 (e) 57 (f) −261

CHAPTER 3

EXERCISE 3·1, Pages 48-49

1. (a) 12.1 (b) 2.1 (c) 50 (d) 160 (e) 265 (f) 125 **2.** 69.84
7. 78.86 (a) 57.6 (b) 49.8 (c) 39 (d) 50.4 (e) 67.2 (f) 28.8 (g) 56.58 (h) 93.44 (i) 69.19

EXERCISE 3·2, Page 50

1. (a) $1\frac{1}{2}$ (b) $1\frac{2}{3}$ (c) $1\frac{1}{2}$ (d) $-2\frac{1}{2}$

EXERCISE 3·3, Page 52

1. $\frac{13}{15}$ **2.** $\frac{1}{8}$ **3.** $\frac{5}{6}$ **4.** $\frac{11}{12}$ **5.** $\frac{8}{15}$ **6.** $2\frac{1}{4}$ **7.** $2\frac{1}{12}$ **8.** $\frac{3}{4}$
9. $1\frac{5}{24}$ **10.** $\frac{1}{4}$ **11.** $\frac{1}{6}$ **12.** $\frac{3}{10}$ **13.** $\frac{5}{12}$ **14.** $\frac{8}{21}$ **15.** $\frac{5}{12}$ **16.** $\frac{11}{36}$
17. $\frac{1}{2}$ **18.** $2\frac{23}{24}$ **19.** $1\frac{29}{40}$ **20.** $1\frac{79}{153}$ **21.** $1\frac{29}{99}$ **22.** $-4\frac{56}{65}$ **23.** $\frac{11}{12}$ **24.** $\frac{7}{8}$
25. 15.37 **26.** 3.064 **27.** 213.21 **28.** 103.36 **29.** 12.772 **30.** 59.28 **31.** 0.378 **32.** 14.2
33. 0.32 **34.** 0.32 **35.** 3.90 **36.** 1.435 **37.** 5.30 **38.** 10 000 **39.** $\frac{1}{3}$ **40.** $\frac{3}{8}$
41. $\frac{1}{12}$ **42.** $\frac{1}{3}$ **43.** $\frac{9}{16}$ **44.** $\frac{7}{10}$ **45.** $2\frac{1}{4}$ **46.** $3\frac{1}{8}$ **47.** $7\frac{7}{8}$ **48.** $2\frac{5}{8}$ **49.** $6\frac{29}{30}$
50. $3\frac{5}{7}$ **51.** 4 **52.** 9 **53.** $4\frac{1}{2}$ **54.** $\frac{2}{3}$ **55.** 2 **56.** $\frac{1}{2}$ **57.** $1\frac{1}{3}$ **58.** $1\frac{1}{8}$

ERCISE 3·4, Pages 55-56

a) $3\frac{11}{24}$ (b) $1693\frac{1}{2}$ (c) $-5\frac{5}{16}$ (d) $11\frac{11}{48}$ (e) $-23\frac{1}{2}$, $-25\frac{1}{2}$
There are indefinitely many points.

ERCISE 3·5, Page 58

a) $0.8\dot{3}$ (b) -0.875 (c) 0.8 (d) $0.\dot{5}$ (e) $-0.5\dot{3}$ (f) $-4.\dot{4}28\,57\dot{1}$
-8.192 (h) 12.025 (i) 6.015 (j) $33.\dot{3}$ (k) -125.125 (l) $16.1\dot{6}$
a), (b), (c), and (f) are equal; the rest are approximately equal.
A (a) 99.08 (b) 103.18 (c) 18.73 (d) 27.40 (e) 29.74 (f) 6.91
a) $99\frac{1}{12}$ (b) $103\frac{7}{40}$ **C.** (a) 99.08 (b) 103.18

ERCISE 3·6, Page 60

a) $\frac{11}{20}$ (b) $\frac{16}{25}$ (c) $\frac{3}{40}$ (d) $\frac{1}{2500}$ (e) $2\frac{7}{10}$ (f) $8\frac{9}{20}$ (g) $10\frac{1}{200}$ (h) $9\frac{7}{50}$
$5\frac{6}{125}$ (j) $12\frac{97}{125}$ (k) $4\frac{1}{16}$ (l) $11\frac{11}{100}$
a) Totals: Black bear, $39 778.74; Beaver, $4 460 006.40; Fox, $409 631.76; Mink, $527 569.74;
skrat, $2 107 665.90; Seals, $1 732 947.84; Squirrel, $152 119.36; Wildcat, $42 361.44 (b) Beaver

ERCISE 3·7, Page 62

a) 23 (b) 20 (c) $6\frac{4}{5}$ (d) $8\frac{2}{3}$ (e) 10 (f) 10 (g) 2 (h) -2
a) $1\frac{7}{8}$ (b) $\frac{11}{12}$ (c) $1\frac{1}{3}$ (d) $\frac{8}{9}$ (e) $11\frac{1}{4}$ (f) -4 (g) 5.3 (h) 11
a) 1 (b) $-\frac{1}{3}$ (c) $2\frac{2}{3}$ (d) 138 (e) $4\frac{1}{2}$ (f) -19 (g) 275 (h) $\frac{6}{19}$ (i) 60.48 (j) 5.76
a) 2 (b) 1 (c) $1\frac{2}{3}$ (d) $\frac{5}{16}$ (e) $1\frac{7}{8}$ (f) $\frac{1}{2}$ (g) $\frac{9}{32}$ (h) $1\frac{1}{2}$ (i) $1\frac{1}{8}$
(k) $-1\frac{1}{5}$ (l) $1\frac{1}{3}$ (m) $\frac{4}{9}$ (n) $-\frac{4}{5}$ (o) $\frac{3}{16}$ (p) $4\frac{1}{2}$ (q) -6 (r) $1\frac{1}{4}$
$\frac{3}{5}$ (t) $-1\frac{1}{2}$ (u) 2 (v) $1\frac{1}{4}$ (w) -6 (x) $\frac{2}{17}$ (y) $\frac{1}{2}$ (z) -5

ERCISE 3·8, Page 63

a) 2 (b) $\frac{2}{3}$ (c) 16 (d) $\frac{1}{9}$ **2.** (a) $\frac{1}{2}$ (b) $\frac{5}{16}$ (c) $\frac{7}{30}$
a) 9.6 m (b) 14.4 m (c) 5 m (d) 46 m (e) 42.4 m
a) 24 cm² (b) 13.5 cm² (c) 21 cm² (d) 23.66 cm² (e) 78.2 cm² (f) 87.78 cm² (g) 138 cm²
a) 15 cm² (b) 28 cm² (c) 10.5 cm² (d) 11.84 cm² (e) 22.32 cm² (f) 27.26 cm² (g) 19.305 cm²
a) -0.44 (b) 2.98 (c) 7.39 **7.** (a) 2.56 (b) 16 (c) 2.8
a) $\frac{4}{13}$ (b) $-\frac{11}{30}$ (c) $-\frac{3}{100}$ **9.** (a) -35.16 (b) 6.95 (c) $\frac{1}{8}$ (d) 0.92 (e) $\frac{11}{16}$ (f) $2\frac{1}{4}$
155.20 (h) 3.54 (i) 78.5 (j) 42.9 (k) 7.1 (l) 21.2 (m) 7.07 (n) 0.05

RCISE 3·9, Pages 66-67

a) 6.2×10^5 (b) 4.698×10^3 (c) 9.9×10^{-3} (d) 5.26×10^{-4} (e) 8.467×10^7
$.5 \times 10^1$ (g) 4.44×10^{-1} (h) 1.2×10^{-7} (i) 7.1124×10^4 **2.** (a) 4.620×10^3 km
5.190×10^3 km (c) 2.87×10^4 km (d) 6.73×10^4 km (e) 1×10^7 km² (f) 2.21×10^7
4.04×10^5 km² (h) 9.65×10^4 km (i) 7.72×10^5 km (j) 7.85×10^3 km (k) 3.66×10^3 km
a) 6.4×10^5 (b) 5.175×10^3 (c) 7.2×10^{-2} (d) 3.2×10^2 (e) 7×10^{-7} (f) 3.2×10^1
3×10^0 (h) 2×10^2 (i) 1.4×10^3 (j) 4×10^{-5} (k) 3.2×10^0 (l) 2×10^7 (m) 5.1×10^2
$.467 \times 10^4$ kg **5.** (a) \$5.4 × 10^8 (b) \$2.40 × 10^2/yr **6.** 2.21 persons/km² **7.** (a) 4600 km
5200 km (c) 29 000 km (d) 67 000 km (e) 10 000 000 km² (f) 22 000 000 (g) 400 000 km²
97 000 km (i) 770 000 km (j) 7900 km (k) 3700 km **8.** 2.2 persons/km²
a) 2.397 78 × 10^7 km² (b) 7.94 times (c) 9.6×10^4 km (d) 8.685×10^5 km
2.45×10^{-4} **11.** 9.4×10^{12} km **12.** 4.3 yr **13.** 6×10^{23} molecules and 3×10^{25}, 1 to 50
3×10^{-13} cm **15.** (a) 1.04×10^1 h (d) 1.24×10^2 h

RCISE 3·10, Page 68

he symbol ≈ means approximately. **2.** (a) 300 (b) 600 (c) 4300 (d) 500
400 (f) 4400 (g) 480 000 (h) 640 000
a) 7 (b) 8 (c) 9 (d) 11 (e) 12 (f) 15 **7.** (a) ≈ 7 (b) ≈ 3.2 (c) ≈ 8.7

RCISE 3·11, Pages 70-72

$\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{4}$ $\frac{3}{40}$ $\frac{3}{5}$ $\frac{4}{5}$ $\frac{1}{3}$ $\frac{5}{7}$ $\frac{7}{8}$ $\frac{3}{4}$ $\frac{5}{9}$ $\frac{1}{4}$

$\frac{0}{0}$, $\frac{25}{100}$, $\frac{40}{100}$, $\frac{70}{100}$, $\frac{20}{100}$, $\frac{60}{100}$, $\frac{35}{100}$, $\frac{26}{100}$, $\frac{95}{100}$, $\frac{46}{100}$, $\frac{80}{100}$, $\frac{85}{100}$, $\frac{76}{100}$, $\frac{65}{100}$, $\frac{96}{100}$

$\frac{5}{0}$, $\frac{35}{100}$, $\frac{41}{100}$, $\frac{62}{100}$, $\frac{10}{100}$, $\frac{5}{100}$, $\frac{1}{100}$, $\frac{3}{100}$, $\frac{100}{100}$, $\frac{110}{100}$, $\frac{300}{100}$, $\frac{500}{100}$, $\frac{2}{100}$, $\frac{38}{100}$

55, 0.63, 0.73, 0.42, 0.10, 0.05, 0.01, 0.03, 1.00, 1.50, 3.00, 1.42, 0.50, 0.75

$\frac{6}{0}$, $\frac{49}{100}$, $\frac{85}{100}$, $\frac{90}{100}$, $\frac{99}{100}$, $\frac{10}{100}$, $\frac{5}{100}$, $\frac{1}{100}$, $\frac{10}{100}$, $\frac{3}{100}$, $\frac{115}{100}$, $\frac{235}{100}$, $\frac{200}{100}$, $\frac{300}{100}$, $\frac{510}{100}$, $\frac{400}{100}$

5%, 98%, 39%, 18%, 40%, 10%, 5%, 1%, 114%, 125%, 155%

7. 41%, 62%, 73%, 84%, 10%, 1%, 3%, 125%, 100%, 400%, 300%, 340%, 40%, 50%
8. (a) 27 (b) 22.5 (c) 53.32 (d) $17\frac{1}{3}$ (e) 11.115 (f) 25.56 (g) 4 (h) 30
9. There are 14 students absent and 21 present. **10.** There are 16 girls and 20 boys in 9B.
11. (a) 17¢ (b) 40¢ (c) 8¢ (d) 4¢ (e) 8¢ (f) 7¢
12. (a) 80% (b) 40% (c) 80% (d) 45.2% (e) 12% (f) 5% (g) 26.9% (h) 2.4%
13. The class has 25% absent and 75% present. **14.** 27.6%, 23.8%, 35.2%
15. (a) 85.71 (b) 415.38 (c) 37.5 (d) 127.78 (e) 40 (f) 119.44 (g) 88 (h) 880
16. 30 students **17.** 775 students **18.** \$60, \$46.50, \$26.50
19. (a) \$3645 (b) 9.1% (c) \$2551.50

REVIEW EXERCISE 3, Pages 72-73

1. $3\frac{5}{16}$ **2.** $2\frac{7}{12}$ **3.** (a) 0.875 (b) −0.3125 (c) $-7.\dot{3}$ (d) $0.\dot{7}14285\dot{}$ (e) 0.575
Answers (a), (b), and (e) are terminating; (c) and (d) are repeating decimals.
4. (a) $1\frac{3}{20}$ (b) $1\frac{13}{16}$ (c) $-\frac{7}{36}$ (d) $1\frac{11}{40}$ (e) 261.66 (f) 0.8
5. (a) $\frac{1}{250}$ (b) $6\frac{3}{8}$ (c) $\frac{7}{125}$ (d) $\frac{3}{25}$ (e) $\frac{3}{400}$
6. (a) $1\frac{7}{20}$ (b) $2\frac{1}{10}$ (c) $1\frac{17}{22}$ (d) $\frac{13}{16}$ (e) $2\frac{3}{5}$ (f) $\frac{7}{15}$
7. (a) $2\frac{1}{2}$ (b) 9.1125 (c) $\frac{247}{300}$ **8.** (a) 1×10^7 (b) 7.268×10^3
(c) 8.4×10^{-2} (d) 5.96×10^5 (e) 8.36×10^{-5}
9. (a) $\frac{1}{12}$ (b) $-3\frac{1}{9}$ (c) 9.6 (d) $-\frac{1}{6}$ (e) 11.1 (f) −6.4 (g) $-\frac{5}{64}$ (h) $-\frac{3}{4}$
10. (a) $\frac{80}{100}$ (b) $\frac{65}{100}$ (c) $\frac{484}{100}$ **11.** (a) $\frac{39}{100}$ (b) $\frac{51}{100}$ (c) $\frac{188}{100}$ (d) $\frac{3}{100}$
12. (a) 0.65 (b) 0.25 (c) 1.33 (d) 0.08 **13.** (a) $\frac{39}{100}$ (b) $\frac{69}{100}$ (c) $5\frac{15}{100}$ (d) $\frac{4}{100}$
14. (a) 36% (b) 49% (c) 125% (d) 7% **15.** (a) 45% (b) 75% (c) 405% (d) 9%
16. (a) 19.25 (b) 171.25 (c) $66.\dot{6}$% (d) 15% (e) 200 (f) 125 **17.** 25%

CHAPTER 4

EXERCISE 4-1, Page 80

1. (a) 7 cm (b) 13 cm (c) 5 m (d) 9 cm (e) 15 m (f) 4 m (g) 6 cm (h) 10 m (i) 12 cm
(j) 11 cm (k) 20 m (l) 30 cm (m) 40 m (n) 60 m (o) 80 m (p) 50 cm (q) 90 cm
(r) 70 m **2.** 10 m × 10 m **3.** 4 cm **4.** 4 m **5.** 12 cm **6.** 42.25 cm² **7.** 16 m × 16 m
8. 4.2 m × 4.2 m **9.** 12 m × 12 m

EXERCISE 4-2, Page 83

1. (a) 3 (b) 6 (c) 12 (d) 9 (e) 5 (f) 11 (g) 20 (h) 4 (i) 10 (j) 13 (k) 15 (l) 25
2. (a) 21 (b) 15 (c) 24 (d) 27 (e) 66 (f) 182 (g) 105 (h) 77 (i) 72
3. (a) $1\frac{1}{4}$ (b) $1\frac{1}{3}$ (c) $1\frac{1}{10}$ (d) $\frac{3}{4}$ (e) $\frac{2}{3}$ (f) 2 (g) $\frac{6}{7}$ (h) $\frac{3}{4}$ (i) $\frac{4}{5}$

EXERCISE 4-3, Page 85

1. (a) 19 (b) 23 (c) 28 (d) 31 **2.** (a) 34 (b) 63 (c) 49 (d) 66
3. (a) 78 (b) 94 (c) 123 (d) 234 **4.** (a) 7.3 (b) 4.6 (c) 7.8 (d) 9.4
5. (a) 0.22 (b) 0.18 (c) 0.53 (d) 0.85 **6.** (a) 1.32 (b) 1.55 (c) 1.86 (d) 3.42 **8.** (a) 1.41
(b) 1.73 (c) 2.45 (d) 3.16

EXERCISE 4-4, Page 86

1. (a) 3 (b) 6 (c) 12 (d) 13 (e) 9 (f) 14 (g) 23 (h) 32 (i) 45 (j) 152 (k) 159 (l) 89
2. (a) 23.0 (b) 22.4 (c) 24.5 (d) 15.8 (e) 17.3 (f) 22.6

EXERCISE 4-5, Pages 89-91

1. (a) 13 cm (b) 10 cm (c) 7.07 cm (d) 11.35 cm (e) 14.21 cm (f) 21.26 cm
(g) 23.43 cm (h) 20.62 cm **2.** (a) 20.12 cm (b) 24.04 cm (c) 31.24 cm (d) 46.53 cm
(e) 27.20 cm (f) 22.36 cm (g) 28.33 cm (h) 22.20 cm **3.** 8.62 m
4. 52 m for each; 208 m in all. **5.** 13.37 m **6.** 5 s, 10 s, 6.32 s, 14.14 s **7.** 24 m²
8. 18.5 Ω **9.** 42 cm **10.** 13.6 km **11.** 240 km **12.** 67.1 m **13.** 10.8 ha
14. 12 ha **15.** Park 0.636 ha, left triangular lot 0.244 ha, trapezoids 0.244 ha, 0.258 ha, 0.216 ha,
0.256 ha, right triangular lot 0.179 ha. The park has the greatest area. **16.** (a) 205 200 000 m²
(b) 20 520 ha (c) 205.2 km² (d) Answers will vary. **17.** (a) Japan (b) Japan (c) Answers will vary.

EXERCISE 4·6, Page 93

Properties	Under Operations	Natural Numbers	Whole Numbers	Integers	Rationals
Closure	+	Yes	Yes	Yes	Yes
	−	No	No	Yes	Yes
	×	Yes	Yes	Yes	Yes
	÷	No	No	No	Yes
Commutative	+	Yes	Yes	Yes	Yes
	−	No	No	No	No
	×	Yes	Yes	Yes	Yes
	÷	No	No	No	No
Associative	+	Yes	Yes	Yes	Yes
	−	No	No	No	No
	×	Yes	Yes	Yes	Yes
	÷	No	No	No	No
Distributive	Multiplication over addition	Yes	Yes	Yes	Yes
Identity Element		1	0,1	0,1	0,1

ENERGY, Page 93

1. Aviation fuel 10 800 m^3, Diesel fuel 32 400 m^3, Other products and industry consumption 54 000 m^3, Heavy fuel oil 72 000 m^3, Light heating oils 79 200 m^3, Motor gasoline 111 600 m^3 **2.** 10 000 m^3 **3.** Alberta 306 000 m^3, Saskatchewan 39 600 m^3, British Columbia 10 800 m^3, Manitoba 3600 m^3 **4.** Venezuela 79 200 m^3, Arab countries 54 000 m^3, Iran 28 800 m^3, Angola 9000 m^3, Nigeria 7200 m^3, Columbia 1800 m^3 **5.** Western Provinces 76 500 m^3, Ontario 85 000 m^3, Quebec 8500 m^3

REVIEW EXERCISE 4, Page 94

1. $\sqrt{78}$ m ≈ 8.83 m **2.** (a) 63 (b) 84 **3.** 33.01 cm **4.** $\frac{11}{25}$ **5.** 18.4 **6.** 31, 37, 41
7. 14 km **8.** 14.5 cm

CHAPTER 5

EXERCISE 5·1, Page 98

1. (a) 2 (b) 3 (c) 3 (d) 1 (e) 1 (f) 2 (g) 3 (h) 2
2. (a) 12 (b) 9 (c) 8 (d) 7 (e) 8 (f) 2 (g) 2 (h) 0 (i) 3 (j) 12 (k) 1 (l) 1 (m) 2 (n) 2 (o) 3 (p) 12

EXERCISE 5·2, Page 100

1. (a) (ii) Subtract 6 from each side. (b) (i) Multiply each side by 2. (ii) Divide each side by 2. (c) (i) Multiply each side by 5. (ii) Divide each side by 5. (d) (i) Subtract 6 from each side. (ii) Add 6 to each side. (e) (i) Multiply each side by 3. (ii) Add 5 to each side. (iii) Subtract 5 from each side. (iv) Divide each side by 3. **2.** (a) (i) Multiply each side by −3. (ii) Add 4 to each side. (iii) $-3x = -33$. (iv) $x = 11$. (b) (i) Multiply each side by 4. (ii) Add 6 to each side. (iii) $4x = 32$. Subtract 6 from each side. (iv) $x = 8$. Divide each side by 4. (c) (i) Multiply each side by −4. (ii) Add 3 to each side. (iii) $-4x = 28$. Subtract 3 from each side. (iv) $x = -7$. Divide each side by −4. (d) (i) Multiply each side by 8. (ii) Add 14 to each side. (iii) $8x = 72$. Subtract 14 from each side. (iv) $x = 9$. Divide each side by 8.

EXERCISE 5·3, Pages 102-103

1. (a) 8 (b) 6 (c) 44 (d) −8 (e) 64.25 (f) −23.5
2. (a) 8 (b) −79 (c) $-59\frac{1}{2}$ (d) 7 (e) −199 (f) −94.8
3. (a) 86 (b) −3 (c) 26 (d) −27 (e) $30\frac{1}{3}$ (f) −32 (g) −10 (h) $53\frac{3}{4}$ (i) −34.5 (j) −9.7
4. (a) 2 (b) 12 (c) −5 (d) −7 (e) −11 (f) $8\frac{4}{13}$ (g) 5 (h) −3 (i) $-\frac{41}{48}$ (j) $-2\frac{2}{3}$

5. (a) 16 (b) 27 (c) 32 (d) −9 (e) −20 (f) −12 (g) 3.4 (h) 12.8 (i) 32 (j) 120 (k) −55 (l) 946 (m) 572 (n) 10 100 (o) 7868 **6.** (a) (i) $3x = 15$. Add 2 to each side. (ii) $x = 5$. Divide each side by 3. (b) (i) $2x = 10$. Add 3 to each side. (ii) $x = 5$. Divide each side by 2. (c) (i) $-2x = 10$. Subtract 4 from each side. (ii) $x = -5$. Divide each side by −2. (d) (i) $5x = 30$. Subtract 4 from each side. (ii) $x = 6$. Divide each side by 5. (e) (i) $-3x = 9$. Subtract 6 from each side. (ii) $x = -3$. Divide each side by −3. (f) (i) $2x = 0$. Add 12 to each side. (ii) $x = 0$. Divide each side by 2.
7. (a) 3 (b) 2 (c) 6 (d) 5 (e) $\frac{2}{5}$ (f) 23 (g) $\frac{1}{48}$ (h) $-\frac{1}{3}$ (i) 3 (j) 3 (k) 7 (l) 8 (m) 2 (n) 5 (o) 6 (p) 5 (q) $15\frac{1}{15}$ (r) 2 (s) 13 (t) 12 (u) 15 (v) 3 (w) 5 (x) 7 (y) 15 (z) 19 (aa) 10 (bb) 10 (cc) 25 (dd) 20 (ee) −12 (ff) 2 (gg) 2 (hh) $1\frac{1}{2}$

EXERCISE 5·4, Page 105

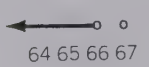
1. (a) 6 (b) 8 (c) 14 (d) 12 (e) 8 (f) 49 (g) 7 (h) −4 (i) −51 (j) 5
2. (a) 6 (b) 1 (c) 3 (d) 8 (e) 12 (f) 2 (g) −9 (h) −19 (i) 28 (j) −3
3. (a) 5 (b) 5 (c) 84 (d) 4 (e) 15 (f) 14 (g) 18 (h) 8 (i) 6 (j) 21 (k) −49 (l) 5
4. (a) 5 (b) −4 (c) 7 (d) 7 (e) 3 (f) $1\frac{7}{15}$ (g) 2 (h) −7 (i) 1 (j) −1 (k) $\frac{5}{2}$ (l) 3 (m) 4 (n) 2 **5.** (a) (i) Multiply each side by 9. (ii) Add 10 to each side. (iii) Subtract $2x$ from each side. (iv) $9x + 10 = 145$. Add $2x$ to each side. (v) $9x = 135$. Subtract 10 from each side. (vi) $x = 15$. Divide each side by 9. (b) (i) Multiply each side by −5. (ii) Add 9 to each side. (iii) Add $2x$ to each side. (iv) $9 - 5x = -11$. Subtract $2x$ from each side. (v) $-5x = -20$. Subtract 9 from each side. (vi) $x = 4$. Divide each side by −5. (c) (i) Multiply each side by −8. (ii) Add 14 to each side. (iii) Subtract $4m$ from each side. (vi) $14 - 8m = 18$. Add $4m$ to each side. (v) $-8m = 4$. Subtract 14 from each side. (vi) $m = -0.5$. Divide each side by −8. (d) (i) Multiply each side by −6. (ii) Add 13 to each side. (iii) Add $7x$ to each side. (iv) $13 - 6x = -17$. Subtract $7x$ from each side. (v) $-6x = -30$. Subtract 13 from each side. (vi) $x = 5$. Divide each side by −6. (e) (i) Multiply each side by −6. (ii) Add $7x$ to each side. (iii) Add 13 to each side. (iv) $x = 7x - 30$. Subtract 13 from each side. (v) $-6x = -30$. Subtract $7x$ from each side. (vi) $x = 5$. Divide each side by −6. (f) (i) Multiply each side by 6. (ii) Add 18 to each side. (iii) Subtract $2x$ from each side. (iv) $18 + 6x = 0$. Add $2x$ to each side. (v) $6x = -18$. Subtract 18 from each side. (vi) $x = -3$. Divide each side by 6. (g) (i) Multiply each side by 6. (ii) Subtract $2x$ from each side. (iii) Add 18 to each side. (iv) $4x = -2x - 18$. Subtract 18 from each side. (v) $6x = -18$. Add $2x$ to each side. (vi) $x = -3$. Divide each side by 6.
6. (a) 7 (b) 7 (c) 3 (d) 3 (e) 7 (f) 5 (g) 3 (h) 11 (i) 6 (j) 3 (k) $\frac{5}{8}$ (l) $13\frac{1}{3}$ (m) $-\frac{1}{4}$ (n) $-\frac{1}{10}$ (o) $-\frac{1}{20}$ (p) −20 (q) −1 (r) −2 (s) −11 (t) 12 (u) 4 (v) 14 (w) 25 (x) $6\frac{6}{7}$ **7.** (a) −9 (b) −4 (c) 11 (d) 11 (e) −7 (f) 0 (g) 16 (h) −4 (i) −14 (j) 2 (k) 2 (l) −3 **8.** (a) 8 N (b) 11 N (c) 6 N

EXERCISE 5·5, Page 107

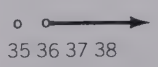
1. $x < 6$

2. $x > \frac{20}{3}$

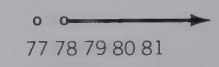
3. $x < 66$



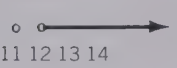
4. $x > 36$



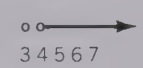
5. $x > 78$



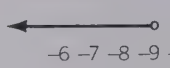
6. $x > 12$



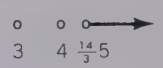
7. $x > 4$



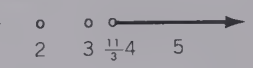
8. $x < -9$



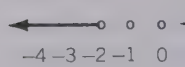
9. $x > \frac{14}{3}$



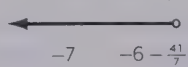
10. $x > \frac{11}{3}$



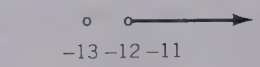
11. $x < -2$



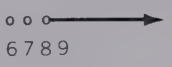
12. $x < -\frac{41}{7}$



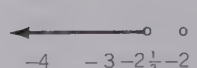
13. $x > -12$



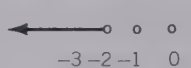
14. $x > 8$



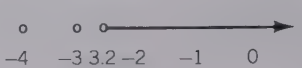
15. $x < -2\frac{1}{2}$



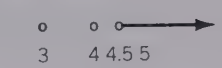
16. $x < -2$



17. $x > -3.2$



18. $x > 4.5$



EXERCISE 5·6, Pages 109-110

1. (a) 1 (b) 1 (c) 1 (d) 1 (e) 1 (f) 1 (g) 1 (h) 1 (i) 1
2. (a) 1 (b) 1 (c) 1 (d) 1 (e) 1 (f) 1 (g) 1 (h) 1 (i) 1
4. (a) 2 (b) $\frac{1}{5}$ (c) $\frac{7}{6}$ (d) $\frac{9}{8}$ (e) $\frac{1}{12}$ (f) 9 (g) $\frac{1}{3}$ (h) $\frac{1}{3}$ (i) $\frac{1}{7}$
(j) $\frac{1}{17}$ (k) 15 (l) $-\frac{1}{5}$ (m) $-\frac{3}{10}$ (n) -4 (o) $-\frac{1}{7}$ **6.** 26 **7.** 80 **8.** 84
9. 32 **10.** 66 **11.** 63 **12.** 39 **13.** 5 **14.** 9 **15.** 25 **16.** 36 **17.** -2 **18.** -2
19. -64 **20.** 25 **21.** -45 **22.** -100 **23.** 21 **24.** 70 **25.** 60 **26.** 36 **27.** 21
28. 63 **29.** 15 **30.** 33 **31.** 65 **32.** 8 **33.** 6 **34.** 20 **35.** -28 **36.** -42
37. 10 **38.** 60 **39.** -48

EXERCISE 5·7, Pages 112-113

1. Width 9 cm, length 27 cm **2.** Width 7.2 cm, length 28.8 cm **3.** Width 7 cm, length 14 cm
4. Width 12 cm, length 16 cm **5.** Mary is 18 and Harry is 6. **6.** 50 cm and 100 cm
7. 15 cm and 25 cm **8.** Jean walked 4 km and Mary walked 2 km. **9.** Harry skied 4 km and Jim 5 km.
10. Ivor is 18 and his father is 45. **11.** $\frac{y}{1.2}$ **12.** Joan is 7 and Alice is 14. **13.** 132 and 216
14. Adele is 12 and Marie is 16. **15.** \$193 **16.** Jane's share was \$277.14 and Kathy's \$158.36
17. \$8 **18.** Width 24 m, length 32 m **19.** (a) 40°, 60°, 80° (b) 40°, 60°, 80° (c) 37.5°, 60°, 82.5°
20. 60 cm and 140 cm
21. The loser gets \$25.50 and the winner \$42.50.
22. The lot cost \$5100 and the house \$28 560. **23.** 60 cm, 90 cm, and 120 cm
24. Lewis is 13, Jane is 17, and Adele is 10.

EXERCISE 5·8, Page 116

1. \$46 **2.** \$20.25, \$170.25 **3.** 3 yr **4.** \$402.50 **5.** \$512.50 **6.** \$86.40 **7.** \$24 000
8. \$1428 **9.** \$11 200

EXERCISE 5·9, Page 118

2. (b), (d), and (e) are correct. **3.** (a) 21 (b) $7\frac{1}{2}$ (c) -4 (d) $\frac{6}{5}$
(e) -25 (f) 8 (g) $5\frac{1}{11}$ (h) 6 (i) 80 **4.** 48 and 56 **5.** 81 and 117 **6.** 143.5 cm
7. 66 sunfish **8.** \$19.50 and \$45.50 **9.** 1350 m and 2250 m **10.** \$5.80 **11.** 50 min
12. 1.5 ℓ of yellow and 3.5 ℓ of brown paint. **13.** 279.4 ℓ **14.** \$10.56
15. 87 480 bricks **16.** \$18.00 to Jim and \$31.50 to Helen. **17.** 36.8 cm

EXERCISE 5·10, Page 120

1. $x + 2$ boys **2.** $y - 8$ **3.** 300 cm, $100x$ cm **4.** 1270 g, $1000x$ g
5. $25k$ cents, $10y$ cents **6.** $y - 6$ **7.** $32 - x$ **8.** $w - 6$ kg **9.** $w - 5z$ kg
10. $\frac{k}{4}$ cm **11.** 19, 11, $n + 1$, $n - 1$ **12.** $k + 1$, $k + 2$ **13.** $m - 1$, $m - 2$
14. $r + 2$ **15.** x^2 **16.** $3y$ m **17.** $x + 5$, $x - 3$, $x + y$, $x - m$ **18.** $\$(d + 20\%d)$
19. $4x + 5$ **20.** $15y$ km, $10y$ km **21.** (a) $xy - pq$ (b) $pq - xy$
22. $(12m + 1400)$ kg **23.** $(12h + 52)$ m **24.** $3m + 5$ **25.** $21x$, $14m - 35$
26. $\frac{1}{4}n$ **27.** $5k$ nickels, $20d$ nickels **28.** $\$(b + 10\%b)$

REVIEW EXERCISE 5, Page 122

1. (a) 7 (b) -2 (c) 12 (d) $x > -4\frac{1}{2}$ **2.** Jean has 50¢ and Sue had \$2.50.
3. (a) 18 (b) 4 (c) 6 (d) 10 **4.** The Bloors took $7\frac{1}{2}$ t and the Trapps $10\frac{1}{2}$ t.
5. \$818.85 **6.** 386 **7.** (a) $31\frac{1}{4}$ (b) -102 (c) 14 **8.** 8, 24, 40 **9.** (a) 12 m (b) 9.25 cm

CHAPTER 6

EXERCISE 6·1, Page 126

1. \$47 **2.** \$121 **3.** \$324 **4.** \$425 **5.** \$450 **6.** \$2050

EXERCISE 6·2, Pages 129-130

1. $\frac{x}{5} = \frac{1}{5} \cdot x$ **2.** Literal coefficients: (a) x (b) x and y (c) x, y, and z (d) x (e) m^3 and p
(f) x^2, y, and z^2. Numerical coefficients: (a) 73 (b) 56 (c) 3 (d) 2×6 (e) 76 (f) 8
3. The following are monomials: (a), (d), (e), and (f). **5.** (a) 2 (b) 0 (c) 3 (d) 5 (e) 6 (f) 0
(g) 3 (h) 5 (i) 104 **6.** (a) x^5 (b) r^5 (c) m^{10} (d) y^{12} (e) a^{19} (f) z^{15} **7.** (a) $75x^4$ (b) $28m^5$

(c) $15x^4y^1$ (d) $102m^6n^4$ (e) $96r^7s$ (f) $40v^9w^9$ **8.** (a) $12x^5$ (b) a^3b^3 (c) $15m^8n^5$ (d) $-3a^5c^2$
(e) $x^3y^3z^3$ (f) $8b^3c^3$ (g) $-12a^3b^5c^5$ (h) $m^9n^9p^3$ **9** (a) $6m^4n^2$ (b) $8m^5$ (c) $7r^3t^3$ (d) $36v^3w^4$
(e) $15x^3y^3z$ (f) $27z^5w^7v^3q$ **10.** (a) b^3 (b) h^2 (c) n^4 (d) p^8 (e) r^{15} (f) x^{16}

11. (a) $2x$ (b) x^3yz (c) $2m$ (d) $\frac{2m^3}{r^2}$ **12.** (a) m (b) $\frac{m}{4}$ (c) $\frac{19r}{st}$ (d) $4\frac{16}{17}tr$ (e) $\frac{1}{10}\frac{v^2w}{z}$ (f) $2\frac{22}{25}yz$

13. (a) $2at^2$ (b) $-3mn^2$ (c) $4x^2y^5$ (d) $-7y^6z^3$ (e) $8ab-3a^2b^2$ (f) $-3x^4y^2z^2$ (g) $4m^2-5n^4$
(h) $6xy^2-7x^2$ (i) $3rt^3+4$ (j) $4x^4+5y^2$ **14.** (a) -12 (b) 144 (c) -6 (d) -10 (e) -48 (f) $-\frac{1}{4}$

EXERCISE 6·3, Pages 132-133

1. They are unlike terms because the degrees of the variables are different.
2. (c) $4x+4y$ (d) $15x-15y$ (e) $14m-21n$ (f) $6r+12s$ (g) $9x-7y$ (h) $12a-16b$
(i) $4r-6t$ (j) $\frac{3}{8}x-\frac{1}{4}z$ (k) $2.55x-3.91y$ (l) $5p^2-5s^2$ (m) $7w-7v^2$ (n) $25x^2-15$
(o) $\frac{5}{9}a-\frac{5}{8}b$ (p) $5x-2.6y$ **3.** (a) m (b) $11r^2$ (c) $16rs$ (d) $7x^2$ (e) $7z^3$
(f) $\frac{2}{3}x$ (g) $\frac{3}{16}t$ (h) $\frac{1}{5}x$ (i) $\frac{13}{30}y^2$ (j) $5t$ (k) $17t^2$ (l) $-62tp$
(m) $-13x^2$ (n) $3m$ (o) -22π (p) $8y$ **4.** (a) $9m$ (b) $2t^2$ (c) $4xy$ (d) $-15a^2b$
(e) $\frac{11}{10}mnq$ (f) $2xy^2z$ (g) $\frac{3}{4}a^2b^3c$ (h) $-\frac{7}{8}t^4$ (i) $10.5a$ (j) $8.65b^3$
(k) $10.83r^2t$ (l) $0.824x^5y^3$ **5.** (a) Binomial (b) Polynomial (4 terms) (c) Monomial
(d) Monomial (e) Trinomial (f) Binomial (g) Polynomial (4 terms) (h) Monomial (i) Monomial
(j) Trinomial (k) Polynomial (4 terms) (l) Trinomial **6.** (a) 1 (b) 1 (c) 2 (d) 2 (e) 3 (f) 2
(g) 2 (h) 1 (i) 3 (j) 2 (k) 3 (l) 3 (m) 1 (n) 4 **7.** (a) $5x+3$ (b) $2m+8n$ (c) -3
(d) $-ab+7c$ (e) $2x^2-2y^2$ **8.** (a) $4a-7b-3c$ (b) $8x-y$ (c) $5a^2b^2-6a-3b$
(d) $7st^2-u^3v+6wx^2$ **9.** (a) $5x-6y-5z$ (b) $8ab+3ac$ (c) $3x^2y+2xy-6xy^2$ (d) $5a^2+6a-4$
10. (a) $6a-4b-7c$ (b) $3xy-4yz+11xz$ (c) $10m^2-4mn+2n^2$ (d) $ab-bc-ac$ (e) $2b+9c-2d$

EXERCISE 6·4, Pages 135-136

1. (a) $6a$ (b) $16y$ (c) $-2c$ (d) $25m$ (e) $-10z$ (f) $-7h$ (g) $5x$ (h) $-20ab$ (i) $6b^2$
(j) $-3xyz$ **2.**(a) $4-3a+4b$ (b) $-a-4b+2c$ (c) $7a+2b-4c+3d$ (d) $3x-4a+5c$
(e) $7x+4y+7a+7b-8c$ (f) $7a-6b+9m$ (g) $+9m$ (h) $-9v+13w^2$ (i) $18a-3b-6x^2-8y^2$
3. (a) $10x+7$ (b) $18m-8n+5p$ (c) $4a+7b$ (d) $6y-5z$ (e) $-8x^2+9xy$
(f) $-4ab+3bc-6ac$ **4.** (a) $2x-4y+4z$ (b) $7ab+13bc+3$ (c) $2x^2y^2+9xy-7xy^2$
(d) $-17pq-5qr-6rt$ (e) $-2x-9y+5z$ (f) $1.5m-6n+4.2p$ (g) $-14y-8z$
(h) $\frac{1}{3}m^2-\frac{17}{12}n-\frac{7}{12}p$ (i) $8.8x-14y-7.5$ (j) $1\frac{2}{3}t-2\frac{1}{4}s+\frac{7}{12}y$
(k) $-\frac{1}{12}a-\frac{13}{12}b+\frac{1}{12}c$ **5.** (a) $6a+4b$ (b) $12m+2n$ (c) $15r+4q-8s$
(d) $12m+8r-4s$ (e) $15m-2r$ (f) $22r-14t$ (g) $34t+13s$
(h) $15a+19b$ **6.** (a) $6a+3b$ (b) $4p^2q^2$ (c) $7r^2$ (d) $4a+2b$ (e) $5a+7b$
(f) $4a$ (g) $2m+8n$ (h) $8m-7n$ (i) $2t-r$ (j) $3q+2r-t$ (k) $2m+n$ (l) $m-n$

EXERCISE 6·5, Page 138

1. (a) $tr+ts$ (b) $ab+a^2$ (c) a^2+3a (d) $rs+2r^2$ (e) $ab+ac+a^2$ (f) $6mn+12mr$
(g) $12ba+8bc$ (h) $6m^2$ (i) $34s^2+51sr$ (j) $4a^2+2ab$
2. (a) length $r+s$, width t, area $tr+ts$ (b) length $b+a$, width a, area $ab+a^2$
(c) length $a+3$, width a, area a^2+3a (d) length $s+2r$, width r, area $rs+2r^2$
(e) length $b+c+a$, width a, area $ab+ac+a^2$
(f) length $2n+4r$, width $3m$, area $6mn+12mr$ (g) length $3a+2c$, width $4b$, area $12ba+8bc$
(h) length $3m$, width $2m$, area $6m^2$ (i) length $2s+3r$, width $17s$, area $34s^2+51sr$
(j) length $2a+b$, width $2a$, area $4a^2+4ab$ **3.** (a)$ab+ac$ (b) $bd+d^2$
(c) $xy+3x$ (d) $4ab+2ac$ (e) $6am+6an$ (f) $6x^2+9xy$ **4.** (a) 63 (b) 248 (c) 448 (d) 266
(e) 414 (f) 1880 (g) 2211 (h) 2688 (i) 7440 (j) 926 (k) 1890 (l) 55 926

EXERCISE 6·6, Page 139

1. $14a-7b+7c$ **2.** $15x+36y$ **3.** $-6mx+8my-2mz$ **4.** $18a^2-24ab-12ac$
5. $4b^3-8ab^2-20b$ **6.** $6p+10p^2-14p^3$ **7.** $-7x^3+21x^2-14x$ **8.** $m^3n+2m^2n^2+3mn^3$
9. $6a^4-3a^3+3a^2$ **10.** $k^8+5k^6-7k^4$ **11.** $2x^4y+2xy^3$ **12.** $-3a^3b^2c-3ab^3c^2$
13. $2a^3+2ab^2+2ac^2+4a^2b+4a^2c+4abc$ **14.** $5m+4n+3p$ **15.** $6y^2+14y$
16. $7x^3+13x$ **17.** $3a^2b-ab^2$ **18.** $11m^3n$ **19.** $4xy^2z+4xyz^2$ **20.** a^3

EXERCISE 6·7, Pages 142-143

1. (a) $(m+n)(m+n)=m^2+2mn+n^2$ (b) $(x+y)(x+y)=x^2+2xy+y^2$
(c) $(2s+t)(s+t)=2s^2+3st+t^2$ (d) $(a+b+c)(a+b+c)=a^2+2ab+2ac+2bc+b^2+c^2$
(e) $(2x+y)(2x+y)=4x^2+4xy+y^2$ (f) $(2a+b)(a+2b)=2a^2+5ab+2b^2$
2. (a) $a^2+2ab+b^2$ (b) $2a^2+5ab+2b^2$ (c) $2x^2+5xy+2y^2$
(d) $9m^2+12mn+4n^2$ (e) $m^2+2mn+2mt+2nt+n^2+t^2$ (f) $4s^2+8st+4t^2$

3. (a) $a^2 + 3a + 2$ (b) $x^2 - 3x + 2$ (c) $r^2 + r - 12$ (d) $b^2 + 2b - 3$ (e) $m^2 + 4m + 4$ (f) $d^2 + 5d - 36$ (g) $y^2 + 8y + 16$
(h) $z^2 - 6z + 9$ (i) $z^2 - 81$ (j) $x^2 + 9x + 18$ (k) $h^2 - 5h - 14$ (l) $m^2 - 8m + 16$ (m) $m^2 + 14m + 45$
(n) $k^2 - 9k + 20$ (o) $m^2 - 24m + 144$ (p) $x^2 + 3x - 130$ (q) $y^2 - 3y - 40$ (r) $y^2 + 50y + 625$
4. (a) $2a^2 + 3a - 2$ (b) $2x^2 - 8x + 8$ (c) $15x^2 + 4xy - 32y^2$ (d) $2m^2 - 11m - 21$ (e) $4y^2 + 20y + 25$
(f) $36y^2 - 24yz + 4z^2$ (g) $9z^2 - 24z + 16$ (h) $25a^2 - 20ab + 4b^2$ (i) $9m^2 - 24mn + 16n^2$
(j) $6b^2 - 17b + 5$ (k) $36m^2 - 4n^2$ (l) $20k^2 + 3k - 2$ (m) $2x^2 - 9xy + 9y^2$ (n) $81z^2 + 54yz - 16y^2$
(o) $4a^2 + 27ab - 7b^2$ (p) $21h^2 - hk - 2k^2$ (q) $121m^2 - 144n^2$ (r) $6h^2 + 5h - 21$ **5.** (a) 312
(b) 4212 (c) 2208 (d) 2146 (e) 3230 (f) 4582 **6.** (a) $2a^2 + 10a + 14$ (b) $-5x - 13$
(c) $-k^2 + 15k - 69$ (d) $-4m^2 - 33m + 108$ (e) $-d^2 - 15d + 48$ **7.** (a) $4a^2 + 12ab + 9b^2$
(b) $16m^2 - 40mn + 25n^2$ (c) $ac + ad + bc + bd$ (d) $xn + xm + yn + ym$ (e) $12a^2 - 23an + 10n^2$
(f) $18m^2 + 18mb - 8b^2$ (g) $12a^2 + 18ax - 12x^2$ (h) $30b^2 - 32by + 8y^2$ (i) $20y^2 + 9yd - 18d^2$
(j) $12y^2 + 3yd - 42d^2$ **8.** (a) $12a^2 + 7ab - 10b^2$ (b) $42m^2 + mn - n^2$ (c) $9a^2 - 4b^2$ (d) $64m^2 - 9y^2$
(e) $144r^2 - 36y^2$ (f) $81x^2 - 16p^2$ (g) $144g^2 - 9h^2$ (h) $a^2 - b^2$ **9.** (a) $2a^2 - ab + 4ac - 3b^2 + 4bc$
(b) $3a^2 + 5ab + 4ac + 2b^2 + 3bc + c^2$ **10.** (a) $3x + 4x = 7x$ (b) $3 \times 4 = 12$ **11.** (a) $-5x + (-6x) = -11x$
(b) $(-5) \times (-6) = 30$ **12.** (a) $4x + (-7x) = -3x$ (b) $(-7) \times (4) = -28$ **13.** (a) $(-11x) + (3x) = -8x$
(b) $(3) \times (-11) = -33$ **14.** (a) $(7x) + (-5x) = 2x$ (b) $(-5) \times (7) = -35$

EXERCISE 6·8, Page 146

1. $b^2 - 1$ **2.** $1 - k^2$ **3.** $x^2 - y^2$ **4.** $a^2 - b^2$ **5.** $m^2 - n^2$ **6.** $4x^2 - 1$ **7.** $y^2 - 4$
8. $a^2 - 9$ **9.** $z^2 - 64$ **10.** $k^2 - 4t^2$ **11.** $a^2 - 25b^2$ **12.** $16s^2 - t^2$ **13.** $49h^2 - k^2$
14. $100x^2 - y^2$ **15.** $9a^2 - 4b^2$ **16.** $16m^2 - 25n^2$ **17.** $9y^2 - 36z^2$ **18.** $36p^2 - 16q^2$
19. $81x^2 - 49y^2$ **20.** $400a^2 - 2500b^2$ **21.** 9999 **22.** 2499 **23.** 4896 **24.** 8091
25. 39 996 **26.** 159 991 **27.** 999 984 **28.** 639 975 **29.** 489 991 **30.** 3 999 991

EXERCISE 6·9, Page 148

1. (a) length $m + n$, width m (b) length $r + s$, width r (c) length $r + s$, width t
(d) length $2a + b$, width a (e) length $2r + 2s$, width r (f) length $x + y$, width x
(g) length $x + y$, width 5 (h) length $a + b$, width 3 (i) length $m + n$, width 7
(j) length $x + y$, width m **2.** (a) 2 (b) 6 (c) 7 (d) 2 (e) 3 (f) 5 (g) 2 (h) 3 (i) 4 (j) a
(k) y (l) n (m) $2x$ (n) $3h$ (o) xy **3.** (a) $3x$ (b) $4b$ (c) $6m$ (d) $5y$ (e) x^2 (f) $3a$ (g) $6m$
(h) $7b$ (i) $8xy$ (j) $4st$ (k) $6h^2k$ (l) $13a^2b^2$ **4.** (a) $3(h - 1)$ (b) $4(x - 2)$ (c) $3(2a - 1)$
(d) $2(m + 3)$ (e) $5(k + 2)$ (f) $2(3t + 4)$ (g) $3(3b - 2)$ (h) $4(2z + 3)$ (i) $3x(y + 1)$ (j) $a(b - 2)$
(k) $n(4 - 3m)$ (l) $4y(x + 2)$ (m) $3h(1 + 3k)$ (n) $ab(7 - 3c)$ (o) $x(x - 1)$ (p) $3m(4m + 3)$
(q) $5y(2y^2 - 3a)$ (r) $bx(x + b)$ (s) $3x^2(1 - 4)$ (t) $2mn(2m + 3n)$ (u) $3ab(6bc - 7)$ (v) $13tx(t + 2x)$
5. For **2.** (a) $2(b + 1)$ (b) $6(1 - m)$ (c) $7(b + 1)$ (d) $2(2x - 1)$ (e) $3(1 + 2k)$ (f) $5(3x - 1)$
(g) $2(3y - 4)$ (h) $3(3z - 2)$ (i) $4(2b + 3)$ (j) $a(3 - b)$ (k) $y(x + 2)$ (l) $n(4 - 3m)$ (m) $2x(1 + 2y)$
(n) $3h(1 - 2k)$ (o) $xy(5 - 3z)$ For **3.** (a) $3x(2x - 3)$ (b) $4b(a + 2)$ (c) $6m(2m + 1)$ (d) $5y(3 - 2x)$
(e) $x^2(x - 1)$ (f) $3a(a - 2b)$ (g) $6m(3m + 2n)$ (h) $7b(2b^2 + 3a)$ (i) $8xy(2z - 3x)$ (j) $4st(7t - 2s)$
(k) $6h^2k(3k + 5h)$ (l) $13a^2b^2(2a + 3b)$

EXERCISE 6·10, Page 150

1. (a) length $m + n$, width $m + n$ (b) length $m + 2n$, width $m + n$
(c) length $a + b$, width $a + b$ (d) length $a + 2b$, width $a + b$
(e) length $x + 2y$, width $x + y$ (f) length $m + 3b$, width $m + b$
2. (a) (6,1) (–6,–1) (2,3) (–2,–3) (b) (8,1) (–8,–1) (2,4) (–2,–4) (c) (9,1) (–9,–1) (3,3) (–3,–3)
(d) (12,1) (–12,–1) (6,2) (–6,–2) (4,3) (–4,–3) (e) (15,–1) (–15,1) (5,–3) (–5,3)
(f) (21,–1) (–21,1) (7,–3) (–7,3) (g) (32,1) (–32,–1) (16,2) (–16,–2) (8,4) (–8,–4)
(h) (20,1) (–20,–1) (10,2) (–10,–2) (5,4) (–5,–4) (i) (12,–1) (–12,1) (4,–3) (–4,3)
(j) (36,1) (–36,–1) (18,2) (–18,–2) (12,3) (–12,–3) (9,4) (–9,–4) (6,6) (–6,–6) (k) (11,–1) (–11,1)
(l) (72,–1) (–72,1) (36,–2) (–36,2) (24,–3) (–24,3) (18,–4) (–18,4) (12,–6) (–12,6) (9,–8) (–9,8)
3. {4,3}, {8,5}, {11,1}, {–11,1}, {–3,2}, {3,–2}, {–6,1}, {6,–1}, {–7,6}, {7,–6},
{–21,2}, {21,–2}, {–14,3}, {14,–3} **5.** $(m + 3)(m + 3)$
6. $(h - 3)(h - 2)$ **7.** $(s + 2)(s + 2)$ **8.** $(x - 7)(x - 7)$ **9.** $(z + 8)(z + 8)$
10. $(a - 1)(a - 1)$ **11.** $(x - 7)(x + 3)$ **12.** $(b - 5)(b + 4)$ **13.** $(m + 5)(m + 2)$
14. $(x - 4)(x - 3)$ **15.** $(y + 6)(y + 4)$ **16.** $(x + 5)(x + 3)$ **17.** $(z - 5)(z + 6)$
18. $(a - 20)(a + 2)$ **19.** $(c - 8)(c + 4)$ **20.** $(k - 4)(k + 10)$ **21.** $(b + 7)(b + 5)$
22. $(x + 6)(x + 6)$ **23.** $(a - 10)(a - 10)$ **24.** $(h - 11)(h - 1)$ **25.** $(c - 18)(c + 3)$
26. $(y - 14)(y + 5)$ **27.** $(m - 4)(m + 18)$ **28.** $(t - 3)(t + 1)$ **29.** $(x - 9)(x + 8)$
30. $(h - 6)(h + 7)$ **31.** $(a - 9)(a + 10)$ **32.** $(c - 9)(c + 6)$

EXERCISE 6·11, Page 151

1. $(x - 2)(x + 2)$ **2.** $(m - 3)(m + 3)$ **3.** $(h - 4)(h + 4)$ **5.** $(m - n)(m + n)$
6. $(x - 7)(x + 7)$ **7.** $(1 - h)(1 + h)$ **8.** $(5 - b)(5 + b)$ **9.** $(z - 7)(z + 7)$
10. $(k - 10)(k + 10)$ **12.** $(p - 9)(p + 9)$ **13.** $(5a - 7b)(5a + 7b)$ **14.** $(4r - 3s)(4r + 3s)$

15. $(2k-h)(2k+h)$ **16.** $(6x-10y)(6x+10y)$ **17.** $(8x-9y)(8x+9y)$
18. $(12a-11b)(12a+11b)$ **19.** $(m-15n)(m+15n)$ **20.** $(20b-50a)(20b+50a)$ **21.** $(9z-7y)(9z+7y)$

REVIEW EXERCISE 6, Page 153

1. $-2x$ **2.** $2b^2-3b$ **3.** $4h^2-5h+7$ **4.** $3x^2+xy-y^2$ **5.** $5a+3b$
6. $3m+9mn-10n$ **7.** $2t^2+5t-5$ **8.** a^8 **9.** b^3 **10.** $-m^3n^8$ **11.** $6b^2x^5$
12. $5xa+4xb$ **13.** $10mp+7mq-23mr$ **14.** $3x^2+x-10$ **15.** $12a^2+36ab-21b^2$
16. $16m^2-24m+9$ **17.** $9x^2+42xy+49y^2$ **18.** $7a^2-30a+11$ **19.** $16t^2-81$
20. $25x^2-9y^2$ **21.** $3m(m-4)$ **22.** $3x^2y^3(5xy^2+2)$ **23.** $4xy(4x-3y)$
24. $(a-6)(a+1)$ **25.** $(x+7)(x+4)$ **26.** $(x-13)(x-3)$ **27.** $(2-t)(2+t)$
28. $(h-6)(h+6)$ **29.** $(9-2s)(9+2s)$

CHAPTER 7

EXERCISE 7·2, Pages 158-159

1. The location of the points is: $A(4,10)$, $B(7,16)$, $C(13,13)$, $D(8,10)$, $E(15,9)$, $F(13,5)$, $G(8,3)$, $H(1,1)$, $I(1,8)$.
3. The ordered pairs of the midpoints are: (a) (3,5) (b) (5,3) (c) (8,9) (d) (8,3) (e) (7,3) (f) (12,7)
4. The points are the following divisions apart: (a) 11 (b) 12 (c) 11 (d) 11 (e) 9 (f) 7
5. (12,9) **6.** (3,13)

EXERCISE 7·3, Page 161

2. First quadrant: (56,39) and (25,32). Second quadrant: (–6,41) and (–75,13). Third quadrant: (–81,–3), (–2,–100), and (–88,–8). Fourth quadrant: (15,–12), (64,–12), (85,–7), and (28,–4).

EXERCISE 7·4, Page 164

5. The graph line slants from the 3rd quadrant upwards into the 1st quadrant when a is positive. The graph line slants downward from the 2nd quadrant into the 4th quadrant when a is negative.

EXERCISE 7·5, Page 165

10. The value of the constant determines whether the graph line will cross the y-axis above or below the origin.

EXERCISE 7·6, Page 168

1. (b) \$500 for 155 guests, \$215 for 55 guests, \$135 for 30 guests, \$400 for 120 guests, \$600 for 200 guests **2.** (a) $53\frac{1}{3}$ km in 40 min, 120 km in $1\frac{1}{2}$ h, $186\frac{2}{3}$ km in 2 h 20 min (b) 15 min for 20 km, 97.5 min for 110 km, 2 h for 160 km **3.** (a) 324 in 54 min, 540 in 90 min, 864 in 144 min, 1620 in $4\frac{1}{2}$ h (b) 11 min for 66, 20 min for 120, 27 min for 162 **4.** (a) 33 in 54 min, 55 in 90 min, 88 in 144 min, 165 in $4\frac{1}{2}$ h (b) 108 min for 66, 198 min for 121, $4\frac{1}{2}$ h for 165 **5.** $C = 25¢\ n\{7 \text{ sheets}\}$
6. (a) 30 min (b) 21 ℓ in 14 min, 33 ℓ in 22 min

EXERCISE 7·7, Pages 171-172

2. (a) When a is positive, the values for y are positive and the curve appears in the 1st and 2nd quadrants. When a is negative, the values for y are negative and the curve appears in the 3rd and 4th quadrants. (b) When the value of a increases, the values of y increase. When a is greater than zero, the values of y are positive. When a is less than zero, the values of y are negative. **3.** (b) In these graphs, the values of y decrease as the values of x increase. The rate of decrease in the y values depends upon the value of the numerator. **4.** (b) The increase in y is the greatest between $x=0$ and $x=1$. There is no increase in the value of y for the interval between $x=4$ and $x=5$. (c) The greatest decrease for the value of y occurs for the interval between $x=8$ and $x=9$.

REVIEW EXERCISE 7, Page 172

2. The midpoint is at (0,1). **3.** The two segments intersect at (4,–1). **4.** The common point is (3,–2). **5.** (0,0)

CHAPTER 8

EXERCISE 8·1, Pages 178-179

3. 82 **4.** 3 **5.** (a) a square (b) 2×2 (c) 4 **8.** 5 **9.** (a) 8 (b) $8+8\sqrt{2}$

EXERCISE 8·2, Pages 181-182

5. The size and shape of an object are unaffected by translation. **6** The figures remain unaffected by the translations. The shapes remain *congruent*. **7.** Two translations can be represented by a single translation.

EXERCISE 8·3, Pages 186-187

2. A, H, I, M, O, T, V, U, W, X, and Y appear the same when the mirror is on the right of the object. B, C, D, E, H, I, K, O, and X appear the same when the mirror is above the object. **3.** A, F, G, J, L, M, N, P, Q, R, S, T, U, V, W, Y **4.** B, C, D, E, F, G, J, K, L, N, P, Q, R, S, Z **5.** (a) M, W (b) None

EXERCISE 8·4, Pages 189-191

1. To *bisect* means to divide into two congruent parts. **2.** (a) The angle is a right angle. (b) Two lines are perpendicular if their intersection forms right angles. Line DE is perpendicular to line AB. **7.** The measure of each new angle is 45°. The newest small angles measure 27.5°. **8.** Each angle of an equilateral triangle measures 60°. Each new angle measures 30°. Six congruent triangles have been formed.
12. $\ell_1 \| \ell_4, \ell_2 \| \ell_6, \ell_3 \| \ell_7$ **13.** All five lines are perpendicular to each other.

EXERCISE 8·6, Pages 196-199

1. (a) (2,1) (b) (3,6) (c) (−3,−2) (d) (5,−2) **2.** (a) (2,0) (b) (−2,0) (c) (0,−2) **3.** (a) (1,2) (b) (3,−1) (c) (2,3) (d) (4,2) **4.** (a) $A(2,-5)$ $B(3,-8)$ $C(7,-1)$ (b) $A(-2,5)$ $B(-3,8)$ $C(-7,1)$ (c) $A(-5,-2)$ $B(-8,-3)$ $C(-1,-7)$ **5.** $K(4,6)$ $M(6,0)$ $N(9,2)$ **6.** $X(1,3)$ $Y(5,0)$ $Z(-2,-5)$ **7.** $G(-9,4)$ $H(-5,1)$ **8.** $Q(-4,6)$ $R(-8,4)$ $S(-3,2)$ **9.** $D(4,-6)$ $E(7,0)$ **12.** The treasure is located at (−12,13)

REVIEW EXERCISE 8, Page 201

1. (a) (−1,−1) (2,−2) (6,−1) (b) (−4,0) (−4,3) (−6,3) (c) (3,1) (4,5) (7,3)
6. (a) The shape of the object and image is the same. (b) The size of the object and image remains the same. (c) The position of the object and image is reversed in reflection, unchanged in translation, and turned upside down in rotation.
7. (a) $m(\overline{AG}) = m(\overline{GB})$ (b) $m(\overline{DK}) = m(\overline{KC})$ (c) $m\angle AGH = m\angle BGH = 90°$

CHAPTER 9

EXERCISE 9·2, Page 208

1. (a) $m\angle ABE = m\angle DBC$, $m\angle ABC = m\angle EBC$ (b) $m\angle PRM = m\angle MRS$, $m\angle SRT = m\angle TRP$
(c) $m\angle STV = m\angle XTW$, $m\angle XTS = m\angle VTW$ (i) Vertically opposite angles have equal measures.
(ii) 360° **2.** (a) $m\angle p + m\angle r = 180°$ (b) $m\angle s + m\angle t = 180°$ (c) $m\angle v + m\angle w = 180°$

EXERCISE 9·5, Pages 219-220

1. (a) $\frac{m(\overline{PA'})}{m(\overline{PA})} = \frac{m(\overline{PB'})}{m(\overline{PB})} = \frac{m(\overline{B'C'})}{m(\overline{BC})}$ (b) (i) $m\angle A = m\angle A'$ (ii) $m\angle B = m\angle B'$ (iii) $m\angle C = m\angle C'$
3. (a) $\frac{m(\overline{M'N'})}{m(\overline{MN})} = \frac{m(\overline{N'R'})}{m(\overline{NR})} = \frac{m(\overline{M'R'})}{m(\overline{MR})}$ (b) (i) $m\angle N = m\angle N'$ (ii) $m\angle M = m\angle M'$ (iii) $m\angle R = m\angle R'$
7. (a) Yes (c) Yes **8.** (a) The measures of corresponding angles are equal.
(b) The ratios of the measures of the corresponding sides are equal.
10. (a) The point P has been chosen to the right of the triangle. (b) The corresponding sides are: TM and $T'M'$, TR and $T'R'$, and RM and $R'M'$. (c) The enlarged triangle is inverted.
(d) The ratios of the measures of the corresponding sides are equal.
(e) The measures of corresponding angles are equal.

EXERCISE 9·6, Pages 222-224

1. (b), (c), (d), and (g) are similar triangles. **2.** (a) 20 (b) 15 and $17\frac{1}{2}$ (c) 25 and $22\frac{1}{2}$
(d) $30\frac{1}{3}$ and 28 **3.** 54 m **4.** 60 m **5.** 125 m **6.** 714.3 m **7.** 56 m **8.** 19.9 m

EXERCISE 9·7, Pages 226-227

1. (a) $m\angle CRY = m\angle TEA$, $m\angle RCY = m\angle ETA$, $m\angle CYR = m\angle TAE$
(b) $m(\overline{CR}) = m(\overline{TE})$, $m(\overline{RY}) = m(\overline{EA})$, $m(\overline{CY}) = m(\overline{TA})$
2. $\triangle CAB \equiv \triangle FDE$, $\triangle TFA \equiv \triangle YPR$ **10.** $m\angle A = 58°$, $m\angle B = 49°$, $m\angle C = 75°$, 180°
11. $m(\overline{GK}) = 11.7$ cm, $m\angle G = 42°$, $m\angle K = 53°$ **12.** $m\angle S = 90°$, $m\angle T = 44.5°$, $m(\overline{ST}) = 8.5$ cm, 180°
13. $m\angle P = 51°$, $m(\overline{PQ}) = 11.2$ cm, $m(\overline{PR}) = 12.7$ cm, 180° **14.** $m(\overline{DE}) = 11.5$ cm, $m(\overline{EF}) = 9.6$ cm, 180°
16. The pairs of congruent triangles are: (a), (b), (d), (e), (f), and (g).

EXERCISE 9·8, Pages 230-232

1. $m\angle P = 118°$, $m\angle R = 72°$ **2.** (a) $m\angle A = 98°$, $m\angle D = 88°$ (b) $m\angle E = 105°$, $m\angle H = 130°$
(c) $m\angle K = 92°$, $m\angle N = 70°$ **3.** All the angles measure 90°. **4.** (a) $m(\overline{DE}) = m(\overline{EF})$
(b) $m\angle ACP = m\angle BCP$ **5.** (a) $m\angle x = 145°$, $m\angle y = 85°$ (b) $m\angle x = m\angle y = 90°$
(c) $m\angle x = m\angle y = 90°$ (d) $m\angle x = m\angle y = 90°$ (e) $m\angle x = 90°$ $m\angle y = 93°$ (f) $m\angle x = 75°$, $m\angle y = 99°$
(g) $m\angle x = 90°$ (h) $m\angle x = 55°$, $m\angle y = 115°$
7. The size of the angles will always be the same since a perpendicular is formed in joining the centre of the circle and the midpoint of any chord. **8.** (a) Right angles are formed at *M*, *N*, *P*, *Q*, and *R*. (b) Yes
(c) Chord *CD* is bisected at *M*. (d) Points *N*, *P*, and *Q* are midpoints of their respective chords. (e) No
11. A parallelogram is formed. **12.** No. In parallelograms, angle bisectors meet at a common point.
13. (a) The angle bisectors meet at a common point. (b) The medians meet at a common point.
(c) The perpendicular bisectors meet at a common point.

EXERCISE 9·9, Page 233

1. $m\angle ABD = 25°$, $m\angle ABC = 65°$ **2.** $m\angle DOY = m\angle TOX = 120°$, $m\angle DOT = m\angle YOX = 60°$
3. $m\angle TOY = 100°$, $m\angle YOZ = 80°$ **4.** $m\angle ADC = 64°$, $m\angle ABC = 116°$
5. $m\angle SRT = 90°$, $m\angle RST = 20°$, $m\angle RTS = 70°$ **6.** $m\angle A = 30°$, $m\angle B = 105°$, $m\angle C = 45°$
7. $m\angle BOP = 64°$, $m\angle POT = 26°$ **8.** $m\angle a = m\angle c = 95°$, $m\angle b = m\angle e = 85°$

REVIEW EXERCISE 9, Pages 234-235

2. $m\angle a = 90°$, $m\angle b = 80°$, $m\angle c = 95°$ **4.** $m\angle KLM = 137°$ **5.** $m\angle RST = 57°$
6. $m\angle g = 52°$, $m\angle f = m\angle h = 128°$ **8.** m(side c) = 17 cm, m(side e) = 24 cm **9.** 7 m
12. The following pairs of triangles are congruent: (a), (b), and (c). **13.** (a) 140° (b) 40° (c) 150°

CHAPTER 10

EXERCISE 10·5, Pages 253-255

1. 10 mm × 10 mm, 100 mm^2 **2.** 100 cm × 100 cm, 10 000 cm^2
3. 1000 mm × 1000 mm, 1 000 000 mm^2 **4.** 1000 m × 1000 m, 1 000 000 m^2
5. (a) 7872 mm^2 (b) 1043.29 cm^2 (c) 0.3456 km^2 (d) 436.8 km^2
(e) 329.94 cm^2 (f) 44 988 mm^2 (g) 174.24 m^2 (h) 27.805 cm^2 (i) 69.3626 m^2
(j) 82 354.8 km^2 (k) 431.505 m^2 (l) 572.265 cm^2 **6.** (a) 78.72 cm^2 (f) 449.88 cm^2
(g) 1 742 400 cm^2 (k) 4 315 050 cm^2 **7.**(b) 0.104 329 m^2 (c) 345 600 m^2 (d) 4.368×10^8 m^2
(f) 0.044 988 m^2 (h) $0.002\,780\,5 \times 10^{-3}$ m^2 **8.** (a) 256 ha (b) 64ha (c) 128 ha
(d) 1 ha (e) 0.0464 ha (f) 4 ha (g) 648 ha (h) 0.045 ha (i) 2.7 ha (j) 0.24 ha
10. 1000 mm^3 **11.** 1 000 000 cm^3 **14.** (a) 584 250 mm^3 (b) 2688 cm^3 (c) 600 m^3
(d) 232 544 m^3 (e) 2.4948 m^3 (f) 720.0648 cm^3 (g) 162 033.15 m^3 (h) 48 412 cm^3
(i) 3590.82 cm^3 **15.** (a) 2 469 348.3 m^3 (b) 0.002 469 348 3 km^3

EXERCISE 10·6, Pages 258-259

1. (a) 250 N (b) 320 N (c) 520 N (d) 610 N (e) 813 N (f) 127 N (g) 4.5 N (h) 7.2 N (i) 15 N
(j) 17.50 N (k) 5 N (l) 7.50 N (m) 2.00 N (n) 1.50 N (o) 10.00 N (p) 15 N **2.** (a) 480 cm^3
(b) 1728 cm^3 (c) 3840 cm^3 (d) 9 m^3 (e) 2.688 m^3 (f) 27.608 m^3 **3.** (a) 480 g or 0.48 kg
(b) 1728 g or 1.728 kg (c) 3840 g or 3.840 kg (d) 9000 kg (e) 2688 kg (f) 27 608 kg
4. (a) 4.8 N (b) 17.28 N (c) 38.40 N (d) 90 000 N (e) 26 880 N (f) 276 080.N **5.** (a) 1.20 N
(b) 0.60 N (c) 4.80 N (d) 12.60 N (e) 60 000 N (f) 186 000 N **6.** 460 N **7.** 560 N
8. (a) 126.4 N (b) 83.2 N **9.** (a) 2085.6 N (b) 1372.8 N

EXERCISE 10·7, Pages 260-261

1. 20 000 Pa or 20 kPa **2.** 30 000 Pa or 30 kPa **3.** 10 000 Pa or 10 kPa
4. 10 000 Pa or 10 kPa **5.** 12 000 Pa or 12 kPa **6.** 400 Pa or 0.4 kPa
7. 1000 Pa or 1 kPa **8.** 2000 Pa or 2 kPa **9.** 800 Pa or 0.8 kPa
10. 3000 Pa or 3 kPa

EXERCISE 10·8, Pages 262-263

1. 12 ℓ/100 km, 8.33 km/ℓ **2.** 9.4 ℓ/100 km, 10.7 km/ℓ **3.** 4.2 ℓ/100 km, 24 km/ℓ
4. $17.50 **6.** $17.20 **7.** $21.89 **8.** $6.46 **9.** 265 mm
10. (a) 37 908 cm^3, 37.908 ℓ (b) 379.08 N (c) 260 Pa or 0.26 kPa
13. (a)(i) 420 mm by 297 mm (ii) 148 mm by 105 mm (iii) 105 mm by 74 mm
(b)(i) 62 370 mm^2 (ii) 31 080 mm^2 (iii) 15 540 mm^2 (iv) 7770 mm^2 (v) 124 740 mm^2 (c) 999 949 mm^2

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